On *PTQ*

In a series of papers published between 1970 and 1973, the late Richard Montague developed a mathematical theory of semantics and its interface with syntax. The theory, later to be called *Montague Grammar* (or *Montague Semantics*), was brought to the attention of a larger linguistic community only after its originator's death, and chiefly through Barbara Partee's (1975). Montague Grammar was not only intended as an alternative to contemporary generative grammar but also designed so broadly as to cover both natural language and systems of formal logic. An outline of the theory was contained in the short and somewhat hermetic essay 'Universal Grammar' (= Montague 1970b). However, it was the slightly more accessible descriptive application in Montague's last publication that was to define the vantage ground of semantic research for at least the next decade: 'The Proper Treatment of Quantification in Ordinary English', *vulgo* (and henceforth) *PTQ*.[1]

The following survey can only provide a rather superficial glimpse of *PTQ* and the theory behind it without doing full justice to its linguistic, philosophical, and logical depth and breadth. For more on the topic readers should consult a decent Montague Grammar text (e.g., Link 1979, Dowty *et al.* 1981, or Partee 1997) or, in case they are not easily scared off by mathematical jargon and density, Montague's original papers.

PTQ was presented as both an illustration of the power of the general theory expounded in Montague (1970b) and as a contribution to the semantic description of English, thereby extending the range of phenomena addressed in Montague's earlier papers, Montague (1970a) and (1970b). So there is both a general theoretical and a more specific descriptive aspect to *PTQ*. For ease of presentation, the following two sections will tease these two aspects apart – sometimes artificially, and not always quite in line with Montague's original views. As we go along, later developments in formal semantics will be addressed that were arguably provoked by inadequacies in the theoretical apparatus and descriptive details of *PTQ*.

1. Theoretical aspects: the framework

In a nutshell, Montague Grammar may be characterized as an (v) *indirect*, (iii) *model-theoretic* approach to the (i) *compositional* behavior of (ii) *extensions and intensions* in a hierarchy of (iv) *functional types*. The following five sub-sections will address these theoretical aspects in the order indicated.

1.1 Compositionality: the algebra of constructions

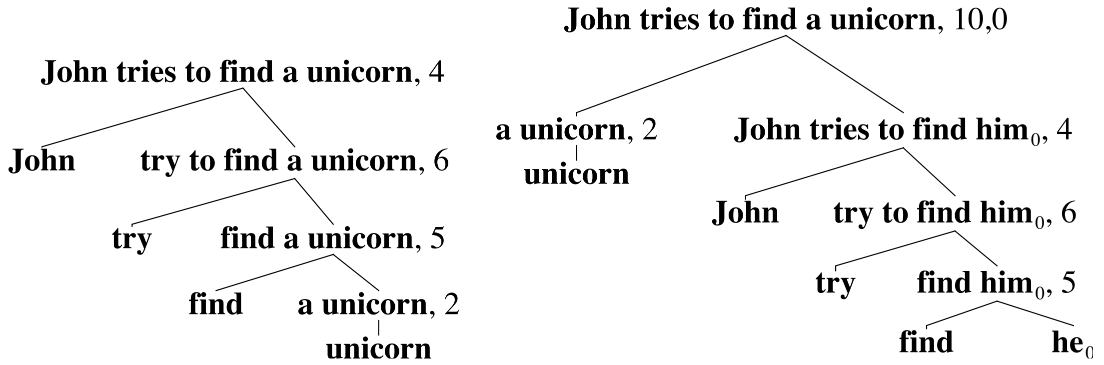
Montague's work is generally seen as marking the beginning of modern *compositional* semantics.[2] In fact, his algebraic characterization of the interface between syntax and semantics basically boils down to the tenet that the meanings of non-lexical expressions can be obtained by combining the meanings of their immediate parts. In order to achieve this, Montague assumed that, underlying any constituent, there is a *disambiguated expression* from which its immediate parts and the way they have been combined can be determined; and it is this underlying disambiguated expression that eventually receives a compositional interpretation. In this respect, Montague's disambiguated expressions are like May's (1977) *LF*s, especially as featured in Heim & Kratzer (1998).

[1] See Partee (2011) and Caponigro (to appear) for the historical and biographical background of Montague Grammar.

[2] The term 'compositionality' only seems to have gained currency in the 1980s though; Thomason (1980) might be the earliest source.

In *PTQ* certain tree graphs were introduced that specify both the (semantically relevant) part-whole structure and the ways in which the (immediate) parts are combined – roughly corresponding to (semantically relevant) grammatical constructions. Thus, the following two trees (redrawn from *PTQ*, p. 228), are disambiguated expressions underlying the surface string **John tries to find a unicorn**:

(1) a. b.



In (1), the numbers n following the boldface surface strings of complex (disambiguated) constituents refer to the pertinent syntactic operation (or construction), called F_n. Thus, F_2 is the operation of pre-posing the appropriate form of the indefinite article – i.e., **a** or **an** – to (the surface string of) a disambiguated expression and it is regulated by a rule S2 to the effect that applying F_2 to a (not necessarily lexical) noun results in a determiner phrase. F_2 is unary: the article does not count as a lexical expression but as 'logical material' that marks the operation of forming an indefinite from a noun. Analogous constructions F_0 and F_1 are used to form DPs of the forms '**every** *N*' and '**the** *N*'. We will get to the semantic effect of these constructions in due course, but already at this point one difference between the Montagovian approach and type-driven frameworks along the lines of Klein & Sag (1985) or Heim & Kratzer (1998) becomes apparent: in disambiguated expressions, unary branching does not always represent a meaning-preserving operation.

The other constructions in (1) are all binary. F_4 brings together subject and predicate to form a sentence. F_5 combines a transitive verb with (the accusative form of) its direct object to construct a verb phrase. F_6 attaches an infinitival object to a control verb, again to produce a verb phrase. Finally, $F_{10,n}$ is the predecessor of May's (1977) Quantifier Raising, where n is the subscript of the trace (which in *PTQ* surfaces as an indexed pronoun); by $F_{10,n}$, a DP is woven into a sentence to form another sentence, in which the DP takes widest scope. The morpho-syntactic details of these binary operations are formulated in a somewhat *ad hoc* manner – which is why they are not worth going into here.[3]

Semantic interpretation in Montague Grammar proceeds by moving up trees like those in (1) that represent derivations by the operations mentioned at their nodes: the way the meanings of the immediate constituents of an expression are combined is determined by the syntactic operation that combines them. In particular, the same expressions may stand in different grammatical constructions that differ in their meanings. As a case in point, in addition to the simple subject-predicate construction F_4, *PTQ* has five operations F_{11} – F_{15}, which introduce negation, non-present tense (future or past), or both.

Generally speaking, to any syntactic operation F there corresponds some semantic operation G on meanings so that the result of combining any disambiguated expressions

[3] Montague (1970b: 388ff.) offers a more principled approach to subject-verb agreement by parallel recursion. See von Stechow (1979) for a general account of feature matching in German along those lines.

$\alpha_1,\ldots,\alpha_n$ by F gets assigned the meaning that results from combining the meanings of $\alpha_1,\ldots,\alpha_n$ by G:

(2) $[\![F(\alpha_1,\ldots,\alpha_n)]\!] = G([\![\alpha_1]\!],\ldots,[\![\alpha_n]\!])$

In mathematical parlance, (2) says that meaning (designated by the 'semantic brackets') is a *homomorphism*. So Montague's (by now standard) criterion of compositionality is that (2) holds for all constructions F operating on underlying disambiguated expressions. These remarks should suffice to give an impression of the general features of the compositional meaning assignment. The following sections will illustrate how the algebraic formulation (2) of the principle of compositionality is specified in *PTQ* in order to give an account of the meanings of disambiguated English expressions.[4]

1.2 Frege's functionality, California style: extensions and intensions

The general approach to meaning pursued in *PTQ* can be traced back to the core piece of Montague's *Universal Grammar*: the 'Theory of Reference' (Montague 1970b: 378ff.). Although presented as a development of Frege's (1892) logical semantics, the roles of sense and reference are played by (generalized versions of) Carnap's (1947) *extensions* and *intensions*. In particular, the intension of an expression α is a function (in the set-theoretic sense) that assigns to any point of evaluation α's extension at that point, where a point of evaluation is an ordered pair of a possible world and a time.[5] As a consequence, Montague's intensions suffer from a certain lack of fine-grainedness vis-a-vis Frege's realm of senses. However, the specific way in which extensions and intensions interact does give the approach a distinctly Fregean flavor: not the meanings, but the extensions of complex expressions are the outputs of compositional operations; and as a rule, the extensions of their parts form their inputs, though at times their intensions may do so. As a case in point, the conjunctive coordination S_1 *and* S_2 of two declarative sentences can be described by a truth-table which combines the extensions, i.e. the truth values, of the two immediate parts, S_1 and S_2 – or, as Frege (1892) would have put it: their ordinary referents ('gewöhnliche Bedeutungen'). On the other hand, in order to determine the truth value of a causal connection S_1 *because* S_2, the informational contents of the two parts need to be taken into account, and these contents are (arguably) provided by their intensions – which thus play the role of indirect referents ('ungerade Bedeutungen') that Frege assigned to their senses. Hence, compositionality is, first and foremost, a matter of extensions, but intensions come to the rescue when everything else fails. In general, then, two fundamentally different ways of compositionally obtaining extensions need to be distinguished: in *extensional* (or direct) environments, extensions are combined, in *intensional* (or indirect) constructions, intensions are employed to obtain the extensions of larger expressions. It is this interplay of the two basic semantic values that Montague took to be central to his semantic theory and that he called 'Frege's functionality principle'.[6]

---

[4] Actually, *PTQ* deviates from the general framework of Montague (1970b) to some extent. In particular, the construction-driven character of compositionality is at times traded for a *rule-driven* account, which comes close to Janssen's (1983: 41ff.) approach in terms of sorted algebras; the interpretation of quantifying determiner phrases and the scope-taking mechanisms are cases in point. We will gloss over these details here.

[5] There is no specific term for such pairs in *PTQ*; I am indebted to Zoltán Szabó for suggesting *point of evaluation*.

