Semantic Values and Model-Theoretic ‘Semantics’

Thomas Ede Zimmermann
Goethe-Uni Frankfurt

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1. Model-theoretic semantics – anything wrong with it?
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2. Semantic values
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2. Semantic values
3. No reference: closure under arbitrary isomorphisms
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4. No content: Model Space vs. Logical Spaces
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3. No reference: closure under arbitrary isomorphisms
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5. Conclusion
1. Model-theoretic semantics – anything wrong with it?
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What we are talking about:
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What we are talking about: Model-theoretic semantics of natural language
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What we are talking about:
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What’s wrong with it:
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What’s wrong with it:

A model-theoretic account of a (given) natural language does not say what their expressions mean
1. Model-theoretic semantics – anything wrong with it?

What we are talking about:

4. Semantics: theory of reference

Let \( e, t, s \) be the respective numbers 0, 1, 2. (The precise choice of these objects is unimportant; the only requirements are that they [...] to objects of type \( \tau \) is in \( T \). In connection with any sets \( E \) and \( I \) and any \( \tau \in T \), we characterize \( D_{\tau, E, I} \), or the set of possible denotations of type \( \tau \) based on the set \( E \) of entities (or possible individuals) and the set \( I \) of possible worlds, as follows: \( D_{e, E, I} = E \); \( D_{t, E, I} = \{ A, \{ A \} \} \) (where \( A \) is as usual the empty set, and \( A, \{ A \} \) are identified with falsehood and truth respectively); if \( \sigma, \tau \in T \), then \( D_{\sigma, \tau, E, I} = D_{\tau, E, I} \cdot D_{\sigma, E, I} \) (where in general \( A^B \) is the set of functions with domain \( B \) and range included in \( A \)); if \( \tau \in T \), then \( D_{\{ \tau \}, E, I} = D_{\tau, E, I}^I \). If \( J \) is also a set, then \( M_{r, E, I, J} \), or the set of possible meaning [...] R> A type assignment for \( L \) is a function \( \sigma \) from \( A \) into \( T \) such that \( \sigma(\delta_0) = t \). A Fregean interpretation for \( L \) is an interpretation \( \langle B, G, f \rangle \) for \( L \) such that, for some nonempty sets \( E, I, J \), and some type assignment \( \sigma \) for \( L \), (1) \( B = \cup_{\tau \in T} M_{\tau, E, I, J} \), (2) whenever \( \delta \in \tau \) and \( \zeta \in X_\delta, f(\zeta) \in M_{\sigma(\theta), E, I, J} \), and (3) whenever \( \langle F, G \rangle, \langle \delta \zeta \rangle, \xi \beta, \zeta \in \beta, \xi \in S \) and \( b_\zeta \in M_{\sigma(\theta), E, I, J} \) for all \( \beta \), then \( G_\theta(\langle b_\zeta \rangle, \xi \beta, \zeta \in \beta) \in M_{\sigma(\theta), E, I, J} \). Here \( I \times J \) is uniquely determined and is called the set of points of reference of the Fregean interpretation. By a Fregean [...] Montague, Richard: 'Universal Grammar'. Theoria 36 (1970), 373–398; pp. 378–380
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What’s **wrong** with it:

A model-theoretic account of a (given) natural language does not say what their expressions mean.

In particular …
1. Model-theoretic semantics – anything wrong with it?

Compositional semantic theories specify the meanings of an expression by assigning a semantic value to it.
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Compositional semantic theories specify the meanings of an expression by assigning a **semantic value** to it.

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Model-theoretic accounts don’t assign (decent) semantic values.
2. Semantic values

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NAIVE answer:
Values are (or represent) meanings.
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**OBJECTION:**
Not clear whether all expressions have (independent) communicative functions.
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**THESIS**

**ANTI-THESIS**
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**EDUCATED** answer: Some values represent communicative functions, some don’t, depending on their **interpretability**.
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The semantic value \( \nu \) of an expression \( E \) is interpretable by a suitable interfacing theory \( T \) [epistemology, pragmatics, communication studies, …] iff \( \nu \) uniquely determines a rôle \( E \) plays according to \( T \).

