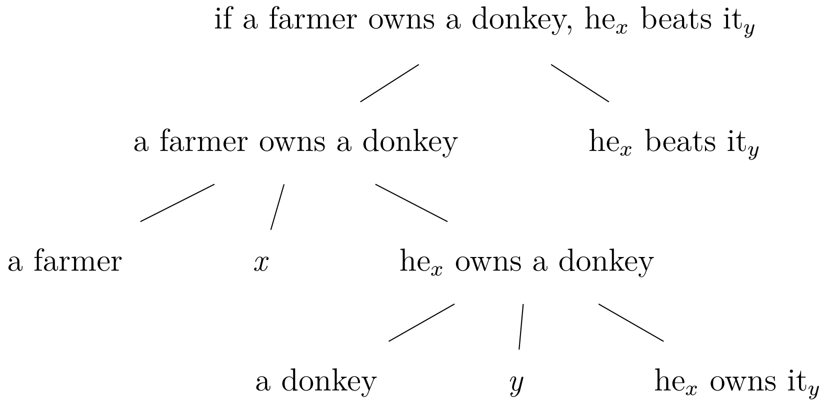
1 Donkey Sentences

1.1 Conditional Donkeys

(1) If a farmer owns a donkey, he beats it.

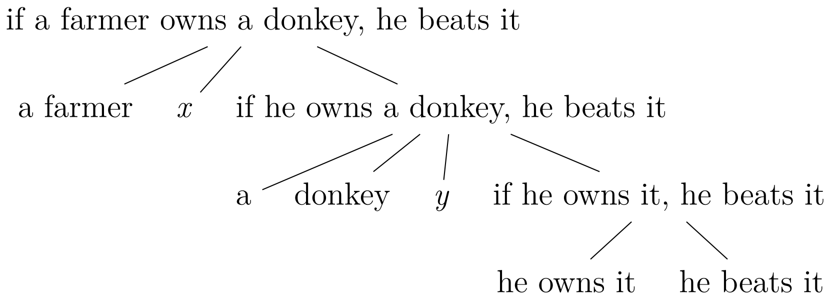
(2) $(\forall x)(\forall y)[[x$ is a farmer & y is a donkey & x owns $y] \rightarrow x$ beats y]

(3) a.



b. $(\exists x)(\exists y)$[farmer'(x) & donkey'(y) & own'(x,y)] $\Rightarrow$ beat'(**x,y**)
["x beats y, if a farmer owns a donkey."]

(4) a.



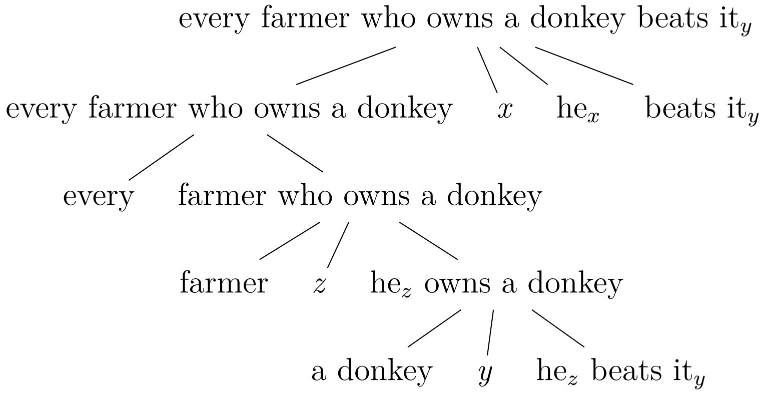
b. $(\exists x)(\exists y)$[farmer'(x) & donkey'(y) & (own'(x,y) $\Rightarrow$ beat'(x,y))]
["A certain farmer beats a certain donkey if he owns it."]

(5) ?If a farmer owns every/each donkey, he beats it.