[6] See Montague (1970c: 75f.), where the principle is characterized with reference to certain formal languages: 'the extension of a formula is a function of the extensions […] of those of its parts not standing within indirect contexts […], together with the intensions […] of those parts that do stand within indirect contexts.' A few paragraphs earlier (*ibid.*, p. 75), Montague makes the connection between extension and Frege's [*gewöhnliche*]

Although extensions are the results of the compositional operations used in semantic analysis, they are not themselves the meanings that the principle of compositionality (2) is about: after all, the extension of a sentence is its truth value, and while speakers ideally know the meanings of all sentences, they can hardly be expected to know of all of them whether they are true, and certainly not solely on the basis of their linguistic competence. Fortunately, though, there is a direct route from extensional compositionality to the compositionality of intensions: if the extension of a complex expression can be determined from the extensions of its parts, then so can its intension from the intensions of its parts. Again, conjunction is a case in point: the fact that the extension of S_1 *and* S_2 can be calculated by a truth table implies that its intension can be obtained by a Boolean operation on propositions (construed as sets of points of evaluation): intersection in the case at hand (or, rather, the corresponding operation on characteristic functions). More generally, writing '$[\![\alpha]\!]^{w,t}$' for the extension of the expression α at point <*w*,*t*>, the extensional compositionality (3) of a construction F (at arbitrary points of evaluation) in terms of some operation H, is easily seen to imply the existence of a corresponding meaning combination G satisfying (2) above – provided that meanings are identified with intensions:

(3) $[\![F(\alpha_1, \ldots, \alpha_n)]\!]^{w,t} = H([\![\alpha_1]\!]^{w,t}, \ldots, [\![\alpha_n]\!]^{w,t})$

For on the basis of (3), G can be defined in a pointwise fashion: the intension G assigns to the intensions $[\![\alpha_1]\!], \ldots, [\![\alpha_n]\!]$ is the function that takes each point <*w*,*t*> to the corresponding H-value:

(4) $G([\![\alpha_1]\!], \ldots, [\![\alpha_n]\!])(< w, t >) = H([\![\alpha_1]\!]^{w,t}, \ldots, [\![\alpha_n]\!]^{w,t})$

Since each extension $[\![\alpha_i]\!]^{w,t}$ is merely the value of $[\![\alpha_i]\!]$ at <*w*,*t*>, i.e., $[\![\alpha_i]\!]$(<*w*,*t*>), (4) is indeed a pointwise definition of G in terms of the intensions of the parts $\alpha_1, \ldots, \alpha_n$. And *a fortiori*, a similar recipe can be given for intensional constructions, where the extensions already depend on the intensions of (some of) the parts. Hence, the (allegedly Fregean) strategy of obtaining extensions from the extensions or intensions of the parts of expressions guarantees the compositionality of meanings – at least to the extent that the meanings of expressions coincide with their intensions.

But do meanings and intensions coincide? In *PTQ*, Montague seems to have given a positive answer to this question, since his semantic account of English largely consists in a recursive characterization of the intensions of expressions. Hence, the *PTQ* framework has rightly been seen as being in the spirit of what Putnam (1975: 133ff.) called 'California semantics': the meanings speakers grasp as part of their mastery of a language are identified with intensions, which in turn enable them to identify corresponding extensions on the basis of further factual knowledge. However, two qualifications to the identification of meaning and intension should be made, even according to Montague:

- The intensions of expressions containing deictic material depend on the utterance context.
- The intensions of formulae with free variables depend on a variable assignment.

In view of these two exceptions, Montague (1970b) actually identified meanings with functions from utterance contexts and (or, rather: including) variable assignments to intensions. However, since *PTQ* features no deictic expressions, the context dependence of intensions has been suppressed throughout. On the other hand, variables do play a role in the interpretation of quantifier scope constructions like $F_{10,0}$ in (1) above; and these variable-binding operations are known to not conform to extensional or Fregean compositionality. For ease of presentation, however, we will follow semantic mainstream

Bedeutung (which he translates as *[ordinary] extension*), and also mentions that Frege did not intend his functionality principle to apply to variable binding.

and ignore the compositionality complications arising from variable binding, for which the interested reader is referred to Chapter 10 of Zimmermann & Sternefeld (2013).

In *PTQ*, then, even though intensional constructions are in focus (and variable binding occurs), it is extensions that play first fiddle: meaning composition is (largely) described in terms of operations deriving the extensions of complex expressions from the extensions, and sometimes the intensions, of their parts. The format in which these extensions and intensions are defined will be addressed in Section 1.5.

1.3 A touch of structuralism: model theory

Whether or not they are identified with intensions, Montagovian meanings ultimately serve the purpose of determining referents and truth values, thereby connecting words and worlds. However, the general theory behind *PTQ* is set up in such a way that any semantic account of any (formal or natural) language is bound to leave out the details of this connection: not the meanings of the expressions under scrutiny but only some of their structural features and relations are specified. This surprising omission is a design feature of the model-theoretic framework, according to which the interpretation of a language is not a mapping from expressions to their meanings but a family of mappings from expressions to objects that share (part of) their set-theoretic structure but may hugely disagree as to what the objects and points of evaluation are. These families of mappings to meaning-like objects are called *models*, and in *PTQ* [p. 230] they come in the form of quintuples $<A,I,J,\leq,F>$ whose components are sets representing objects that play the roles of: potential referents of individual terms (A); possible worlds (I); moments of time (J); the temporal order ($\leq$); and the lexical meaning assignment (F), whose extrapolation to complex expressions will be addressed in Section 1.5 below. All of these components are arbitrary sets, which means that they do not have to contain any physical objects, worlds, or times. Yet while none of the A-components in a model M of the above form may contain wolves or sheep, the roles of these animals (at certain points of evaluation) will be played by the elements of the extensions determined by F(**wolf**) and F(**sheep**); yet in another model of the same structure at least some of the wolves according to F play the roles of sheep. And both of these models count as equally legitimate representations of the form-meaning mapping in English. In terms of classic structuralism, it is only (set-theoretic) *structure* that counts in model-theoretic interpretation, not the (set-theoretic) *material* that displays it.[7] On the other hand, the model-theoretic interpretation of English in *PTQ* does not fully specify the structure of the meaning assignment. In fact, it comes with a huge class of models that not only vary as to their substance but are also structurally divergent. Thus, e.g., apart from demanding non-emptiness of the domains A, I, and J, nothing is said about their sizes. As a consequence, there are models with only one (object playing the role of an) individual in their domain as well as infinite sets of (representatives of) individuals of any cardinality, and similarly for worlds and times. Moreover, given a selection of domains A, I, and J, a lexical item α may receive any meaning constructed from them, as long as $F(\alpha)$ is of the type matching α's category (about which more in the next subsection) and certain 'reasonable' *meaning postulates* [*PTQ*, p. 235] are respected to take care of semantic effects like:[8]

[7] Cf. Saussure's (1916: 122) famous dictum that 'language is a form, not a substance' [*la langue est une forme et non une substance*].

[8] To be sure, the labels are anachronistic. The rigidity schema can already be found in Montague (1970b: 393); hence Montague's treatment of proper names as ''logically determinate'' [*PTQ*, p. 235, in scare-quotes] foreshadows Kripke's (1972) rigid designators. – Transparency blocks both non-specific and intensional readings in verbs like **find** (and unlike **seek**) and in prepositions like **in** (and unlike **about**); it should be noted,

- *Rigidity*: the intensions of proper names should be constant across all points of evaluation;
- *Transparency*: the complements of certain verbs and prepositions should take scope over them;
- *Lexical decomposition*: **seek** can be paraphrased in terms of **try** and **find**.

The *PTQ* meaning postulates are formulated in terms of higher-order intensional logic and exert their effect on the expressions of English by way of the indirect interpretation mechanism to be addressed in Section 1.5; in particular, they are not expressed in the object language itself, nor can they be without loss of substance (cf. Zimmermann 1985).

Even given the above meaning postulates (and others to be addressed in Section 2), the type assignment and the nine schematic postulates leave a lot of wiggle room for potential meaning assignments. As a case in point, according to some models with a singleton domain of individuals, the sentence **Some man is not Bill** comes out as false at every point of evaluation; the same effect will arise in a model according to which the extension of **man** coincides with the (characteristic function of) the singleton of the (rigid) extension of **Bill** at every point of evaluation. Clearly, such 'degenerate' models (Rooth 1985: fn. 13) can be eliminated by *ad hoc* postulates but there seems to be no general systematic way for doing so short of assuming a fixed *intended* model whose meaning assignment would be fully isomorphic to the real thing – thereby in effect giving up the model-theoretic approach (cf. Zimmermann 2017).

Though rarely explicitly questioned, the value of the model-theoretic approach to natural language semantics is moot. In particular, in some of the most influential works in the (post-) Montagovian tradition, model classes are traded for material truth conditions and meaning assignments. As a case in point, the meaning assignments in Heim & Kratzer (1998) are given in terms of reference to persons and other concrete objects, as opposed to arbitrary set-theoretic constructions. In the remainder of this brief survey, reference to model-space will be largely suppressed too.

1.4 The hierarchy of extensions: functional types

According to Montague Grammar, the category of an expression not only determines its syntactic behavior but also the *kind* of meaning it has. The rationale for carving up the realm of meanings into different types goes back to Frege's (1891) strategy of distinguishing *saturated* ['gesättigt'] expressions which, like names and sentences, have truth values or (individual) referents, from those whose extensions compositionally contribute to the referents or truth values of the expressions in which they occur, where these contributions are construed as functions assigning extensions of mother constituents to extensions of sister constituents. Thus, e.g., the extensions of verb phrases *V* come out as characteristic functions assigning truth values (of sentences with *V* as a predicate) to individuals (as the extensions of referential subjects of said sentences). By iteration, this strategy, which has become part of many textbook accounts of modern semantics, eventually leads to a whole hierarchy of ever more complex functional extensions, including: functions from individuals to truth values (VP-extensions); functions from individuals to VP-extensions (extensions of transitive verbs); functions from VP-extensions to truth values (quantifier extensions); etc.

---

though, that non-specificity and intensionality are both theoretically and empirically independent, as pointed out, respectively, in Zimmermann (1983: 73, fn. 9) and Mador-Haim & Winter (2015). – The lexical decomposition of **seek** goes back to Quine (1956; 1960: §32), where it is presented as a reductive analysis rather than an accidental observation; see Montague (1969: 174ff.) for criticism and den Dikken *et al.* (2018) for a recent revival of the Quinean spirit in syntactic terms.

For notational convenience, Montague used mnemonic labels for the layers of this hierarchy – his system of *(functional) types*, starting with the extension types *e* [*entities*] and *t* [*truth values*] for referential expressions and sentences, and labelling the layer of functions from layer *a* to layer *b* by the ordered pair of *a* and *b*: <*a*,*b*>. Hence, just for the record, the by now ubiquitous angular brackets *per se* do not have anything to do with types but merely serve to indicate ordered pairs; in particular, Montague did not use them to mark the ground types *e* and *t* themselves. In order to cope with intensional constructions by way of Fregean compositionality, the type hierarchy also needs to contain intensions, which serve as the compositional contributions of those constituents that give rise to substitution problems. Since, according to Montague (and presumably Frege, too), expressions of any category might in principle contribute their intensions, the functional type hierarchy contains intensions corresponding to any type of extension, i.e., functions from points of evaluation to any layer *a* of extensions, which, paying reverence to Frege (1892), Montague (1970b: 379) dubbed <*s*,*a*>; again for the record: the '*s*' in the intension types stands for *sense* (or *Sinn*), not for *situation*, even though arguably this is what the points of evaluation that constitute their domains come down to.

The set *Type*, then, contains the labels *e* and *t* as well as all pairs <*a*,*b*> and <*s*,*b*>, where *a* and *b* are members of *Type*, and nothing else [*PTQ*, p. 229]; and the (model-dependent) domains D_a of objects of any type – the *possible denotations* of that type – are then defined by the following recursion [*PTQ*, p. 230]: D_e is the set of individuals provided by the model; D_t is the set {0,1} of truth values; $D_{<a,b>}$ is the set of total functions from D_a to D_b, i.e., those functions that assign to each argument of type *a* a value of type *b*; and $D_{<s,b>}$ is the set of total functions from the (model-dependent) set of evaluation points to D_b, i.e., those functions that assign to each pair <*i*,*j*>, where *i* is a world and *j* is a time, a value of type *b*.