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- The Frege-Carnap intension of a declarative sentence is its informational content.
- The GQ extension of *every* is likely to be uninterpretable.
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Q: What about uninterpretable values? What is their rôle?
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Q: What’s wrong with model-theoretic semantics again?
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Q: What’s wrong with model-theoretic semantics again?
A: It assigns *local* [= model-dependent] and *global* values [obtained by abstracting from models], but:
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A: It assigns *local* [= model-dependent] and *global* values [obtained by abstracting from models], but:

- the former are not unique
- the latter are (for the most part) uninterpretable
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BUT: No interpretable values, no interface
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... or less dramatically:
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A: It assigns *local* \([= \text{model-dependent}]\) and *global* values \([\text{obtained by abstracting from models}]\), but:
• the former are not unique
• the latter are (for the most part) uninterpretable
BUT: No interpretable values, no interface
... or less dramatically:
Uninterpretability may lead to serious restrictions in applying semantic theory.
3. No reference: closure under arbitrary isomorphisms
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In particular, despite the section heading
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4. Semantics: theory of reference

Let $e, t, s$ be the respective numbers $0, 1, 2$. (The precise choice of these objects is unimportant; the only requirements are that they
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In particular, despite the section heading global extensions do not determine reference because the class of all (admissible) models – **Model Space** for short – is closed under _arbitrary_ (model-) isomorphisms.

> A *type assignment* for \( L \) is a function \( \sigma \) from \( \Lambda \) into \( T \) such that \( \sigma(\delta) = t \). A *Fregean interpretation* for \( L \) is an interpretation \( \langle B, G_\gamma, f \rangle \gamma \in \Gamma \) for \( L \) such that, for some nonempty sets \( E, I, J \), and some type assignment \( \sigma \) for \( L \), (1) \( B \equiv \cup_{\gamma \in \Gamma} M_{\sigma(\delta), E, I, J} \), (2) whenever \( \delta \in \Lambda \) and \( \zeta \in X_\delta \), \( f(\zeta) \in M_{\sigma(\delta), E, I, J} \), and (3) whenever \( \langle F_\gamma, \langle \delta_\zeta \rangle \zeta < \beta, \varepsilon \rangle \in S \) and \( b_\zeta \in M_{\sigma(\delta_\zeta), E, I, J} \) for all \( \xi < \beta \), then \( G_\gamma(\langle b_\zeta \rangle \zeta < \beta) \in M_{\sigma(\delta), E, I, J} \). Here \( I \times J \) is uniquely determined and is called the set of _points of reference_ of the Fregean interpretation. By a *Fregean*
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This may have repercussions on the interface with syntax [= the theory of individuating expressions]
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Example
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Example

*Only John likes Mary*

∴ Bill doesn’t like Mary
3. No reference: closure under arbitrary isomorphisms

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Should this come out valid?
3. No reference: closure under arbitrary isomorphisms

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Example

Only John likes Mary
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Should this come out valid?
Maybe not: John and Bill could be the same person.
3. No reference: closure under arbitrary isomorphisms

This may have repercussions on the interface with syntax [= the theory of individuating expressions]

Example

\textit{Only John likes Mary}

\textbf{∴} \textit{Bill doesn’t like Mary}

Should this come out valid?
Maybe not: John and Bill could be the same person.
And indeed, it is safe to assume:
\[
\llbracket\text{John}\rrbracket_{M,i} = \llbracket\text{Bill}\rrbracket_{M,i}
\]
for at least some admissible models
3. No reference: closure under arbitrary isomorphisms

This may have repercussions on the interface with syntax [= the theory of individuating expressions]

Example

*Only John likes Mary*
\[\therefore \text{ Bill doesn’t like Mary}\]