1.2 Relative Donkeys

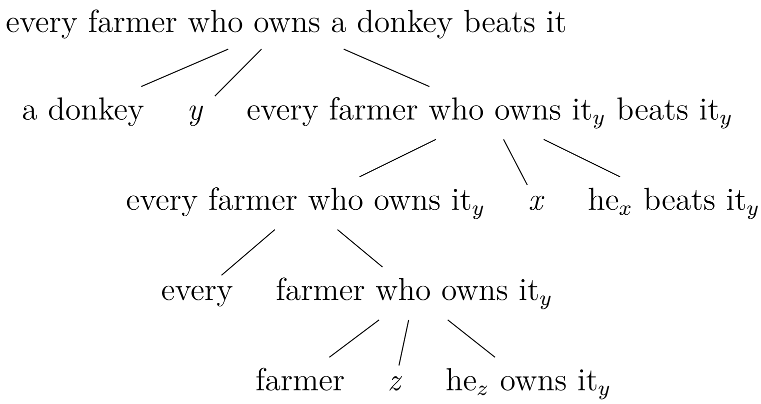
(6) Every farmer who owns a donkey beats it.

(7) a.



b. (∀x)([farmer'(x) & (∃y) [donkey'(y) & own'(x,y)]] → beat'(x, **y**))
['Every farmer who owns a donkey, beats *y*']

(8) a.



b. (∃y)[donkey'(y) & (∀x) ([farmer'(x) & own'(x,y)] → beat'(x,y))]
['Every farmer who owns a certain donkey, beats it.']

(9) ?Every farmer who owns every/each donkey beats it.

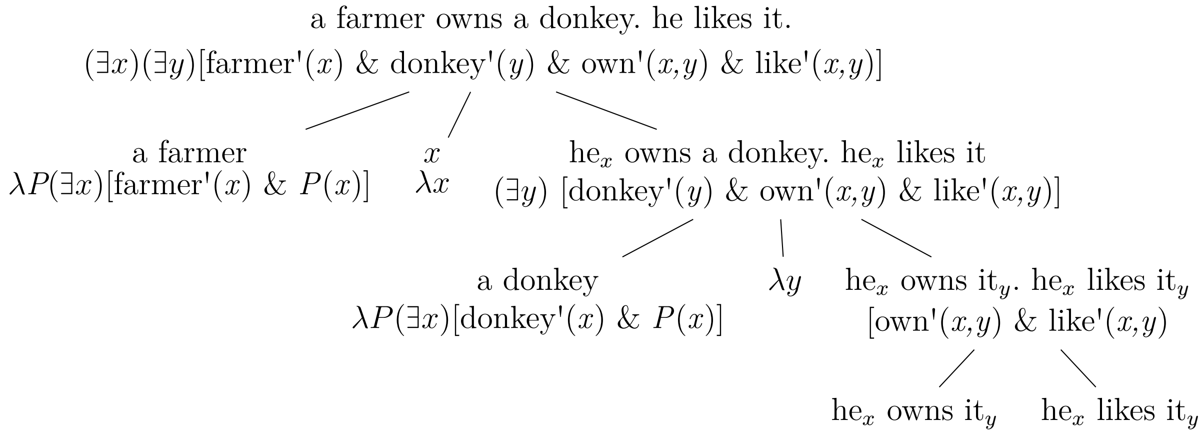
2 Discourse Anaphora

(10) A farmer owns a donkey. He likes it.

(11)

$$\underbrace{(\exists x)(\exists y)}_{?}\;[\underbrace{\text{farmer}'(x)\ \&\ \text{donkey}'(y)\ \&\ \text{own}'(x,y)}_{(\mathbf{a})}\ \&\ \underbrace{\text{like}'(x,y)}_{(\mathbf{b})}]$$

(12)



(13) [?]Every farmer owns a donkey. He likes it.

(14) The man who gave his paycheck to his wife was wiser than the one who gave it [∼ ***his*** paycheck] to his mistress.