It should be noted that Montague's type hierarchy differs from later versions in that it does not treat *s* as a type of its own: due to their special function of determining extensions relative to logical space, points of evaluation do not count as objects of reference. By the same token, there are no functional extensions that would assign points of evaluation to extensions of potential sister nodes, and as a consequence, there appears to be no need for any types of the form <*a*,*s*>. It would certainly have been possible (for Montague or anyone) to include *s* in the type hierarchy as well as all pairs formed from the so-extended types – thus arriving at the more inclusive set *Type2* of *two-sorted types* containing *e, t*, and *s*, as well as all pairs of members of *Type2*, and nothing else.[9] In this extended hierarchy, intensions are merely special cases of functional extensions. One reason why Montague did not take this somewhat more straightforward route lies in his motivation to build the hierarchy of extensions as part of a reconstruction of Frege's functionality principle in terms of Carnapian intensions. A further potential reason for this restriction on *Types* will be brought up in the next subsection.

Given the motivation of types in terms of compositional contributions to extensions, it should not come as a surprise that expressions that behave syntactically alike also have extensions of the same type. In Montague Grammar, this connection between syntax and semantics is made by a *type assignment* called 'f', which (as the term indicates) associates syntactic categories with corresponding functional types. One obvious condition on this association is that the category of sentences is matched with the type *t* of truth values. The theoretical framework of Montague (1970b) imposes no further restrictions on type

---

[9] The term *two-sorted* was introduced by Gallin (1975: 58) in this connection to reflect the fact that in *Type2* and its applications to type logic (more about which below), the sets D_e and D_s play a role analogous to that of sorts of individuals in many-sorted predicate logic.

assignments in general. However, in *PTQ*, Montague borrowed some semantically motivated notation from categorial grammar (Ajdukiewicz 1935, Bar-Hillel 1953, Lambek 1958) to replace the arbitrary category labels: sentence predicates (or verb phrases) fall under the category t/e, where (non-italicized) t stands for the category of sentences and e for the category of individual-denoting expressions. In classical categorial grammar, this categorization would have the consequence that putting a verb next to an expression of category e results in a sentence. But in *PTQ*, the notation merely indicates the type of extensions of expressions of a category. On the basis of numerous descriptive applications and extensions of Montague Grammar as well as several later developments, it is natural to assume that this type should be the type $<e,t>$ of (characteristic functions of) sets of individuals.[10] The reasons why, in *PTQ*, Montague decided otherwise, lies in his attempt to achieve a maximum of semantic *uniformity* in syntactic categories, to which we will now turn.

Since in general, expressions with functional extensions may give rise to substitution problems, their sister constituents may have to contribute their intensions (rather than their extensions) to the extension of the mother constituent. Sentence adverbs are a case in point. In *PTQ*, they naturally fall under the category t/t, because they combine with a sentence to produce another sentence, with **necessarily** (construed in the logicians' sense of *everywhere in Logical Space*) serving as the sole example in *PTQ.* Clearly, in this case the corresponding type needs to be $<<s,t>,t>$: the truth value of a sentence of the form '**Necessarily** *S*' cannot be determined on the basis of the truth value of *S* alone (not if *S* is true, that is). So the type assigned to sentence adverbs is that of functions taking *intensions* of the sister type to *extensions* of the mother type. However, this does not make them *per se* intensional: even negation (if it were a sentence adverb) could be regarded as an operator of type $<<s,t>,t>$: given a point of evaluation, its extension would assign the opposite truth value of whatever its sister assigns to that point.[11] Of course, there is something hybrid to this construal of negation as operating on propositions, since it would do the very same job as the familiar truth table of type $<t,t>$. But it would unify the types of intensional and extensional sentence adverbs. And it is readily seen that, quite generally, at any point $<i,j>$, any function $g \in D_{<a,b>}$ could be 'simulated' by a hybrid function $g^+ \in D_{<<s,a>,b>}$ that does the same job: $g^+(x) = g(x(<i,j>))$, whenever $x \in D_{<s,a>}$. This elementary observation gives rise to *PTQ*'s [p. 232] unified treatment of the types ƒ(A/B) of expressions of a category A/B that combine with category B expressions to produce expressions of category A: whether or not they create intensional contexts, they are treated as operating on the intensions of their sisters (in LF, as it were) and thus receive the extension type $<<s, ƒ(B)>,ƒ(A)>$. Hence verb phrases, whose category is t/e, come out as having extensions of type $<<s,e>,t>$. Thus, at a given point of evaluation $<i,j>$, the extension of **walk** contains all functions of type $<s,e>$ whose value for that point is an individual that walks at that point. If one thinks of such functions as intensions of descriptions or modes of presentation of objects, then the extension of **walk** may be said to contain each walking individual in any way this individual can be described or presented. This is certainly an overkill vis-à-vis the simpler extension that would merely collect the walking individuals themselves. But then the type $<e,t>$ of that simpler extension would not work for intensional predicates. And, according to Montague, there are such predicates; we will turn to them in Section 2.3.4.

---

[10] See, e.g., Dowty *et al.* (1981) or Heim & Kratzer (1998).

[11] Cf. Thomason & Stalnaker (1973: 203), where **actually** is given as another example of an extensional sentence adverb besides negation.

The unification of the semantic types of extensional and intensional verb phrases is part of a general strategy that has aptly been described as 'generalizing to the worst case' (Partee 1997: 75): if a class *K* of expressions is syntactically homogenous enough to form a category, *K* is assigned the most complex type observed among the compositional contributions of its members, all of whose extensions are adapted to that type. The adaptions from the originally observed (or stipulated) types – <*e*,*t*> in the case at hand – to the most complex (or 'worst') one are known as *type shifts*. The map from sets of individuals to a corresponding set of individual concepts (of type <*s*,*e*>) is a case in point. Another type shift is the so-called *Montague Lift* that maps the referents of proper names to corresponding quantifiers. Here the relevant syntactic category is that of a Term, i.e., a nominal expression that may combine with a predicate (of category IV) to form a sentence (t), which is reflected in the label of this category: t/(t/e), abbreviated as T. Given the above intensionality-sensitive pattern of mapping categories to types, ƒ(T) comes out as ƒ(t/(t/e)) = <<*s*,ƒ(IV)>,*t*>, which spells out as: <<*s*,<<*s*,*e*>,*t*>>,*t*>. As a consequence, the extensions of names according to *PTQ*, are (characteristic functions of) sets of properties of individual concepts. More will be said about the exact nature of these sets later; for the time being we only note that they need to be defined in such a way that they fit the quantificational subject construal F_4.

While differences in the semantic behavior of syntactically similar expressions can be evened out by generalizing to the worst case, the semantic similarity among expressions does not imply syntactic uniformity. In particular, two distinct categories may be assigned the same extension type. In *PTQ* the category of nouns, whose extensions come out as sets of functions of type <*s*,*e*>, is a case in point. Given their different syntactic behavior, their category thus needs to be distinguished from the category of verb phrases while still wearing its extension type on its notational sleeves. Since the categorial label t/e has already been used up, Montague came up with the *ad hoc* device of adding another slash to distinguish the category of nouns from that of Intransitive Verbs and defined it as t//e. The same hack is used to distinguish adverbs (IV/IV) from control verbs (IV//IV), and again it is quite arbitrary which is which: one or two slashes, the effect on the extension type is the same: ƒ(A/B) = ƒ(A//B) = <<*s*, ƒ(B)>,ƒ(A)>, for any categories A and B [*PTQ*, p. 232].

1.5 Translating into *IL*: indirect interpretation

Montague Grammar owes much of its flavor to the particular format used to specify extensions (and intensions), viz. by formulae of the language of intensional type logic – *IL*, for short.[12] In the *PTQ*-grammar, as already in Montague (1970b), an algorithm is specified that translates the expressions of English into *IL*-formulae, whose (model-dependent) denotations are specified independently and carry over to the English sources. Crucially, the algorithm proceeds in a compositional fashion so as to guarantee the compositional behavior of the English expressions it translates. This means that the translation into *IL* constitutes a detour on the way from natural language to meaning – and hence an *indirect interpretation*. Yet despite its eliminability, the procedure offers a number of advantages, not the least of which is that *IL* (or whichever language is chosen as a medium) may serve as a *lingua franca* for the semantic community. However, indirect interpretation must not be

[12] This by now common name appears to be due to Gallin (1971). In Montague (1970b), the system was called L_0, and no shorthand name was used for the slightly extended version in *PTQ*. We will also follow semantic practice and use the term *(IL-) formula* for all terms or 'meaningful expressions' [*PTQ: passim*] of that language, not just for truth-valued ones (of type *t*: see below), as logicians would prefer.

misconstrued as representationalism: the meanings of English expressions are the model-theoretic interpretations of their *IL*-translations, not the *IL*-formulae themselves.

The algebraic details of indirect interpretation, and particularly its compositionality and eliminability, are beyond this short survey, in which the translation algorithm will only be presented by way of examples.[13] The same goes for the language *IL* itself. The following remarks are mainly meant as a help to decipher the formulae used in the *PTQ*-translation.

The formulae of *IL* are stratified by their *Types*; the idea is that the translation of an English expression of any category A will be an *IL*-formula of type ƒ(A) denoting an object of that type. Thus, sentences translate as formulae of type *t*; verb phrases as formulae of type <<*s*,*e*>,*t*>; etc. As a rule of thumb, lexical expressions get translated by corresponding constants, whereas the translations of complex expressions are obtained by combining the translations of their immediate parts in accordance with the syntactic constructions by which they have been built. Thus, e.g., the verb **try** and the verb **sleep** translate as certain constants of types <<*s*,<<*s*,*e*>,*t*>>,<<*s*,*e*>,*t*>> (= ƒ(IV//IV)) and <<*s*,*e*>,*t*> (= ƒ (IV)), which in *PTQ* are referred to as **try**′ and **sleep**′, respectively. As a consequence, their extensions may vary wildly across the class of all models (though not necessarily across the worlds within one model; cf. Zimmermann 2017). While such wide ranges of extensions appear to reflect the inferential behavior of non-logical lexical items, it would be inappropriate in worst case generalizations. Thus, e.g., a type-shifted name should not be able to denote an arbitrary quantifier: if, say, the extension of **John** (at some point of evaluation in some model) were the same as that of **no entity**, the inference from **John walks** to **Some entity walks** would not go through. To capture their inferential behavior, then, proper names like **John** are translated by type-theoretic formulae of the form **c***, where **c** is a constant of type *e* and the *-operator expresses Montague Lifting in *IL*. Before getting into the specifics of this type shift, it will be instructive to take a brief look at how complex expressions of English are translated into *IL*. As a first step in this direction, let us adorn the tree in (1a) with type-logical translations:

[13] The algebraic aspects of indirect interpretation are expounded in Montague (1970b: 383f.). An interesting generalization (in terms of 'safe derivers') can be found in Chapter II of Janssen (1983).

