Varieties of Compositionality — ***Emmonfest***

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According to the Principle of Compositionality, the meanings of complex expressions can be systematically derived from the meanings of their immediate parts. For more than a century, varieties of this principle have played a central role in logic and semantic theory. I will take a look at the role of compositionality within Frege-Carnap semantics.

1. *Compositionality*

[Model-theoretic semantics: no, thanks]

1.1 Compositionality Principles

General compositionality: about meaning

Generalized compositionality: about semantic values

1.2 *What is compositionality?*

Not: Top-down procedure for breaking up meanings into parts, answering:

```
  ✓
 / \
?   ?
```

But: [[non-lexical expression]] = [[complex linguistic form]]
=> no (unique) decomposition

<u>In (algebraic) fact</u>:
(General) decomposition is incompatible with (non-trivial) compositionality

Rather: Bottom-up procedure for determining meanings/values from parts, answering:

```
  ?
 / \
✓   ✓
```

2. *Extension and Intension*

QUESTION: What are extensions?
ANSWER: **Not** referents
But: contributions to reference

Example: *the German chancellor* has a referent, but *the* does not, though it helps *the German chancellor* referring: it contributes to reference. Extension of *the*: that which *the* contributes to the reference of expressions in which it occurs.

QUESTION: What are intensions?
ANSWER: **Not:** contributions to content (these would be Russellian donotations)
But: ways of specifying referents … if you're Frege
Or: extensions varying across logical space (functions) … for Carnap

Hence intensions are
- not (directly) obtained by compositionality
- systematically related to extensions

3. *Extensional Compositionality*

QUESTION: What are contributions?
ANSWER: *Compositional* contributions, … i.e., answers to:

$$\checkmark_{12}$$
$$/ \quad \backslash$$
$$?_1 \quad \checkmark_2$$

HENCE: Intensions presuppose extensions; extensions presuppose compositionality.

Canonical constructing (extensions as) contributions (assuming compositionality):

Step 1: Determine basic values (as starting points):
- referents, including multiple ones and/or truth values

Step 2: Subtraction by abstraction:
2a) Determine canonical environment
2b) Generalize (and idealize)

$$[[\checkmark_{12}]] = [[\boldsymbol{F}_1]]\,([[\checkmark_2]])$$
$$/ \quad \backslash$$
$$[[\boldsymbol{F}_1]] \quad [[\checkmark_2]]$$

LIMITATION: Construction presupposes *extensional* compositionality.

4. *Intensional vs. Fregean Compositionality*

Substitution arguments:
(S)

$$[[\checkmark_{12}]] \neq [[\checkmark_{13}]]$$
$$/ \quad \backslash$$
$$[[\boldsymbol{F}_1]] \quad [[\checkmark_2]] = [[\checkmark_3]]$$

Classical case: Attitude verbs are not truth-functional

Obvious solution: Global repair, i.e.,
Replace extensional compositionality by *intensional compositionality*.

(IC)

$$\begin{array}{c} [[\checkmark_{12}]]^\wedge \\ / \quad \backslash \\ [[?_1]]^\wedge \quad [[\checkmark_2]]^\wedge \end{array}$$

… and keep your fingers crossed that (S) won't recur.

Remark: Extensional compositionality won't be affected as long as extensions determine intensions (pointwise evaluation):

From: $[[\checkmark_{12}]] = [[\boldsymbol{F}_1]]\,([[\checkmark_2]])$

To: $[[\checkmark_{12}]]^\wedge(i) = [[\boldsymbol{F}_1]](i)\,([[\checkmark_2]]\,(i))$, for all i (from Logical Space),

Or: $[[\checkmark_{12}]]^\wedge = \lambda i.\ [[\boldsymbol{F}_1]]^i\,([[\checkmark_2]]^i)$

Less obvious solution: Local repair, i.e.,
Replace extensional compositionality by *Fregean compositionality*.

(FC)

$$\begin{array}{c} [[\checkmark_{12}]] = [[\boldsymbol{F}_1]]\,([[\checkmark_2]]^\wedge) \\ / \quad \backslash \\ [[\boldsymbol{F}_1]] \quad [[\checkmark_2]]^\wedge \end{array}$$

So the *extension* of the complex expression is determined by the intension of one of its parts, which thus contributes to the extension and is therefore the extension of the part (in Frege's paradoxical parlance).

Question: How do intensional and Fregean compositionality relate?
Answer: Fregean compositionality implies intensional compositionality:

IF $[[\alpha\ \beta]]^i = O_i([[\alpha]]^i, [[\beta]]^\wedge)$ [for all i]

THEN $[[\alpha\ \beta]]^\wedge = \lambda i.\ O_i([[\alpha]]^I, [[\beta]]^i)$

… but not vice-versa, with (artificial counterexamples of the form:

$[[\alpha\ \beta]]^i = A_i([[\alpha]]^i, [[\beta]]^\wedge)$ = if $i = i_0$

$[[\alpha\ \beta]]^i = B_i([[\alpha]]^i, [[\beta]]^\wedge)$ = if $i \neq i_0$

Fregean compositionality adds some uniformity to semantic composition – as in Shake&Bake Semantics.