(15) a. A boy owns a guinea-pig
$(\exists x)(\exists y)$[boy'(x) & guinea-pig'(y) & own'(x,y)]

b. He [∼ ***the*** boy who owns a guinea-pig] likes it [∼ ***the*** guinea-pig that he, the boy who owns a guinea-pig, owns].

like'((ιx)[boy'(x) & $(\exists y)$ [guinea-pig'(y) & own'(x,y)]],
(ιy)[guinea-pig'(y) & $(\exists x)$ [boy'(x) & own'(x,y)]]])

(16) a.

$$\text{A farmer} \left\{\begin{array}{l}\text{rides on a bicycle}\\ \text{cycles}\end{array}\right\}.$$

b.

$$\text{It} \left\{\begin{array}{l}\text{[the bicycle that the farmer who rides on a bicycle rides on]}\\ \text{[the bicycle that the cycling farmer rides on]}\end{array}\right\} \text{does}$$

$$\text{not belong to him} \left\{\begin{array}{l}\text{[the farmer who rides on a bicycle]}\\ \text{[the cycling farmer]}\end{array}\right\}.$$

(17) a. A farmer rides on a bicycle.
λR $(\exists x)(\exists y)$[farmer'(x) & bicycle'(y) & ride-on'(x,y) & $\underline{R(x,y)}$]

b. It does not belong to him.
$\hat{x}\hat{y}$ [¬ belong'(y,x) & *R(x,y)*]

(17′) a. $\hat{x}$ $\hat{y}$ [farmer'(x) & bicycle'(y) & ride-on'(x,y)]
b. $\hat{x}$ $\hat{y}$ ¬ belong'(y,x)
c. $\hat{x}$ $\hat{y}$ [farmer'(x) & bicycle'(y) & ride-on'(y,x) & ¬ belong'(y,x)]

(18) a. A farmer cycles.
($\exists x$)($\exists y$) [farmer'(x) & bicycle'(y) & ride-on'(x,y) & *R(x)*]
b. It does not belong to him.
$\hat{x}$ $\boldsymbol{\hat{y}}$ [¬ belong'($\boldsymbol{y}$,x) & *R(x)*]

(18′) a. $\hat{x}$ ($\exists y$) [farmer'(x) & bicycle'(y) & ride-on'(x,y)]
b. $\hat{x}$ $\boldsymbol{\hat{y}}$ ¬ belong'($\boldsymbol{y}$,x)
c. $\hat{x}$ $\boldsymbol{\hat{y}}$ [($\exists y$)] [farmer'((x) & bicycle'(y) & ride-on'(x,y) & ¬ belong'($\boldsymbol{y}$,x))]

(19) a. A man loves a woman. He kisses her.
b. A man loves a woman. A man kisses her.

3 Adverbial Quantification

(20)

$$\text{If a farmer owns a donkey, he} \left\{ \begin{array}{c} \text{always} \\ \text{sometimes} \\ \text{never} \\ \ldots \end{array} \right\} \text{beats it.}$$

(21)

$$\left\{ \begin{array}{c} (\forall x)(\forall y) \\ (\exists x)(\exists y) \\ \neg(\exists x)(\exists y) \\ \ldots \end{array} \right\} ([x \text{ is a farmer } \& \ y \text{ is a donkey } \& \ x \text{ owns } y] \left\{ \begin{array}{c} \rightarrow \\ \& \\ \& \\ \ldots \end{array} \right\} x \text{ beats } y)$$

(21′)

$$\left\{ \begin{array}{c} \forall xy \\ \exists xy \\ \neg\exists xy \\ \ldots \end{array} \right\} ([x \text{ is a farmer } \& \ y \text{ is a donkey } \& \ x \text{ owns } y] \left\{ \begin{array}{c} \rightarrow \\ \& \\ \& \\ \ldots \end{array} \right\} x \text{ beats } y)$$

(21″)

$$\left\{\begin{array}{c}\forall\\ \exists\\ \neg\exists\\ \ldots\end{array}\right\}(\hat{x}\;\hat{y}\;([x \text{ is a farmer } \&\ y \text{ is a donkey } \&\ x \text{ owns } y]\left\{\begin{array}{c}\rightarrow\\ \&\\ \&\\ \ldots\end{array}\right\} x \text{ beats } y))$$

(22) If a farmer owns a donkey, he usually beats it.