(5) *IL*-translation of (1a)

John tries to find a unicorn, 4

$$[\lambda P.\ [{}^{\vee}P](^{\wedge}\mathbf{j})]\ (^{\wedge}\mathbf{T}(^{\wedge}\mathbf{F}(^{\wedge}\lambda P.\ (\exists y)\ [\mathbf{U}(y) \wedge [{}^{\vee}P](y)])))$$
$$\equiv$$
$$\mathbf{T}(^{\wedge}\mathbf{j},^{\wedge}\lambda x.\ (\exists y)\ [\mathbf{U}(y) \wedge \mathbf{F}_{*}(x,y)])$$

John

$$\lambda P.\ [{}^{\vee}P](^{\wedge}\mathbf{j})$$

try to find a unicorn, 6

$$\mathbf{T}(^{\wedge}\mathbf{F}(^{\wedge}\lambda P.\ (\exists y)\ [\mathbf{U}(y) \wedge [{}^{\vee}P](y)]))$$
$$\equiv$$
$$\mathbf{T}(^{\wedge}\lambda x.\ (\exists y)\ [\mathbf{U}(y) \wedge \mathbf{F}_{*}(x,y)])$$

try

$\mathbf{T}$

find a unicorn, 5

$$\mathbf{F}(^{\wedge}\lambda P.\ (\exists y)\ [\mathbf{U}(y) \wedge [{}^{\vee}P](y)])$$
$$\equiv$$
$$\lambda x.\ (\exists y)\ [\mathbf{U}(y) \wedge \mathbf{F}_{*}(x,y)]$$

find

$\mathbf{F}$

a unicorn, 2

$$\lambda P.\ (\exists x)\ [\mathbf{U}(x) \wedge [{}^{\vee}P](x)]$$

unicorn

$\mathbf{U}$

Apart from a few minor notational variations,[14] (5) displays the stepwise *IL*-translation of (1a) according to the *PTQ*-algorithm, plus some logical reductions (following the triple bars). Readers unfamiliar with the *IL*-formalism may prefer the following equivalent representation in terms of a more transparent notation that makes explicit reference to points of evaluation by way of variables, a notation known as *Ty2* (for *two-sorted type theory*):

[14] Thus, Montague uses capitalized versions of the connectives '$\wedge$' and '$\vee$' to denote universal and existential quantification, where we use the more common inverted 'A's and 'E's. Moreover, we have given mnemonic names to the constants and omitted predictable brackets around λ-terms.

(6) *Ty2*-translation of (1a)

John tries to find a unicorn, 4
$[\lambda P.\ P_i(\lambda i.\mathbf{j}_i)](\lambda i.\ \mathbf{T}_i(\lambda i.\ \lambda x.\ (\exists y)\ [\mathbf{U}_i(y) \wedge \mathbf{F}_i(x,y)]))$
$\equiv$
$\mathbf{T}_i(\mathbf{j}, \lambda i.\ \lambda x.\ (\exists y)\ [\mathbf{U}_i(y) \wedge \mathbf{F}_i(x,y)])$

John
$\lambda P.\ P_i(\lambda i.\ \mathbf{j}_i)$
$\equiv$
$\lambda P.\ P_i(\mathbf{j})$

try to find a unicorn, 6
$\mathbf{T}_i(\lambda i.\ \lambda x.\ (\exists y)\ [\mathbf{U}_i(y) \wedge \mathbf{F}_i(x,y)])$

try
$\mathbf{T}_i$

find a unicorn, 5
$\lambda x.\ (\exists y)\ [\mathbf{U}_i(y) \wedge \mathbf{F}_i(x,y)]$

find
$\mathbf{F}_i$

a unicorn, 2
$\lambda P.\ (\exists x)\ [\mathbf{U}_i(x) \wedge P_i(x)]$

unicorn
$\mathbf{U}_i$

The ubiquitous index *i* is a variable ranging over points of evaluation; and subscripting is merely a (common) notational variant for functional application, $\mathbf{U}_i$ being shorthand for $\mathbf{U}(i)$, etc. The logical formulae in (6) are built up using bold-face constants of the appropriate *intension* types. Thus, given that the extension of the noun **unicorn** is of type f(t/e), **U** is of type <*s*,<<*s*,*e*>,*t*>>. Slightly deviating from the *PTQ* type assignment (see below), the constant **F** in (6) denotes the intension of a binary relation between individual concepts and is thus of type <*s*,<<*s*,*e*>,<<*s*,*e*>,*t*>>>. And given that the verb **try** is of category (IV//IV), the constant **T** is of type <<s,f(IV)>, f(IV)> (= <<s,<<*s*,*e*>,*t*>>,<<*s*,*e*>,*t*>>); as a consequence (to which we will also return), the constant **j** that fills the 'subject' argument of **T**, is of type <*s*,*e*>. The rest of the notation is hopefully self-explanatory. In particular, the triple bar between the type-logical translations and their reduced forms stands for logical equivalence; the suggested reductions all make use of the familiar laws of λ-conversion.[15]

Readers may feel that the formulae in (5) are somewhat simpler than those in (6), particularly because they seem to do without the evaluation-point variable *i*. As a matter of fact, though, rather than being absent, it is cleverly hidden. Its suppression is the main feature that distinguishes *IL* from the seemingly more complicated notation in (6). Closer inspection of the translations of all parts of (1a) but the embedded verb reveals how the evaluation-point variable can be made to reappear by a systematic procedure:

- *IL*-constants **c** of any extension type *a* correspond to constellations of the form '$\mathbf{c}_i$', where **c** is a homographic *Ty2*-constant of the corresponding intension type <*s*,*a*> and *i* is a fixed variable of the (non-Montagovian but) two-sorted type *s*.
- The *cup* operator preceding the variable '*P*' (of type <*s*,<<*s*,*e*>,*t*>>) corresponds to application of the homographic *Ty2*-variable '*P*' to the evaluation-point variable *i*; in

[15] The leftmost equivalence (below **John**) also takes advantage of the fact that $\lambda i.\,\mathbf{j}_i$ reduces to **j**, due to the logical law of η-conversion, which reflects the extensionality of set-theoretic functions.

fact, any *IL*-formula of the form '$[^{\vee}\alpha](\beta)$' corresponds to $\alpha^*(i)(\beta^*)$, where α^* and β^* correspond to α and β.[16]

- The *cap* operator preceding intensional arguments corresponds to the prefix '$\lambda i.$'.

Using these three correspondence rules and leaving everything else as is, almost any *IL*-formula can be translated into the more transparent *Ty2*-notation.[17] The one remaining difference between (5) and (6) then concerns the translation of **find**, which is a constant of the worst case for transitive verbs, which take Term intensions of the dazzlingly complex type <*s*,f(T)> = <*s*,<<*s*,<<*s*,*e*>,*t*>>,*t*>> as their arguments. The justification for this type assignment will be supplied later (Section 2.3.2). However, like its namesake in (6), the $\mathbf{F}$ in (5) merely relates individual concepts represented by x and y – except that the type of the latter has been adapted by a Montague Lift. It is this binary relation between *x* and y that features in the translation tree (5):[18]

(7) $\mathbf{F}_*:=[\lambda y.\lambda x.\mathbf{F}(x,y^*)]$

where the upper asterisk again indicates a Montague Lift, the definition of which can be gleaned from the translation of the subject in (5):

(8) $\alpha^* := [\lambda P.\ [^{\vee}P](^\wedge\alpha)]$

In *PTQ*, equation (7) is implemented as (a consequence of) a meaning postulate – an *IL*-formula that is supposed to be true at points of evaluation of all ("reasonable") models. The same effect could have been achieved by taking the right-hand side of (7) as the translation of **find**, instead of having a constant $\mathbf{F}$ (which, it should be recalled, is the *IL*-version of the Ty2-formula $\mathbf{F}_i$). Indeed, for the translation of proper names, Montague [*PTQ*, p. 233, T1.(d)] followed this alternative strategy and put:

(9) $\mathbf{John}' := \mathbf{j}^*$,

where $\mathbf{j}$ is a constant of type *e* (and thus, again, abbreviates $\mathbf{j}_i$) and the asterisk is defined as in (8).

The type reduction in (7) can be used to express that any quantificational object of **find** will be interpreted so as to take scope over the verb – which is precisely what the transparency postulate demands [*PTQ*, p. 235, (4)]:

(10) $\mathbf{F}=[\lambda\wp.\lambda x.[^{\vee}\wp](^\wedge\lambda y.\mathbf{F}_*(x,y))]$

(10) offers an alternative route to what Heim & Kratzer (1998: 178) famously dubbed 'the problem of quantifiers in object position'. The Term is interpreted *in situ* (and by functional application) but still manages to quantify over objects found by the subject. This is possible because the verb (or its *IL*-translation $\mathbf{F}$) does not denote the relation of finding between individuals but a type-shifted relation between individual concepts x and Term intensions $\wp$, viz. the relation that holds if the property of <u>being a y that is found by x</u> is in the extension of the quantifier – where the underlined condition is expressed in terms of $\mathbf{F}_*$, not $\mathbf{F}$.

These illustrations must suffice to give readers a glimpse of how the indirect translation process works. For the specifics of the translation algorithm and the target language *IL*, they are referred to *PTQ* itself or the surveys mentioned in the Introduction.

---

[16] In *PTQ* the constellation '$[^{\vee}\alpha](\beta)$' is usually abbreviated as '$\alpha\{\beta\}$'.

[17] Tense and modal operators need a special treatment, thence 'almost'; see, e.g., Köpping & Zimmermann (to appear). – The correspondence, which is a special case of the so-called *standard translation* of modal logic (Fine 1975: 19), was first described in Gallin (1975: 61); given that the system *Ty2* is the obvious substratum underlying the very construction of *IL*, Montague was certainly aware of it, though.

[18] In fact, by the meaning postulate (4) given in *PTQ* [p. 235], $\mathbf{F}_*$ even downgrades finding to a relation between *individuals* (instead of individual concepts); however, we refrain from doing so here for expository reasons. Similar remarks apply to (10).

Before we get to the descriptive aspects of the *PTQ* grammar, a word is in order about the very choice of an intensional target language over its obvious – and obviously more transparent – two-sorted rival. For apart from being shorter and at times closer to predicate logic, the *IL*-formulae have little to recommend them. In particular, they are much harder to manipulate: the principle of λ-conversion, which is vital when it comes to reduction for readability, is not valid in its familiar form, the reason being that the suppressed evaluation point variable may lead to a variable clash but cannot be renamed. Pertinent examples can be found in any good reference text.[19] Another, less frequently noticed deficiency of *IL* is revealed in connection with the rigidity of proper names, which in *Ty2* can be dealt with by type shifting constants from *e* to <*s*,*e*> – but not in *IL*, because *IL*-constants have intension types only (Zimmermann 1993: 160, fn.20).

Given these problems with Montague's *IL*, one may wonder whether the simplification is worth the trouble.[20] One potential reason for still using the language – and possibly Montague's own main motivation – may be seen in its restricted expressivity: *IL* avoids apparently unnecessary types like <<*s*,*t*,>,*s*> and has only one 'invisible' evaluation-point variable (*i*) at its disposal. So if the intensional language suffices for indirect interpretation, its limitations might tell us something about the complexity of natural language, viz. that its combinatory power is below full two-sorted type theory. There are, however, two reasons that might speak against such an interpretation. On the one hand, it is quite uncertain that the two-sorted types omitted by *IL* or its avoidance of multiple (free) variables of type *s* is even descriptively adequate (cf. Cresswell 1990). On the other hand, the expressivity gap between *IL* and *Ty2* is much smaller than it may appear: ignoring multiple evaluation-point variables and non-intensional types, anything that can be expressed in *Ty2* can also be expressed in *IL* – but usually in a much more roundabout and much less conspicuous way (cf. Zimmermann 1989). Hence there is every reason to do away with Montague's elegant but hard to read and manipulate intensional logic and use the much more transparent two-sorted version in indirect interpretation – as many semanticists have done since the 1980s.