5. *Fregean vs. Baroque Compositionality*

(BC)

$$\begin{array}{c} [[\checkmark_{12}]] = [[\boldsymbol{F}_1]]\,([[\checkmark_2]]^\wedge) \\ / \quad \backslash \\ [[\boldsymbol{F}_1]] \quad [[\checkmark_2]]^\wedge = [[\boldsymbol{F}_3]]\,([[\checkmark_4]]^{\wedge\wedge}) \\ / \ \backslash \\ [[\boldsymbol{F}_3]]^\wedge \quad [[\checkmark_2]]^{\wedge\wedge} \end{array}$$

Baroque compositionality in (Montague's Version of) Frege-Carnap Semantics

- *Notation*

${}^{\wedge 0}\alpha = \alpha$

${}^{\wedge n+1}\alpha = [{}^{\wedge\wedge n}\alpha]$

${}^{\vee 0}\alpha = \alpha$

${}^{\vee n+1}\alpha = [{}^{\vee\vee n}\alpha]$

$A^n(\alpha,\beta) = {}^{\wedge n}[{}^{\vee n}\alpha]([{}^{\vee n}\beta])$

- *Translation*

$|\,\mathit{Anna}\,|^n = {}^{\wedge n}\mathbf{a}$

$|\,\mathit{plays}\,|^n = {}^{\wedge n}\mathbf{P}$

$|\,\mathit{George\ believes\ that}\,|^n = {}^{\wedge n}\mathbf{\Gamma}$

$|\,\mathit{Name\ Pred}\,|^n = A^n(|\,\mathit{Pred}\,|^n, |\,\mathit{Name}\,|^n)$

$|\,\mathit{Op\ Sent}\,|^n = A^n(|\,\mathit{Op}\,|^n, |\,\mathit{Sent}\,|^{n+1})$

- *Example*

$|\,\mathit{Gbt\ Gbt\ Gbt\ Anna\ plays}\,|^0$

$= A^0(|\,\mathit{Gbt}\,|^0, A^1(|\,\mathit{Gbt}\,|^1, A^2(|\,\mathit{Gbt}\,|^2, A^3(|\,\mathit{plays}\,|^3, |\,\mathit{Anna}\,|^3))))$

$= A^0(\mathbf{\Gamma}, A^1([{}^{\wedge}\mathbf{\Gamma}], A^2([{}^{\wedge\wedge}\mathbf{\Gamma}], A^3([{}^{\wedge\wedge\wedge}\mathbf{P}], [{}^{\wedge\wedge\wedge}\mathbf{a}]))))$

$= A^0(\mathbf{\Gamma}, A^1([{}^{\wedge}\mathbf{\Gamma}], A^2([{}^{\wedge\wedge}\mathbf{\Gamma}], {}^{\wedge\wedge\wedge}[{}^{\vee\vee\vee\wedge\wedge\wedge}\mathbf{P}]({}^{\vee\vee\vee\wedge\wedge\wedge}\mathbf{a}))))$

$= A^0(\mathbf{\Gamma}, A^1([{}^{\wedge}\mathbf{\Gamma}], {}^{\wedge\wedge}[{}^{\vee\vee\wedge\wedge}\mathbf{\Gamma}]({}^{\vee\vee\wedge\wedge\wedge}[{}^{\vee\vee\vee\wedge\wedge\wedge}\mathbf{P}]({}^{\vee\vee\vee\wedge\wedge\wedge}\mathbf{a}))))$

$= A^0(\mathbf{\Gamma}, {}^{\wedge}[{}^{\vee\wedge}\mathbf{\Gamma}]({}^{\vee\wedge\wedge}[{}^{\vee\vee\wedge\wedge}\mathbf{\Gamma}]({}^{\vee\vee\wedge\wedge\wedge}[{}^{\vee\vee\vee\wedge\wedge\wedge}\mathbf{P}]({}^{\vee\vee\vee\wedge\wedge\wedge}\mathbf{a}))))$

$= \mathbf{\Gamma}({}^{\wedge}[{}^{\vee\wedge}\mathbf{\Gamma}]({}^{\vee\wedge\wedge}[{}^{\vee\vee\wedge\wedge}\mathbf{\Gamma}]({}^{\vee\vee\wedge\wedge\wedge}[{}^{\vee\vee\vee\wedge\wedge\wedge}\mathbf{P}]({}^{\vee\vee\vee\wedge\wedge\wedge}\mathbf{a}))))$

$\equiv \mathbf{\Gamma}({}^{\wedge}\mathbf{\Gamma}({}^{\wedge}\mathbf{\Gamma}({}^{\wedge}\mathbf{P}(\mathbf{a}))))$

- *Motivation?*

George believes that every national player lives in a 5* hotel. (Bäuerle 1983)

- Bäuerle constellation

$\lambda \boldsymbol{i}.\ \boldsymbol{F}(\boldsymbol{i})(\lambda \boldsymbol{j}.\ \boldsymbol{G}(\boldsymbol{j})(\lambda \boldsymbol{k}.\ \boldsymbol{H}(\boldsymbol{i},\boldsymbol{j},\boldsymbol{k})))$

- *Variants*

${}^{\wedge 3}\mathbf{P} = [{}^{\wedge\wedge\wedge}\mathbf{P}]$

$[\lambda \boldsymbol{i}.\ [\lambda \boldsymbol{j}.\ [\lambda \boldsymbol{k}.\ \mathbf{P}(\boldsymbol{k})]]]$

$\widetilde{{}^{\wedge}}{}^{3}\mathbf{P} = [{}^{\wedge}[\ {}^{\wedge}[\lambda \mathbf{X}^{et}.\ [{}^{\wedge}\boldsymbol{X}]]\ ({}^{\vee}\mathbf{P})]]$

$[\lambda \boldsymbol{i}.\ [\lambda \boldsymbol{j}.\ [\lambda \boldsymbol{k}.\ \mathbf{P}(\boldsymbol{j})]]]$

$A^0(\mathbf{\Gamma}, A^1([{}^{\wedge}\mathbf{\Gamma}], A^2([{}^{\wedge\wedge}\mathbf{\Gamma}], A^3([\widetilde{{}^{\wedge}}{}^{3}\mathbf{P}], [{}^{\wedge\wedge\wedge}\mathbf{a}]))))$