(23)

$$\left\{\begin{array}{c}\forall\\ \exists\\ \neg\exists\\ \text{MOST}\\ \ldots\end{array}\right\}(\hat{x}\;\hat{y}\;x \text{ is a farmer } \&\ y \text{ is a donkey } \&\ x \text{ owns } y,\ \hat{x}\;\hat{y}\;x \text{ beats } y)$$

(24)

$$\text{If a boy draws a picture of a girl, he}\left\{\begin{array}{c}\text{always}\\ \text{sometimes}\\ \text{never}\\ \text{usually}\\ \ldots\end{array}\right\}\text{gives it to her.}$$

(25) If φ(a N_1, …, a N_n), [then] ADV $\psi(\text{it}_1, \ldots, \text{it}_n) \mapsto$
$\text{ADV}'(\hat{x}_1, \ldots, \hat{x}_n\ [N_1'(x_1)\ \&\ \ldots\ \&\ N_n'(x_n)\ \&\ \varphi'(x_1, \ldots, x_n)],\ \hat{x}_1, \ldots, \hat{x}_n\ \psi'(x_1, \ldots, x_n))$

(26)

$$\text{If a farmer owns a donkey, he}\left\{\begin{array}{c}\text{always}\\ \text{sometimes}\\ \text{never}\\ \text{usually}\\ \ldots\end{array}\right\}\text{beats it.}$$

$$\underbrace{\left\{\begin{array}{c}\text{always}\\ \text{sometimes}\\ \text{never}\\ \text{usually}\\ \ldots\end{array}\right\}}_{\textsc{Quantifier}}\quad \underbrace{\text{a farmer}_x \text{ owns a donkey}_y}_{\textsc{Restrictor}}\quad \underbrace{\text{he}_x \text{ beats it}_y}_{\textsc{Scope}}$$

(27) a farmer$_x$ owns a donkey$_y$ $\mapsto$ [farmer′(x) & donkey′(y) & own′(x,y)]
a farmer$_x$ $\mapsto$ farmer′(x) (‘Indefinites as variables’)
a farmer owns a donkey $\mapsto$ [(farmer′ × donkey′) ∩ own′]
a farmer$_x$ $\mapsto$ farmer′(x)(‘Indefinites as properties’)

(28) every farmer who owns a donkey beats it

every — Quantifier
farmer$_x$ who owns a donkey$_y$ — Restrictor
he$_x$ beats it$_y$ — Scope

(29) If φ(a N_1, ..., a N_n), [then] ADV ψ (it$_1$,..., it$_n$) $\mapsto$
ADV’($\hat{x}_1$,..., $\hat{x}_n$ [x_1 is a N_1 & ...& x_n is a N_n & φ'(x_1, ..., x_n)],
$\hat{x}_1$,..., $\hat{x}_n$ ψ'(x_1, ..., x_n))

(30) a.
If a farmer own a donkey, he {always / sometimes / never / usually / ...} beats it <u>with a stick</u>.

b.
{Every / No / Most / ...} farmer[s] who own[s] a donkey beat it <u>with a stick</u>.

4 Asymmetries

(31) Most farmers who own a donkey beat it.

(29′) MOST($\hat{x}$ $\hat{y}$ farmer'(x) & donkey'(y) & own'(x,y), $\hat{x}$ $\hat{y}$ beat'(x,y))

(32) Every person who has a dime will put it in the meter.

(30′) ∀ ($\hat{x}$ $\hat{y}$ person'(x) & dime'(x) & have'(x,y), $\hat{x}$ $\hat{y}$ put-in-the-meter’(x,y))
[≡ ($\forall x$) ($\forall y$) ([person'(x) & dime'(x) & have'(x,y)] →
put-in-the-meter'(x,y))
≡
($\forall x$) ([person'(x) →
<u>($\forall y$) ([dime'(x) & have'(x, y)] →</u> put-in-the-meter'(x,y)])])

(30″) ($\forall x$)(($\exists y$) [person'(x) & dime'(x) & have'(x,y)] →
<u>($\exists y$) [person'(x) & dime'(x) & have'(x,y) &</u> put-in-the-meter'(x,y)])

(33) every$^n_{\textbf{universal}}$’ =
$\lambda R\ \lambda S\ (\forall x)\ (\forall y_2) \ldots (\forall y_n)\ [R(x, y_2, \ldots, y_n) \rightarrow S(x, y_2, \ldots, y_n)]$

(34) $\text{every}^n_{\textbf{existential}}$' =
$\lambda R\ \lambda S\ (\forall x)\ [(\exists y_2)\ \ldots (\exists y_n)\ R(x, y_2, \ldots, y_n) \rightarrow$
$(\exists y_2) \ldots (\exists y_n)[R(x, y_2, \ldots, y_n)\ \&\ S(x, y_2, \ldots, y_n)]]$

(35) Most persons who have a dime will put it in the meter.