2. Descriptive aspects: the fragment

The semantic phenomena covered by the *PTQ fragment* of English are too varied to all be addressed here. We will therefore concentrate on a few of the most innovative and influential descriptive contributions. It needs to be mentioned though that, apart from Montague's previously published work, as a precise account of compositional semantics, *PTQ* was without precedent. In that sense, practically everything in that paper was both original and innovative.

2.1 Extensional constructions

Although Frege (1879) had characterized quantification as second-order predication and Russell (1905) had extended Frege's approach to definite descriptions, thereby rejecting Frege's (1892) own, referential treatment, Richard Montague is usually credited with the

[19] See, e.g., Dowty *et al.* (1981: 167). The correct, restricted version of λ-conversion in *IL* has first been formulated in Gallin (1975: 19) as an axiom schema.

[20] The questions in this paragraph are addressed in more detail in Zimmermann (to appear), where relevant references can be found too.

first systematic compositional interpretation of nominal quantification.[21] Given this achievement, it is somewhat surprising that his treatment of quantificational Terms does not proceed by a binary composition of determiner and noun, but rather lifts the latter to quantifier status by one unary operation per determiner. We thus have:

(11) $F_0(\mathbf{unicorn}) = \mathbf{every\ unicorn}$,

which translates as:

(12) $\lambda P.(\forall x)[\mathbf{U}(x) \rightarrow [\ ^{\vee}P](x)]$,

which in turn is an instance of a general rule applying to translations α of nouns lifted by F_0 [*PTQ*, p. 233, T2]:

(13) $\lambda P.(\forall x)[\alpha(x) \rightarrow [\ ^{\vee}P](x)]$

And we do not have anything like:

(14) $F_{100}(\mathbf{every}, \mathbf{unicorn}) = \mathbf{every\ unicorn}$,

which would translate as:

(15) $\mathbf{every}'(\mathbf{U})$,

where $\mathbf{every}'$ is the following *IL*-formula of type <<<*s*,*e*>,*t*>,<<<*s*,*e*>,*t*>,*t*>>:

(16) $\lambda Q.\lambda P.(\forall x)[[\ ^{\vee}Q](x) \rightarrow [\ ^{\vee}P](x)]$

– and similarly for the determiners $\mathbf{a[n]}$ and $\mathbf{the}$.[22] In other words, instead of forming one lexical category, determiners are treated as each indicating its own unary syntactic operation. Hence Montague's analysis of nominal quantification does not lend itself so easily to the study of determiner denotations as the generalized quantifier theory it inspired, starting with Barwise & Cooper (1981). The reason why Montague preferred (12) over (15) may have to do with the *logicality* of determiners, which shows in their definability in terms of parameter-free type-logical formulae (i.e. formulae that do not contain any non-logical constants or free variables) and suggests that they be treated as part of the grammatical structure of language. However, the characterization of the distinction between lexical and grammatical meaning in terms of logicality is not part of the general theory of Montague (1970b).[23]

As pointed out by Thomason (1974: 261, fn. 12), the translation (13) (like its variants for the other determiners) leads to undesired results: if α happens to contain a free pronoun with the same index n as the (fixed) variable 'x', the universal quantifier would accidentally bind it – a case in point being: α = **unicorn such that it loves him**$_n$, which according to (12), would result in quantification over the set of narcissist unicorns. The following corrected version of (13) takes care of this nuisance:[24]

(17) $[\lambda N.\lambda P.(\forall x)[N(x) \rightarrow [\ ^{\vee}P](x)]](\alpha)$

[21] Essentially the same account of nominal quantification (also covering the determiner **no**) was already given in Montague (1970b). There is evidence (Barbara Partee, p.c.) that it was motivated by, and started with, the compositional analysis of intensional transitives, as announced in Montague (1969: 177).

[22] A comparison of (12) and (16) nicely brings out the asymmetry between constants and variables in *IL*: in (12) the dependence on the point of evaluation is implicit in the constant $\mathbf{U}$, which corresponds to $\mathbf{U}_i$ in *Ty2*, whereas in (16) it needs to be marked by the cup operator preceding Q.

[23] A distinction is still made though: according to the theory of reference in Montague (1970b), abstraction from context, with variable binding as a special case, is confined to semantic composition. – Definability in a suitable logical system is one way of characterizing logicality. Another, more popular approach goes back to Tarski (1986) and uses invariance with respect to permutations on the ground domains (i.e., the individuals and the points of evaluation); this criterion was first proposed for (lexical) determiners in Keenan & Stavi (1986: 310f.).

[24] Thomason (*ibid.*) proposes to choose the variable x as depending on α, which is however not in line with Montague's (1980b: 383f.) 'polynomial' construal of indirect interpretation.

Using (17), Montague's analysis turns out to be identical to the by now standard generalized quantifier account (15) + (16). Moreover, it helps bringing out an elementary feature of the unary constructions F_0 – F_2: they are *extensional* in that the extensions of the quantifying Terms so constructed are determined solely on the basis of the extensions of their (unique) nominal parts. In (17) this shows in the position of the meta-variable α, which does not stand in the scope of any variable binder (λ, $\forall$, $\exists$) or intensional operator ($^\wedge$, $\Box$, W, H).

Apart from the Term formation operations F_0 – F_2, there is only one other family of extensional constructions in the *PTQ* fragment of English, viz. the coordinations of sentences, verb phrases (IVs), and Terms; their *PTQ* treatment formed the starting point for numerous investigations into (generalized) Boolean conjunction and disjunction.[25] We will not go into them here except for noticing that Term coordination in *PTQ* is restricted to disjunction (by T13 on p. 234), thereby evading any semantic problems posed by plural formation.[26]

Interestingly though, the (later) so-called *in situ* interpretations of quantificational subjects and objects, both of which are listed among the rules of functional application [*PTQ*, p. 233], fall prey to Montague's strategy of generalizing to the worst case, ending up as intensional constructions, which is why we will only turn to them in Section 2.3. Finally, the reader should bear in mind that, on account of the extension type <<*s*,*e*>,*t*> assigned to nouns and verbs, the *IL*-variable x in (13) ranges over individual concepts of type <*s*,*e*>. This peculiarity of the *PTQ*-treatment of nominal quantification will also be addressed in Section 2.3.

2.2 Binding and Scoping

Failure of extensional substitution is not always caused by intensionality in the narrow sense. For apart from the point of evaluation, extensions also depend on context; and according to Montague's (1970b) theory of reference, this dependence can be abstracted from by variable binding, where the assignment plays the role of context. In the *PTQ* fragment, there are two kinds of construction that are interpreted as variable-binding operations: (restrictive) relative clause formation and quantifier raising. On the syntactic side, both involve removing subscripts on pronouns, thus indicating their bound status; according to Montague's (1970b) theory of reference, any remaining subscripted ('free') pronouns need to be interpreted by a contextually given assignment.[27]

The *PTQ* fragment does not really cover relative clauses but only circumlocutions like **unicorn such that it speaks**. The reason for this is presumably that, unlike the traces of relative pronouns, the bound anaphors in these circumlocutions do not call for any syntactic restrictions, as witnessed by **unicorn such that it or John speaks.** On the semantic side, Montague implemented Quine's (1960: 110f.) suggestion that restrictive relative clause formation and modification correspond to set abstraction and intersection, respectively, which are rolled into one construction $F_{3,n}$, translated by the following rule [*PTQ*, p. 233, T3]:

---

[25] See, in particular, Partee & Rooth (1983) and Hendriks (1990); Geach (1970) is an important predecessor.

[26] Barbara Partee (2011: 26) reports that 'there are handwritten pages in [Montague's] files from 1970 when he was working on PTQ that show failed attempts to treat quite a number of phenomena that never made it into PTQ. For example, he had intended to include a much larger class of quantifiers than the three (*a, the, every*) that ended up being treated in PTQ. But he abandoned the attempt to include a treatment of plural expressions in PTQ, which eliminated most quantifiers and eliminated term phrases conjoined with *and*, leaving only those three singular determiners and term phrases conjoined with *or*.'

[27] This suggestion has since been taken up by many semanticists – see, e.g., Kaplan (1989b: 527) or Heim & Kratzer (1998: 242ff.); still, its coherence is debatable: cf. Wehmeier (2018).

(18) $[\lambda x_n.[\alpha(x_n) \wedge \varphi]]$

where α and φ are the respective *IL*-translations of the modified noun and the relative clause (bar the **such that**-prefix), and n is the index of the pronoun corresponding to the trace. It may be noted in passing that (18) suffers from similar problems as (13) above, and that analogous cures may be applied.[28]

As to scope construal, the *PTQ* fragment distinguishes three cases according to the landing site of the quantifying Term, which may take scope over (a) a sentence, (b) a noun, or (c) a predicate. Case (a), which applies in (1b) above, naturally generalizes the quantifier construal of predicate logic, by binding a variable in a 'matrix' sentence and then applying the quantifier to the resulting set:

(19) $\Theta(\lambda x_n.\varphi)$,

where Θ translates the quantifying Term and φ the sentence from which it has been scope-raised. To be sure, in (19) it is the λ-operator and not the quantifier itself that binds the variable; but then the predicate logic notation (20) may be taken as shorthand for (19) anyway – it's just that predicate logic lacks λ-abstraction:

(20) $(\Theta x_n)\ \varphi$

Due to the uniform type assignment, (19) is not quite the *PTQ* translation for scope-raised quantifiers of type (a) yet: since Terms are of category t/IV, their extension type comes out as <<*s*,ƒ(IV)>,*t*>, and thus their argument needs to be intensionalized. So we have (21) as translating case (a) – even though no intensionality effect is observed here:[29]

(21) $\Theta({}^\wedge\lambda x_n.\varphi)$

Whenever a Term translation Θ is inserted in (21), it so happens that the cap operator eventually washes out by a variant of λ-reduction. (22), which is (equivalent to) the translation of (1b), is a case in point:

(22) $[\lambda P.(\exists x)[\mathbf{U}(x) \wedge [{}^\vee P](x)]]([{}^\wedge\lambda x_0.\mathbf{T}({}^\wedge\mathbf{j},{}^\wedge\lambda z.\mathbf{F}_*(z,x_0))])$

As readers should verify for themselves, using the above *Ty2*-representation of *IL*, (22) can be λ-converted to (23), where the underlined formula has taken the place of the λ-bound variable P:

(23) $(\exists x)\left[\mathbf{U}(x) \wedge [{}^\vee\underline{[{}^\wedge\lambda x_0.\mathbf{T}({}^\wedge\mathbf{j},{}^\wedge\lambda z.\mathbf{F}_*(z,x_0))]}](x)\right]$

The underlined part of (23) is in the scope of the cup operator, thus producing a formula of the form: $[{}^\vee[{}^\wedge\alpha]]$, which in *Ty2* spells out as:

(24) $[\lambda i.\alpha^*](i)$

which in turn λ-reduces to α^*.[30] As a consequence, the cup-cap sequence turns out to be redundant and we get the following logical law:[31]

---

[28] More specifically, (18) could either be traded for the more cumbersome (i), in analogy to (17), or the relative clause could be constructed independently, translated as (ii), in analogy to (15), and then get attached by predicate modification (in the sense of Heim & Kratzer 1998: 65):

(i) $[\lambda P.\lambda x_n.[P(x_n) \wedge \varphi]](\alpha)$

(ii) $[\lambda x_n.\varphi]$

A third possibility is offered in Montague (1970b: 393, def. of H_7), where the relative clause gets generalized to the worst case of an intensional modifier.