However, if names $N$ are disambiguated by their bearers $b$ [as at least some semanticists have suggested], then the inference should be valid on the reading:
3. No reference: closure under arbitrary isomorphisms

This may have repercussions on the interface with syntax [= the theory of individuating expressions]

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However, if names $N$ are disambiguated by their bearers $b$ [as at least some semanticists have suggested], then the inference should be valid on the reading:

Only John_{Johnny} likes Mary
∴ Bill_{Billy} doesn’t like Mary
3. No reference: closure under arbitrary isomorphisms

This may have repercussions on the interface with syntax [= the theory of individuating expressions]

A straightforward disambiguation policy could take care of this:

- The referent of $N_x = x$.

\[ \text{Only John_{Johnny} likes Mary} \]
\[ \therefore \text{Bill_{Billy} doesn’t like Mary} \]
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This may have repercussions on the interface with syntax [= the theory of individuating expressions]

A straightforward disambiguation policy could take care of this:
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However, this strategy is inconsistent with model-theoretic interpretation, where the referent of a name cannot be determined from its global extension (and shifts with its local extensions).

Only John $\text{Johnny}$ likes Mary

∴ Bill $\text{Billy}$ doesn’t like Mary
4. No content: Model Space vs. Logical Spaces

Closure under arbitrary isomorphisms also leads to problems with cross-linguistic comparison (as hinted at in K&K’s intro):

Adapting a classical argument (by Heringer?) against structuralist phonology, it follows that no two languages can be distinguished if one results from the other by permuting (lexical) expressions of the same category (e.g., cat and mouse): the Model Spaces are the same!
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In fact, since model-theoretic semantics is essentially structuralist, it can only account for language-internal sense relations.
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...like *entailment*.
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...like entailment:

\[ \langle \text{thou art hungry}, \langle i, \langle \text{Smith, Jones} \rangle \rangle \rangle. \] The precise characterizations are the following. If \( \langle \varphi, p \rangle \) and \( \langle \psi, q \rangle \) are tokens in \( L \), then \( \langle \varphi, p \rangle \) \textit{K-entails} \( \langle \psi, q \rangle \) in \( L \) if and only if \( \varphi, \psi \in DS_L \) and, for every Fregean interpretation \( \mathcal{B} \) for \( L \), if \( \langle \mathcal{B}, p \rangle \) is in \( K \) and \( \varphi \) is a true sentence of \( L \) with respect to \( \langle \mathcal{B}, p \rangle \), then \( \langle \mathcal{B}, q \rangle \) is in \( K \) and \( \psi \) is a true sentence of \( L \) with respect to \( \langle \mathcal{B}, q \rangle \). If \( \varphi, \psi \in DS_L \), then the sentence \textit{type} \( \varphi \) \textit{K-entails} the sentence \textit{type} \( \psi \) in \( L \) if and only if \( \langle \varphi, p \rangle \) K-entails \( \langle \psi, p \rangle \) for every ordered pair \( p \). (It is clear

Montague (1970: 381f.)
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Note that there are two kinds of entailments given a class $K$ of admissible models:
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A sentence $S_1$ *locally entails* a sentence $S_2$ according to a model $M \in K$ iff $S_2$ is true [= has extension 1] at every point of reference (of $M$) at which $S_1$ is true:

- $(\forall i \in W_M) \left[ S_1 \right]^{M,i} = 1 \Rightarrow \left[ S_2 \right]^{M,i} = 1$
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A sentence $S_1$ **globally entails** a sentence $S_2$ iff $S_1$ locally entails $S_2$ according to every model $M \in K$:

- $\left( \forall M \in K \right) \left( \forall i \in W_M \right) [S_1]^{M,i} = 1 \Rightarrow [S_2]^{M,i} = 1$
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An expression $E_1$ is *locally synonymous with* an expression $E_2$ *according to* a model $M \in K$ iff, at every point of reference (of $M$), the extension of $E_1$ coincides with the extension of $E_2$:

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An expression $E_1$ is *locally synonymous with* an expression $E_2$ *according to* a model $M \in K$ iff, at every point of reference (of $M$), the extension of $E_1$ coincides with the extension of $E_2$:

- $(\forall i \in W_M) \ [E_1]^{M,i} = [E_2]^{M,i}$

An expression $E_1$ is *globally synonymous with* an expression $E_2$ iff $E_1$ is locally synonymous to $S_2$ according to every model $M \in K$:

- $(\forall M \in K)(\forall i \in W_M) \ [E_1]^{M,i} = [E_2]^{M,i}$
4. No content: Model Space vs. Logical Spaces
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...or:
4. No content: Model Space vs. Logical Spaces

...or:

An expression $E_1$ is *locally non-synonymous with* an expression $E_2$ according to a model $M \in K$ iff, at some point of reference (of $M$), the extension of $E_1$ does not coincide with the extension of $E_2$:

- $(\exists i \in W_M) \ [E_1]^{M,i} \neq [E_2]^{M,i}$
4. No content: Model Space vs. Logical Spaces

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An expression $E_1$ is \textit{locally non-synonymous with} an expression $E_2$ \textit{according to a model $M \in K$} iff, at some point of reference (of $M$), the extension of $E_1$ does not coincide with the extension of $E_2$:

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In general, a local sense relation $R$ is defined in terms of the set of all points of reference of a given model – its Logical Space – and the corresponding global relation $R^*$ holds iff $R$ holds according to every model.

Given the structuralist spirit of model-theoretic semantics, one would expect the global relations to be the ones that predict ‘observed’ sense relations. However, they don’t …
4. No content: Model Space vs. Logical Spaces
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The smallness of Model Space
4. No content: Model Space vs. Logical Spaces

The smallness of Model Space

If Model Space is large enough, it will block many desirable global sense relations. As a case in point, unless the relevant counter-examples are not declared inadmissible (e.g., by means of meaning postulates), the entailment between

Everyone is married

and

Nobody is a bachelor

does not come out.
4. No content: Model Space vs. Logical Spaces

The smallness of Model Space
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So Model Space has to be small enough for the global sense relations to come out right.
4. No content: Model Space vs. Logical Spaces

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Q: How small?
4. No content: Model Space vs. Logical Spaces

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So Model Space has to be small enough for the global sense relations to come out right.

Q: How small?
A [without argument]: Ideally so as to contain only one single ‘intended’ model and its isomorphic copies
4. No content: Model Space vs. Logical Spaces

The smallness of Model Space

So Model Space has to be small enough for the global sense relations to come out right.

Q: How small?
A [without argument]: Ideally so as to contain only one single ‘intended’ model and its isomorphic copies

… in which case local and global relations coincide.
4. No content: Model Space vs. Logical Spaces

The vastness of Logical Space(s)
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The vastness of Logical Space(s)
Despite the (intended) smallness of Model Space, Logical Space ought to be vast so as to allow for a maximum of variation among the possible combinations of extensions …
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… which is needed to get the global sense relations right: models with small Logical Spaces could be counter-examples to, say, the non-synonymy of John loves Mary and Bill loves Mary.
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The vastness of Logical Space(s)
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*The vastness of Logical Space(s)*

The elimination of small models [or 'degenerate' models, to use Mats Rooth's term] is largely a matter of formulating principles to account for the vastness of Logical Space.
4. No content: Model Space vs. Logical Spaces

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No – the theory of Logical Space is a metaphysical background theory. Logical Space should be assumed to be given in all its vastness, ready to be made use of whenever needed
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5. Conclusion
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A Farewell to Model Theory, then?
Of course not. But its place is outside semantics textbooks.
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There is in my opinion no important theoretical difference between natural languages and the artificial languages of logicians; indeed, I consider it possible to comprehend the syntax and semantics of both kinds of languages within a single natural and mathematically precise theory. On this point I differ from a

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Maybe, but then that theory is not model theory …
THANK YOU FOR YOUR ATTENTION