(33′) MOST (x̂ ($\exists y$) [person'(x) & dime'(x) & have'(x,y)],
x̂ ($\exists y$) [person'(x) & dime'(x) & have'(x,y) & will-put-in-the-meter'(x,y))]

(36) EXISTENTIAL$^n(Q)$ =
$\lambda R\ \lambda S\ (Qx)((\exists y_2)\ldots(\exists y_n)\ R(x,y_2,\ \ldots,\ y_n),$
$(\exists y_2)\ldots(\exists y_n)\ [R(x,y_2,\ \ldots,\ y_n)\ \&\ S(x,y_1,\ldots,y_n)])$
$[= \lambda R\ \lambda S\ Q(\hat{x}\ (\exists y_2)\ \ldots(.\exists y_n)\ R(x,y_2,\ \ldots,\ y_n),$
$\hat{x}\ (\exists y_2)\ldots(\exists y_n)[R(x,y_2,\ \ldots,\ y_n)\ \&\ S(x,y_1,\ldots,y_n)]])]$

(37) Every farmer who owns a donkey, beats it.

(35′) $(\forall x)((\exists y)$[farmer'(x) & donkey'(x) & have'(x,y)] $\rightarrow$
$(\exists y)$[farmer'(x) & donkey'(x) & have'(x,y) & beat'(x,y)])

(38) Most people that owned a slave also owned his offspring.

(36′) MOST($\hat{x}\ \hat{y}$ [person'(x) & slave'(y) & own'(x,y)],
$\hat{x}\ \hat{y}$ [person'(x) & slave'(y) & own'(x,y) & own'(x,y's offspring)])

(39) MOST($\hat{x}$ $(\exists y)$ [person'(x) & slave'(y) & own'(x,y)],
$\hat{x}$ $(\forall y)$ [[person'(x) & slave'(y) & own'(x,y)] $\rightarrow$ own'(x,y's offspring)])

(40) UNIVERSAL$^n(Q)$ =
$\lambda R\ \lambda S\ (Qx)((\exists y_2)\ldots(\exists y_n)\ R(x,y_2,\ldots,y_n),$
$(\forall y_2)\ \ldots(\forall y_n)\ [R(x,y_2,\ldots,y_n) \rightarrow S(x,y_1,\ldots,y_n)])$

(41) If a farmer owns a donkey he is usually rich.

(39′) (MOST $\hat{x}\ \hat{y}$) ([farmer'(x) & donkey'(y) & own'(x,y)], rich'(x))

(39″) (MOST $\hat{x}$) ($(\exists y)$[farmer'(x) & donkey'(y) & own'(x,y)], rich'(x))
(=(MOST $\hat{x}$)($(\exists y)$[farmer'(x) & donkey'(y) & own'$(x,y$]),
$(\exists y)$ [farmer'(x) & donkey'(y) & own'(x,y) & rich'(x)])
=
(EXISTENTIAL^n(MOST) $\hat{x}$)([farmer'(x) & donkey'(y) & own'(x,y)], rich'(x)))

(42) If a DRUMMER lives in an apartment complex, it is usually half empty.

(40′) (MOST $\hat{x}$) ($(\exists y)$ [apartment-complex'(x) & drummer'(y) & live-in'(y,x)],
$(\exists y)$[apartment-complex'(x) & drummer'(y) & live-in'(y,x) & half-empty'(y)])
["The majority of tenement houses, where a drummer lives, is half empty."]

(43) If a drummer lives in an APARTMENT COMPLEX, it is usually half empty.

(41′) (MOST $\hat{y}$) ($(\exists x)$ [apartment-complex'(x) & drummer'(y) & live-in'(y,x)],
$(\exists y)$ [apartment-complex'(x) & drummer'(y) & live-in'(y,x) & half-empty'(y)])
["The majority of drummers, who live in rented houses, live in semi-empty ones."]

5 Uniqueness

(44) Every farmer who owns a donkey, beats it.