[29] In *PTQ* notation, (21) gets contracted to $\Theta(\hat{x}_n\ \varphi)$.

[30] β-reduction always applies to constellations of the form $[\lambda x.\alpha](x)$ because x cannot be accidentally bound when it replaces a free occurrence of itself; moreover, the replacement $\alpha[{}^x/_x]$ is obviously the same as α.

[31] The law – whose name goes back to (Dowty et al. 1981: 154) – features as an axiom schema in Gallin (1975: 19) but is not mentioned in *PTQ* or Montague (1970b), although it is crucial for many of the reductions given there. – Incidentally, there is no 'reverse' law of up-down cancellation: $[{}^\wedge[{}^\vee\alpha]]$ is generally not equivalent to α; where it is, it boils down to an instance of η-conversion in *Ty2*. Readers are invited to construct their own counterexamples, starting by inspecting corresponding *Ty2*-constellations $[\lambda i.\alpha(i)]$.

(25) *Down-up cancellation*

For any *IL*-formula α: $[\,^{\vee}[^{\wedge}\alpha]] \equiv \alpha$

With the help of (25), (22) ultimately reduces to:

(26) $(\exists x)[\mathbf{U}(x) \wedge \mathbf{T}(^{\wedge}\mathbf{j}, ^{\wedge}\lambda z.\, \mathbf{F}_{*}(z,x))]$

The same kind of spurious intensionality is built into the variants (b) and (c) of the scope-mechanism, which allow Terms to quantify into nouns and IVs, respectively. These possibilities are needed to account for readings that cannot be obtained by quantifying into sentences, as in the following examples:

(27) **John wishes to find a unicorn and eat it**

(28) **Every man such that a woman loves him such that he loves her loves a woman**

(27) is the example adduced in *PTQ* [p. 240] to motivate rule T16 [p. 234] that translates $F_{10,n}(T,P)$ as applying to a Term *T* and a predicate *P*, which themselves respectively translate as Θ and α:

(29) $\lambda y.\ \Theta(^{\wedge}\lambda x_n.\ \alpha(y))$

Using (29), the indefinite **a unicorn** in (27) can receive a non-specific reading and at the same time bind the pronoun **it** by taking scope over the embedded infinitive. – (28) illustrates how quantifier raising to the noun level allows for anaphora across 'stacked' relative clauses.[32] We leave it to the reader to figure out the details.

The flexibility concerning the landing sites is but one of a number of major differences between the *PTQ* construal of quantifier scope and later, more sophisticated approaches. Most importantly, it is completely unrestricted and thus allows for numerous unattested scope constellations such as (30), one *PTQ* translation of which reduces to (31):

(30) **A woman such that every man loves her loves him**

(31) $(\forall x)[\mathbf{man}'(x) \rightarrow (\exists y)\ [\mathbf{woman}'(y) \wedge \mathbf{love}'_{*}(x,y) \wedge \mathbf{love}'_{*}(y,x)]]$

Moreover, due to restrictions on scope constellations corresponding to what was later going to be called c-command (Reinhart 1983), *PTQ* has no way of capturing the apparent covariation between indefinites and pronouns in so-called donkey sentences like:

(32) **Every farmer who owns a donkey beats it**

Though the problem had been known since Geach (1962: 117ff., 128f), Montague apparently made no attempts at solving it; indeed, legend has it that he refused to even recognize the relevant reading of (32) [Terence Parsons, p.c.]. As it turns out, its solution seems to require a revision of several basic assumptions of Montague Grammar, including the treatment of indefinites as existential quantifiers, as first pointed out in Lewis (1975) and later systematized in so-called *dynamic* accounts starting with Kamp (1981) and Heim (1982).

2.3 Intensionality

The above examples already offered a glimpse of how intensionality – in the usual sense of substitution resistance on the level of extension – is treated in *PTQ*: in an intensional construction, substitution objectors get marked by a cap operator in front of their usual, extensional translation. Thus, in particular, object clauses of the form **that** *S* translate as *IL*-formulae $[^{\wedge}\varphi]$, where φ is of type *t* and translates the embedded sentence *S*. Since the cap operator merely abbreviates λ-abstraction from the invisible variable *i*, intensionality boils down to binding *i*, and substitution resistance turns out to be the result of a variable clash: if the translations $\mathbf{L}_i(\mathbf{j},\mathbf{b})$ and $\mathbf{L}_i(\mathbf{b},\mathbf{j})$ of **Jane loves Bill** and **Bill loves Jane** happen to have the

[32] No justification for raising Terms above nouns is given in *PTQ*. (28) is inspired by Partee's (1975: 236) **Every man who has lost a pen who does not find it will walk slowly**, which however happens to receive the intended reading by **such that** circumlocution if the second relative clause is attached to the noun **pen**.

same truth value under a given assignment, they may replace each other *salva extensione* – though not in the scope of λi, $\forall i$ or $\exists i$.

In a similar vein, tense morphology is translated into *IL*-operators that act on the proposition expressed by the whole sentence. As already mentioned in Section 1.1, this is achieved by a number of alternative subject-predicate construals that introduce temporal operators and/or negation. However, Frege's functionality principle does not restrict or reduce intensionality to clausal embedding: any expression has an intension, which may in principle contribute to the extension of the mother constituent when combined with an intensional sister. Indeed, *PTQ* has several intensional constructions that do not operate on propositions, i.e., objects of type <*s*,*t*>, to which we now turn.

2.3.1 Infinitival Complements

As can be gleaned from the above examples, in *PTQ*, control verbs like **try** and **want** take uninflected predicates (as opposed to clauses with PRO-subjects) as arguments and their extension operates on the intension of the embedded infinitive. Interestingly, the ensuing complex type of control verbs has the semantic potential of differentiating infinitival embedding from propositional attitude reports. In particular, it is consistent with this approach that the relation denoted by $\mathbf{T}$ – the *IL*-constant of type <<*s*,f(IV)>,f(IV)> that translates **try** – cannot be reduced to a relation $\mathbf{T}_*$ of type <<*s*,*t*>,f(IV)> in the sense that the following equivalence would hold (for any individual concept x and any property Q of individual concepts):[33]

(33) $\mathbf{T}(x, Q) = \mathbf{T}_*(x, [^\wedge[{}^\vee Q](x)])$

In effect – and ignoring the fact that the 'x' stands for individual concepts – (33) says that for any individual x and property Q, trying to have Q means trying for it to be the case that x has Q – i.e., if Q happens to be the property of performing a certain action A: x tries to do A means x tries to bring it about that x does A. Crucially, *PTQ* is *not* committed to (33). This is good news insofar as it has been argued that, quite generally, the implicit subjects of infinitival objects – aka PRO – receive an irreducibly *de se* reading, according to which (roughly) the subject portrays the attitude holder as his or her own self (Chierchia 1989; Higginbotham 2003), which in turn means, following Lewis' (1979) influential account of attitudinal self-identification, that the object of the attitude ought to be a property rather than a proposition – the property denoted by the infinitive, more likely than not.

That the *PTQ* treatment of control verbs is consistent with them irreducibly expressing *de se* attitudes does not mean that it implies it: the bad news about (33) is that it is as consistent with the *PTQ* approach as its negation. And though its absence from the list of meaning postulates might suggest that Montague had his doubts about it or even reasons to reject it, there is no indication that he did so, let alone to account for the *de se* reading of PRO.

Whether or not control verbs are interpreted as denoting propositional attitudes, infinitival embedding may be regarded as a variant of clausal embedding: with only the subject missing, the intension operated on is almost a proposition. In fact, given Lewis' (1979) account, since properties are the proper objects of mental attitudes, they may well be regarded as epistemic propositions [David Kaplan, p.c.]. On that view, then, the *PTQ*

[33] Readers can test their *IL* proficiency by verifying that, according to the correspondence established in Section 1.5, (33) is just shorthand for the following *Ty2*-formula: $\mathbf{T}_i(x, Q) = \mathbf{T}_*(i)(x, [\lambda i. Q_i(x)])$.

analysis of infinitival embedding confirms the *propositionalist* prejudice that intensionality is a matter of propositional attitude.[34]

2.3.2 Intensional Transitive Verbs

The connection between intensionality and propositional embedding is less clear in the case of intensional transitives, which to the semantically uninitiated seem to denote relations between individuals and objects of desire (aka *intentional* objects). As already pointed out in Montague (1969: 175), though, a compositional treatment of the non-specific (in Quine's parlance: *notional*) reading of objects of intensional transitives can be obtained on the basis of Quine's (1956) proposal to paraphrase them in terms of propositional embedding, and more specifically by: (a) analyzing the paraphrase, (b) isolating the contribution the unspecific object makes to that paraphrase, and (c) identifying the remainder of the paraphrase as the contribution of the intensional verb. As a case in point, (a) the *IL*-translation (35) of (34) is taken to carry over to Quine's paraphrase (36):

(34) **try to find a unicorn**

(35) $\mathbf{T}({}^{\wedge}\lambda x.(\exists y)[\mathbf{U}(y) \wedge \mathbf{F}_*(x,y)])$

(36) **seek a unicorn**

Next, (b) the infinitival complement may be dissected into the contributions of its immediate constituents, rendering (35) equivalent to:

(37) $\mathbf{T}({}^{\wedge}\lambda x.\underline{[\lambda P.(\exists y)[\mathbf{U}(y) \wedge [{}^{\vee}P](y)]]}({}^{\wedge}\lambda z.\mathbf{F}_*(x,z)))$

The underlined formula marks the contribution of the indefinite **a unicorn** to the infinitival complement in (34), which however does not coincide with its contribution to the entire predicate. This becomes clear if one tries to λ-abstract from it: the following formula is *not* equivalent to (37):

(38) $[\lambda\wp.\mathbf{T}({}^{\wedge}\lambda x.\wp({}^{\wedge}\lambda z.\mathbf{F}_*(x,z)))](\lambda P.(\exists y)[\mathbf{U}(y) \wedge [{}^{\vee}P](y)])$

As the reader is invited to verify, the main argument in (38) contains two invisible occurrences of i, which would be bound by the cap operator in the body of the λ℘-functor. To avoid this kind of variable clash, Montague frequently employed redundant reformulations in terms of Down-Up-Cancellation (25) – which also helps in the case at hand, turning (37) into:

(39) $\mathbf{T}({}^{\wedge}\lambda x.\ {}^{\vee}\underline{[{}^{\wedge}[\lambda P.(\exists y)[\mathbf{U}(y) \wedge [{}^{\vee}P](y)]]]}({}^{\wedge}\lambda z.\mathbf{F}_*(x,z)))$

Now the underlined functor of type <*s*,ƒ(T)>, which refers to the *in*tension of **a unicorn**, can be λ-abstracted, thereby isolating its contribution to the extension of the predicate **try to find a unicorn**:

(40) $\underline{[\lambda\wp.\mathbf{T}({}^{\wedge}\lambda x.[{}^{\vee}\wp]({}^{\wedge}\lambda z.\mathbf{F}_*(x,z)))]}({}^{\wedge}\lambda P.(\exists y)[\mathbf{U}(y) \wedge [{}^{\vee}P](y)])$

Since, by assumption, (40) also represents the extension of **seek a unicorn**, (c) the (underlined) main functor may be taken to be the contribution made by the intensional verb **seek**; and the construction of feeding it its (non-specific) object is interpreted by (intensional) functional application. Closer inspection of (40) reveals that the type of the underlined functor is <<s,ƒ(T)>,ƒ(IV)>, which thus features as the worst case for intensional

[34] The term 'propositionalism' originates with Forbes (2000), which marks the starting point of a still ongoing debate; cf. the contributions in Grzankowski & Montague (2018). – Lewis' account also raises the question of how to deal with clausal embedding as reporting attitudes towards properties; see Schlenker (2011) for a survey.

verbs. In the end, then, **seek** gets translated by a constant **S** of that type, which is postulated to be equivalent to the functor in (40):[35]

(41) $\mathbf{S} = \lambda\wp.\mathbf{T}(\hat{}\lambda x.[{}^{\vee}\wp](\hat{}\lambda z.\mathbf{F}_*(x,z)))$

In a nutshell, then, the *PTQ* treatment of **seek** is a compositional reformulation of Quine's (1956) paraphrase in terms of possible worlds semantics. Montague would not have put it that way, though, pointing out that his account of intensional transitives does not necessitate their reducibility by propositionalist paraphrase. However, as in the above case of control verbs, even in the absence of reduction postulates like (41), Montague's analysis of intensional transitives does not imply their irreducibility: contrary to Montague's (1969: 177) suggestions, an analysis without reduction is not an analysis of irreducibility but merely incomplete. Moreover, as Larson (2002) argued, given Montague's abstract analysis, the fact that clear cases of irreducibly higher-order verbs seem hard to find appears somewhat mysterious.[36]

Due to the non-specific reading of its object, **seek** represents the worst case of a transitive verb. On the specific reading though, it behaves like an ordinary, extensional verb. In *PTQ* this ambiguity is captured by the general scope mechanism $F_{10,n}$: as in the paraphrase (1b) above, the object **a unicorn** can be raised above the verb (and its referential subject), leaving a trace of type <*s*,*e*>. Hence, using the notation introduced in (7) for ordinary transitive verbs like **find**, the specific reading of (42) comes out as in (43), which in the presence of (41), is indeed equivalent to the reduced translation (44) [≡ (26)] of (1b):

(42) **John seeks a unicorn.**

(43) $(\exists y)[\mathbf{U}(y) \wedge \mathbf{S}_*(\hat{}\mathbf{j}, y)]$

(44) $(\exists y)[\mathbf{U}(y) \wedge \mathbf{T}(\hat{}\mathbf{j}, \hat{}\lambda x.\mathbf{F}_*(x,y))]$

Readings like (26) and (44), which are derived by raising a quantifying Term out of an intensional position (or, from a top-down perspective, *quantifying* it *into* that position), raise a vexing problem concerning the truth conditions. Due to the asymmetry of variables and constants in *IL*, the *PTQ* treatment of such constellations invariably predicts that the embedded proposition or property is *about the object* (= lat. *de* re) that the bound variable denotes, independently of how that object is described. When spelt out in possible worlds semantics and using Kripke's (1972) familiar term, this means that the variable acts as a *rigid designator*.[37] This assumption has been argued to be inadequate for various reasons; see, e.g., Kaplan (1968: 192ff.) or Lewis (1981). As it turns out, more adequate truth conditions for *de re* reports require a number of restrictions on the descriptive content contributed by the bound variable; see, e.g., Kaplan (1968: 197ff.), Lewis (1979: 538ff.), Aloni (2001: ch. 2), and many others. As a result, post-Montagovian semanticists have tried to implement these conditions in the semantics of *de re* reports, which has proven to be quite tricky; see, e.g., Cresswell & von Stechow (1982), Percus & Sauerland (2003), or Maier (2009).

---

[35] Actually, (41) merges the transparency postulate (4) for **find** and the reduction postulate (9) (of *PTQ*, p. 235); the latter is formulated as a bi-conditional rather than an equation. – According to Montague, the higher-order type <s,f(T)> corresponding to the object position of intensional transitives can also be found in the subject position of raising verbs like **appear**. Generalizing to the worst case, then, all verbs end up as denoting operators acting on the intensions of their quantifying subjects. Apparently for reasons of presentation, this complication of the type assignment was avoided in *PTQ* [cf. p. 222] – though not in Montague (1970b).

[36] Another problem with the *PTQ* type of intensional transitives was noticed in Zimmermann (1993): given its generality, non-specific readings are expected to also occur with non-existential quantifiers – contrary to fact. See Schwarz (to appear) for a survey of alternative analyses.

[37] One may have hoped that this descriptive content is taken care of since the variable ranges over individual concepts. However, for one thing, the sole function of that type in *PTQ* is to deal with the data discussed in Section 2.3.4; and for another thing, the flexibility needed to cover *de re* constellations exceeds that provided by locally bound individual concepts.

2.3.3 Adverbial Modification

In *PTQ* adverbs like **slowly** are in the syntactic category (IV/IV) of (potentially intensional) modifiers of verb phrases, which they share with PPs like **in London**. The latter are interpreted by having the preposition operate on the intension of its Term argument. Rather than going into the details of this approach to adverbial modification, which was further elaborated by Thomason & Stalnaker (1973), it is worth pointing out some of its shortcomings that eventually led to various proposals to seriously revise the framework of Montague Grammar by invoking event arguments, in the spirit of Davidson (1967) and Parsons (1990).

Since attitude predicates, including intensional transitives and control verbs, are supposed to have extensional subject positions, prepositions happen to be the only expressions in the *PTQ* fragment that may reveal double intensionality, a case in point being **about**. This seems well motivated. For one thing, the predicates in (45) and (46) are likely to differ in extension even though the underlined (non-specifically construed) indefinites do not:

(45) **talk about <u>a unicorn</u>**

(46) **talk about <u>a centaur</u>**

For another thing, even if the talkers and the thinkers are the same persons, those who think about a unicorn do not have to talk about a unicorn (again in the non-specific sense):

(47) **<u>talk</u> about a unicorn**

(48) **<u>think</u> about a unicorn**

As in previous cases, the intensionality of prepositions is only treated by omission: the absence of any transparency postulate for **about** blocks any extensional substitution. And again, neither does this treatment imply intensionality nor does it explain or describe just how the intensions of the arguments (Term and predicate) manage to determine the resulting extensions of the predicates in (45) – (48).

Moreover, the *PTQ* analysis of PPs as predicate modifiers encounters serious problems when it comes to apparently extensional prepositions like **in** (in its local reading), which, unlike **about**, is subject to a transparency postulate to the effect that its Term argument outscopes the VP argument, in analogy to (7) and (10):[38]

(49) $\mathbf{I}_{*} := [\lambda y.\lambda Q.\mathbf{I}(Q, y^{*})]$

(50) $\mathbf{I} = [\lambda\wp.\lambda Q.[{}^{\vee}\wp](^{\wedge}\lambda y.\mathbf{I}_{*}(Q, y))]$

To see why (50) cannot be the full story about the local preposition **in**, one may consider (51) whose *in situ* construal comes out as (52) (assuming a transparency postulate for **meet**):

(51) **John meets a woman in Paris.**

(52) $\mathbf{I}_{*}(^{\wedge}\lambda x.(\exists y)[\mathbf{W}(y) \wedge \mathbf{M}_{*}(x, y)], \mathbf{p}^{*})(^{\wedge}\mathbf{j})$

(52) roughly says that John's meeting of some (unspecific) woman took place in Paris, but does not imply that John did meet a (specific) woman in Paris; this is easily repaired by the following, apparently harmless veridicality postulate:

(53) $\mathbf{I}_{*}(Q, p)(x) \rightarrow [{}^{\vee}Q](x)$

According to (53), what is done in a specific place (by a specific individual), is done *simpliciter* (by that individual). Given (53), (52) does imply that John meets a woman and that his meeting of some woman takes place in Paris; but this does not mean that there is a woman

[38] For reasons unknown to the author, the relevant *PTQ* postulate (8) (on p. 235), which is again formulated in terms of a bi-conditional and does not make use of the $_{*}$-notation, does not reduce the predicate argument to a *set* but only to a *property*, albeit one of individuals. For simplicity, it has been left untouched in (50).

he meets in Paris. In other words, (52) does not imply (54); in fact, (52) is consistent with the negation of (54) even in the presence of (53):

(54) $(\exists y)[\mathbf{W}(y) \wedge \mathbf{I}_*(^\wedge\lambda x.\mathbf{M}_*(x,y),\mathbf{p})(^\wedge\mathbf{j})]$

The problem is that the local adverbial **in Paris** only seems to target the lexical verb **meet**, not its (existentially) quantifying object; and, as readers are invited to verify, this observation is independent of the kind of quantifier in object position. Again, it would seem that this gap could be closed, too; in fact, the following scope principle seems to do so:

(55) $\mathbf{I}_*(^\wedge\lambda x.[^\vee\wp](^\wedge\lambda y.[^\vee R](x,y),p))(x) = [^\vee\wp](^\wedge\lambda y.\mathbf{I}_*(^\wedge\lambda x.R(x,y),p)(x))$

Roughly, (55) says that a quantificational object of a transparent verb takes scope over a referential local PP that modifies it. As it turns out, however, (53) and (55) together imply (54), which says that predicate modifiers like **in Paris** are redundant – clearly an unwelcome result:[39]

(56) $\mathbf{I}_*(Q,p)(x) = [^\vee Q](x)$

The only known safe way to avoid this embarrassment is to interpret adverbial modifiers as predicates on event arguments introduced by lexical verbs.

2.3.4 The temperature paradox

The intensional constructions discussed so far have in common that they all operate on *predicative* (or *Boolean*, or *conjoinable*) *types*, i.e. types $<a_1,\ldots,<a_n,t>\ldots>$ that end in a t and may thus be construed as hosting relations between objects of (not necessarily predicative) types $a_1,\ldots,a_n$. One may wonder whether intensionality is more general than that and also occurs with arguments of non-predicative types, i.e. those that end in e (given that all Montagovian Types end in either e or t). According to Montague, this is indeed so: in *PTQ*, [p. 239], he describes intensionality in first-order predicates, i.e. those that may combine with referential expressions, as 'a kind of intensionality not previously observed by philosophers', crediting Barbara Partee for its discovery.[40] As already indicated, it is this type of intensionality that is responsible for the worst-case analyses of nouns and predicates as denoting sets of individual concepts rather than merely individuals. The following type of example (taken from *PTQ*, p. 239) motivates this decision:

(57) **The temperature is ninety.**

(58) **The temperature rises.**

(59) **Ninety rises.**

Like all Terms in *PTQ*, the definite descriptions in (57) and (58) are treated as generalized quantifiers and thus give rise to truth conditions à la Russell (1905). Moreover, following Quine (1960: 118), Montague decided to interpret the copula verb **to be** uniformly as denoting identity. If, in addition, the extensions of predicates and nouns were sets of individuals, (57) and (58) would jointly entail (59). To escape this absurdity, Montague proposed to interpret the noun **temperature** as denoting a set of *individual concepts* – functions whose (numerical) values change across points of evaluation, thus in particular reflecting fluctuations over time. By the same token, the verb **rise** would have only certain

[39] See Zimmermann (1987), where Engesser (1980) is credited with the basic observations on adverbial modification in *PTQ* and (56) is proved for **slowly** in lieu of **in Paris**.