(42′) $(\forall x)$ ([farmer'(x) & $(\exists y)$ [donkey'(y) & own'(x,y)]],
beat'(x,($\underline{\iota y}$)[donkey'(y) & own'(x,y)]))
[“Every farmer slaps ***the*** donkey he owns.”]

(45) Every woman who bought a sage plant bought eight others along with it.

(46) No parent with a teenage son lends him the car.

(47) If a woman buys a sage plant here, she always buys eight others along with it.

(48) If a woman has a teenage son, she never lends him the car.

6 Discourse Representation

6.1 Relations

(47*a*) A farmer wanted to introduce a priest to a doctor. He told him that he already knew him.

(47*b*) $\hat{x}\ \hat{y}\ \hat{z}\ \mathbf{WI}(\boldsymbol{x,y,z})\ \&\ \mathbf{F}(\boldsymbol{x})\ \&\ \mathbf{P}(\boldsymbol{y})\ \&\ \mathbf{D}(\boldsymbol{z})$

(47*c*) $\hat{x}\ \hat{y}\ \hat{z}\ \mathbf{TK}\ (\boldsymbol{x,y,z})$

(47*d*) $[\hat{x}\ \hat{y}\ \hat{z}\ \mathbf{WI}(\boldsymbol{x,y,z})\ \&\ \mathbf{F}(\boldsymbol{x})\ \&\ \mathbf{P}(\boldsymbol{y})\ \mathbf{D}(\boldsymbol{z})\ ] \cap [\ \hat{x}\ \hat{y}\ \hat{z}\ \mathbf{TK}\ (\boldsymbol{x,y,z})\ ]$
$\equiv [\hat{x}\ \hat{y}\ \hat{z}\ \mathbf{WI}(\boldsymbol{x,y,z})\ \&\ \mathbf{F}(\boldsymbol{x})\ \&\ \mathbf{P}(\boldsymbol{y})\ \mathbf{D}(\boldsymbol{z})\ \&\ \mathbf{TK}\ (\boldsymbol{x,y,z})]$

(47*e*) $[\hat{x}\ \hat{y}\ \hat{z}\ \mathbf{WI}(\boldsymbol{x,y,z})\ \&\ \mathbf{F}(\boldsymbol{x})\ \&\ \mathbf{P}(\boldsymbol{y})\ \mathbf{D}(\boldsymbol{z})\ \&\ \mathbf{TK}\ (\boldsymbol{x,y,z})]$

(48*a*) $[\![\varphi]\!] \subseteq U^n$

(48b) $\underbrace{U \times \cdots \times U}_{n\ times}$

(48*c*) $B^A = \{f \subseteq (A \times B) \mid f : A \to B\}$

(48*d*) $n = \{m \in \omega \mid m < n\} = \{0, \ldots, n-1\}$

(48*e*) $\{f \subseteq (n \times U) \mid f : n \to U\}$

(48*f*) $U^n \cong \{f \subseteq (X \times U) \mid f : X \to U\} \Leftrightarrow \mid X \mid = n$

6.2 Representations

Basic symbols:

Predicates with -arities: $P, Q, R \ldots$
Variables (discourse markers)
Quantifiers: $\forall, \mathbf{MOST}, \mathbf{NO}$
Auxiliary symbols: **(**, **)**, **[**, **]**, **,**, **:**

Categories:
Main category: *DRS*
Others: *Pred[n], Var, Cond, Quant*

Formation rules

(i) $X \subseteq Var, \Phi \subseteq Cond \Rightarrow [X\textbf{:}\ \Phi] \in DRS$

(ii) $\mathbf{R} \in Pred[n], \mathbf{x}_1, \ldots, \mathbf{x}_n \in Var \Rightarrow \mathbf{R}(\mathbf{x}_1, \ldots, \mathbf{x}_n) \in Cond$

(iii) $\mathbf{Q} \in Quant, K, K' \in DRS \Rightarrow [K\mathbf{<Q>}\ K'] \in Cond$

Interpretation:

Model $\mathcal{M} = (U_\mathcal{M}, F_\mathcal{M})$, where: $U_\mathcal{M} \neq \varnothing, F_\mathcal{M} : \text{Pred}[n] \to \wp(U_{\mathcal{M}^n})$
Assignment g for model $\mathcal{M}$: Var $\rightharpoonup U_\mathcal{M}$

Extension $[\![\text{A}]\!]^{\mathcal{M},g}$ of A relative to $\mathcal{M}$ and g:

(i') $[\![\ [\text{X} : \Phi]\ ]\!]^{\mathcal{M},g} = \{f :\ X \to U_\mathcal{M} \mid (\forall\varphi \in \Phi)\ [\![\varphi]\!]^{\mathcal{M},g\cup f} = 1$

(i^*) $dom(g) \cap X\ =\ \varnothing$

(ii') $[\![\mathbf{R}(\mathbf{x}_1, \ldots, \mathbf{x}_n)]\!]^{\mathcal{M},g} = \begin{cases} 1 \text{ if } (g(\mathbf{x}_1), \ldots, g(\mathbf{x}_n)) \in F_\mathcal{M}(\mathbf{R}) \\ 0 \text{ if } (g(\mathbf{x}_1), \ldots, g(\mathbf{x}_n)) \notin F_\mathcal{M}(\mathbf{R}) \end{cases}$

(ii^*) $\{\mathbf{x}_1, \ldots, \mathbf{x}_n\} \subseteq dom(g)$

$(iiia)$ $[\![[K\mathbf{<}\forall\mathbf{>}\ K']]\!]^{\mathcal{M},g} = 1 \Leftrightarrow (\forall f \in [\![K]\!]^{\mathcal{M},g})(\exists h) h \in [\![K']\!]^{\mathcal{M},g\cup f}$

$(iiib)$ $[\![[K\mathbf{<NO>}\ K']]\!]^{\mathcal{M},g} = 1 \Leftrightarrow \neg(\exists f \in [\![\text{K}]\!]^{\mathcal{M},g})\ (\exists h) h \in [\![K']\!]^{\mathcal{M},g\cup f}$

$(iiic)$ $[\![[K\mathbf{<MOST>}\ K']]\!]^{\mathcal{M},g} = 1 \Leftrightarrow$
$|\ \{f \in [\![\text{K}]\!]^{\mathcal{M},g} : (\exists h) h \in [\![K']\!]^{\mathcal{M},g\cup f}\}\ | > |\ \{f \in [\![\text{K}]\!]^{\mathcal{M},g} : \neg(\exists h) h \in [\![K']\!]^{\mathcal{M},g\cup f}\}\ |$

(iii^*) $(\forall f \in [\![K]\!]^{\mathcal{M},g})\ [\![K']\!]^{\mathcal{M},g\cup f}$ def. & $[\![K]\!]^{\mathcal{M},g}$ def.

Asymmetric quantification

$F_\mathcal{M} : Quant \to \wp(\wp(\text{U}_\mathcal{M}) \times \wp(\text{U}_\mathcal{M}))$

(iv) $\mathbf{Q} \in Quant, \mathbf{x} \in Var, K, K' \in DRS \Rightarrow [K\mathbf{<Qx>}\ K'] \in Cond$

(iv') $[\![\ [K\mathbf{<Qx>}\ K']]\!]^{\mathcal{M},g} = 1 \Leftrightarrow$

$(\{u \in U_{\mathcal{M}} : (\exists f) f \cup \{(\mathrm{x}, u)\} \in [\![K]\!]^{\mathcal{M},g}\},$
$\{\mathrm{u} \in U_{\mathcal{M}} \mid (\exists f, h)[f \cup \{(\mathrm{x}, u)\} \in [\![K]\!]^{\mathcal{M},g} \& (\exists h) h \in [\![K']\!]^{\mathcal{M}, f \cup g \cup \{(\mathrm{x},u)\}}]\})$
$\in F_{\mathcal{M}}(\mathbf{Q})$
$\Leftrightarrow (\{\mathrm{u} \in U_{\mathcal{M}} : (\exists f \in [\![K]\!]^{\mathcal{M},g}\}) f(\mathrm{x}) = u\},$
$\{\mathrm{u} \in U_{\mathcal{M}} \mid (\exists f \in [\![K]\!]^{\mathcal{M},g}\})[f(\mathrm{x}) = u \ \& [\![K']\!]^{\mathcal{M}, f \cup g\}} \neq \varnothing]\})$
$\in F_{\mathcal{M}}(\mathbf{Q})$

(iv'') $[\![[K < \boldsymbol{Qx} > K']]\!]^{\mathcal{M},g} = 1 \Leftrightarrow$
$(\{u \in U_{\mathcal{M}} : (\exists f) f \cup \{(\mathrm{x}, u)\} \in [\![K]\!]^{\mathcal{M},g}\},$
$\{u \in U_{\mathcal{M}} : (\forall f)[f \cup \{(\mathrm{x}, u)\} \in [\![K]\!]^{\mathcal{M},g} \Rightarrow (\exists h) h \in [\![K']\!]^{\mathcal{M}, f \cup g \cup \{(\mathrm{x},u)\}}]\})$
$\in F_{\mathcal{M}}(\mathbf{Q})$
$\Leftrightarrow (\{u \in U_{\mathcal{M}} : (\exists f \in [\![K]\!]^{\mathcal{M},g}\}) f(\mathrm{x}) = u)\},$
$\{u \in U_{\mathcal{M}} : (\forall f \in [\![K]\!]^{\mathcal{M},g}\})[f(\mathrm{x}) = u \Rightarrow [\![K']\!]^{\mathcal{M}, f \cup g} \neq \varnothing]\})$
$\in F_{\mathcal{M}}(\mathbf{Q})$

(iv^*) Like (iii^*)

Truth Values

(49) $|(44)| = [\varnothing| \ [\{x, y\}|\{F(x), D(y), O(x, y)\}] < \forall x > [\varnothing| \ \{B(x, y)\}] \ ]$

(50) $[\![[\{x, y\}|\{F(x), D(y), O(x, y)\}] < \forall x > [\varnothing| \ \{B(x, y)\}]]\!]^{\mathcal{M},g}$
$= [\![[\{x, y\}|\{F(x), D(y), O(x, y)\}] < \forall x > [\varnothing| \ \{B(x, y)\}]]\!]^{\mathcal{M}}$
$= 1 \Leftrightarrow (F_{\mathcal{M}}(F) \times F_{\mathcal{M}}(D)) \cap F_{\mathcal{M}}(O) \subseteq F_{\mathcal{M}}(B)$

(51) $[\![[\varnothing| \ [\{x, y\}|\{F(x), D(y), O(x, y)\}] < \forall x > [\varnothing| \ \{B(x, y)\}] \ ]]\!]^{\mathcal{M},g}$
$= \{f : \varnothing \to U_{\mathcal{M}} | [\![[\{x, y\}|\{F(x), D(y), O(x, y)\}] < \forall x > [\varnothing| \ \{B(x, y)\}]]\!]^{\mathcal{M}, g \cup f} = 1\}$
$= \{f : \varnothing \to U_{\mathcal{M}} | (F_{\mathcal{M}}(F) \times F_{\mathcal{M}}(D)) \cap F_{\mathcal{M}}(O) \subseteq F_{\mathcal{M}}(B)\}$
$\Leftrightarrow \Phi$

HENCE:

(52) a. IF $\mathrm{A}^{\varnothing} = \varnothing$ (for any A): ↯
$[\![|(44)|]\!]^{\mathcal{M},g} = \varnothing$
b. IF $\mathrm{A}^{\varnothing} = \{\varnothing\}$ (for any A): CARNAPIAN TRUTH VALUES
$[\![|(44)|]\!]^{\mathcal{M},g} = \{\varnothing\}$ iff Φ, and $\ldots = \varnothing$ otherwise
c. IF $\mathrm{A}^{\varnothing} = A$ (for any A):: 'ALGEBRAIC' TRUTH VALUES
$[\![|(44)|]\!]^{\mathcal{M},g} = U_{\mathcal{M}}$ iff Φ, and $\ldots = \varnothing$ otherwise
d. etc.

Anaphors

(v) $\mathbf{x}, \mathbf{y} \in Var \Rightarrow \mathbf{(x = y)} \in Cond$

(v') $[\![\ \mathbf{(x = y)} \ ]\!]^{\mathcal{M},g} = 1 \Leftrightarrow g(\mathbf{x}) = g(\mathbf{y})$