[40] Partee (2004: 17) reminisces: 'I first gave this example [i.e., (57)–(59), TEZ] […] to David Lewis […] He had stated (Lewis 1970) that intransitive verbs are never intensional […] So I […] came up with examples about the price of milk rising or changing and wondered why they didn't count as intensional. And David told them to Montague […]'. The example may have had an influence on Montague's decision to give up the Russellian approach to types advocated in Montague (1970a) in favor of a Fregean system – which would however have been premature in view of later equivalence proofs: see Kaplan (1975), Muskens (1989), and Liefke (2015).

such functions in its extension[41] – but crucially no constant functions like the one denoted by **ninety**. As a consequence, (58) may be true while (59) comes out false. In order for (57) to also be true under the same circumstances, it would not express identity between individual concepts, but rather identity between their values (at a given point). The following *IL*-translation of the copula achieves this [*PTQ*, p. 233: T1.(b)]:

(60) $\lambda\wp.\lambda x.[{}^{\vee}\wp](^\wedge\lambda y.[{}^{\vee}x] = [{}^{\vee}y])$

(60) generalizes the binary relation of value equivalence (between individual concepts) to the worst-case type of **seek**; in this way, the copula can be treated as an extensional transitive verb (for better or for worse). Given this construal and the Russellian approach to definite descriptions, (57) – (59) come out as follows:

(61) $(\iota y.\mathbf{T}(y))\ [{}^{\vee}y] = \mathbf{n}$

(62) $(\iota y.\mathbf{T}(y))\ \mathbf{R}(y)$

(63) $\mathbf{R}(^\wedge\mathbf{n})$

In the interest of readability, (61) and (62) make use of a common abbreviatory convention not found in *PTQ*:

(64) $(\iota y{:}\ \varphi)\ \psi := (\exists y)[(\forall x)[[\lambda y.\varphi](x) \leftrightarrow x = y] \wedge \psi]$

(64) expresses the Russellian truth condition that there is precisely one individual concept y that satisfies φ and that every (or equivalently: some) y satisfying φ also satisfies ψ.

In (61) and (63), $\mathbf{n}$ is a constant of type *e*, whose extension is the value 90 at any point of evaluation; as a consequence, [$^\wedge\mathbf{n}$] denotes a constant function, thus falsifying (63) throughout any decent model. However, for (61) and (62) to be true at one point, the (unique) temperature curve only needs to take the value 90 at a point and still increase. Such models, then, provide counter-examples to the alleged inference (57) – (59) if construed along the lines of (61) – (63).

As repeatedly pointed out above, in *PTQ* individual concepts are required for the extensions of a few nouns (viz. **temperature** and **price**) as well as the verb **rise** which display the inference behavior observed in (57) – (59). For all 'ordinary' nouns and verbs, sets of individuals would have sufficed as extensions. However, due to Montague's strategy of generalizing to the worst case, the latter receive the extension type <<*s*,*e*>,*t*>, too. Interestingly, though, extensional nouns and verbs do not come out alike in *PTQ*. First of all, a closer look at the interpretation of the definite description **the temperature** reveals that the Russellian uniqueness condition concerns individual concepts, not individuals: the noun needs to have a singleton extension whose member must be in the extension of **rise**, which in turn only contains individual concepts (by the very idea of the *PTQ* approach to the puzzle). For 'ordinary' nouns like **woman** that get lifted from (extension) type <*e*,*t*> to <<*s*,*e*,>,*t*>, this means that each individual in the original extension needs to be represented by precisely one individual concept in the type-lifted version; otherwise the uniqueness condition would not carry over. To achieve this, each individual u gets represented by the constant (or rigid) function that assigns *u* to any point of evaluation; thence the following schematic meaning postulate (*PTQ*, p. 235, (2)):

(65) $[\delta(x) \rightarrow (\exists u)\ x = {}^\wedge u]$,

where δ translates an 'ordinary' noun and x and u are variables of types <*s*,*e*> and *e*, respectively. So sets of individuals get represented by corresponding sets of rigid intensions. However, this representation does not work for 'ordinary' intransitive verbs like **sleep**, which also need to be type-adjusted from <*e*,*t*> to <<*s*,*e*,>,*t*>. For otherwise prices and

[41] To wit, those whose value at a given point of reference increases at that point. Moreover, the place, which is not part of the point of evaluation, needs to be fixed to make the point about (57) – (59). Such obvious details have been omitted from the *PTQ* analysis.

temperatures could never have mundane properties, like being even or, indeed, identical to 90. As a consequence, the lifted predicate extensions need to contain all individual concepts whose value at a given point is in the original set of individuals. Again, this feature is captured by a postulate schema:[42]

(66) $[\ {}^{\vee}x = {}^{\vee}y \rightarrow [\delta(x) \leftrightarrow \delta(y)]]$,

where δ translates an 'ordinary' verb and x and y are variables of type <*s*,*e*>. In particular, then, in *PTQ* two distinct roads from sets of individuals to sets of individual concepts co-exist: 'strategies of intensionalization' (in van Benthem's 1988 words) need not be uniform – *pace* de Groote & Kanazawa (2013). Still, as pointed out in *PTQ* (p. 236), the original set δ_* of individuals can be uniformly retrieved from an extension δ, no matter whether it satisfies (65) or (66):

(67) $\delta_* = \lambda u.\,\delta({}^{\wedge}u)$

A further strange feature of the *PTQ* solution to Partee's temperature puzzle had apparently escaped its author: the *double intensionality* of nouns like **temperature** – both their intensions and the elements of their extensions abstract from the Logical Space of evaluation points – leads to the unwelcome consequence that the inference from (68) and (69) to (70) is not predicted to be valid, as argued by Anil Gupta and reported in Dowty *et al.* (1981: 284f.):

(68) **Necessarily the temperature is the price.**

(69) **The temperature rises.**

(70) **The price rises.**

The source of this gap has been located in the intensional overkill indicated (Hansen 2016): since the temperatures and price values are already fully recorded within the extension of the nouns, their intension should not vary with the points of evaluation but rather needs to be rigid, thus in effect dislocating the original variation of the extension from the entire set to its individual members. The shift is possible because of the *functionality* of the noun in question: at any given point there is precisely one (relevant) temperature or price; on the other hand, in view of a number of related phenomena, it seems unlikely that it is lexically based, as the *PTQ* analysis would have it (Janssen 1984, Schwager 2007).

Acknowledgment
I would like to thank Kristina Liefke, Louise McNally, Jürgen Pafel Zoltán Szabó, and Dina Voloshina for detailed comments and corrections that helped to improve the quality of this article. – In December 2018, while I was working on it, I learnt the sad news that Theo Janssen passed away. Theo's pioneering work on Montague Grammar had helped shaping my own views on the subject to a considerable extent. I therefore dedicate this piece to his memory.

References

Ajdukiewicz, Kazimierz (1935): 'Die syntaktische Konnektizität'. *Studia Philosophica* **1**, 1 – 27.

Aloni, Maria (2001): *Quantification under Conceptual Covers*. University of Amsterdam PhD dissertation.

Bar-Hillel, Yehoshua (1953): 'A Quasi-Arithmetical Notation for Syntactic Description'. *Language* **29**, 47 – 58.

Barwise, Jon; Cooper, Robin (1981): 'Generalized Quantifiers and Natural Language'. *Linguistics and Philosophy* **4**, 159 – 219.

Caponigro, Ivano (to appear): *Richard Montague: The Simplicity of Language, the Complexity of Life*. Oxford.

Carnap, Rudolf (1947): *Meaning and Necessity*. Chicago.

Chierchia, Gennaro (1989): 'Anaphora and Attitudes *De Se*'. In: R. Bartsch *et al.* (eds.), *Semantics and Contextual Expression*. Dordrecht: 1 – 32.

Cresswell, Maxwell J. (1990): *Entities and Indices*. Dordrecht.

[42] In *PTQ* (p. 235, (3)), a slightly different but equivalent postulate is given: $(\exists M)\ (\forall x)\ \Box[\delta(x) \leftrightarrow [\ {}^{\vee}M](\ {}^{\vee}x)]$.

Cresswell, Maxwell J.; Stechow, Arnim von (1982): '*De Re* Belief Generalized'. *Linguistics and Philosophy* **5**, 503 – 535.
Davidson, Donald (1967): 'The logical form of action sentences'. In: N. Rescher (ed.), *The Logic of Decision and Action*. Pittsburgh. 81 – 95.
de Groote, Philippe; Kanazawa, Makoto (2013): 'A Note on Intensionalization '. *Journal of Logic, Language and Information* **22**, 173 – 194.
den Dikken, Marcel; Larson, Richard; Ludlow, Peter (2018): 'Intensional Transitive Verbs and Abstract Clausal Complementation'. In: A. Grzankowski & M. Montague (2018): 46 – 94.
Dowty, David; Wall, Robert; Peters, Stanley (1981): *Introduction to Montague Semantics*. Dordrecht.
Engesser, Kurt (1980): *Untersuchungen zur Montaguegrammatik*. University of Konstanz PhD dissertation.
Fine, Kit (1975): 'Some Connections Between Elementary and Modal Logic'. *Studies in Logic and the Foundations of Mathematics* **82**, 15 – 31.
Forbes, Graeme (2000): 'Objectual Attitudes'. *Linguistics and Philosophy* **23**, 141 – 183.
Frege, Gottlob (1879): *Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens*. Halle.
– (1884): *Die Grundlagen der Arithmetik*. Breslau.
– (1891): *Function und Begriff*. Jena.
– (1892): 'Über Sinn und Bedeutung'. *Zeitschrift für Philosophie und philosophische Kritik* (NF) **100**, 25-50.
Gallin, Daniel (1971): 'Systems of intensional logic'. *Journal of Symbolic Logic* **36**, 585.
– (1975): *Intensional and Higher-order Modal Logic*. Amsterdam.
Geach, Peter T. (1962): *Reference and Generality*. Ithaca, New York.
– (1970): 'A Program for Syntax'. *Synthèse* **22**, 3–17.
Grzankowski, Alex; Montague, Michelle (eds.) (2018): *Non-Propositional Attitudes*. Oxford.
Hansen, Casper Storm (2016): 'The Temperature Paradox and Russell's Analysis of the Definite Determiner'. *Linguistic Inquiry* **47**, 695 – 705.
Heim, Irene (1982): *The Semantics of Definite and Indefinite Noun Phrases*. University of Massachusetts at Amherst PhD dissertation.
Heim, Irene; Kratzer, Angelika (1998): *Semantics in Generative Grammar*. Oxford.
Hendriks, Herman (1990): *Flexible Montague Grammar*. University of Amsterdam PhD dissertation.
Higginbotham, James (2003): 'Remembering, imagining, and the first person.' In: A. Barber (ed.), *Epistemology of Language*. Oxford: 496 – 533.
Janssen, Theo (1983): *Foundations and Applications of Montague Grammar*. University of Amsterdam PhD dissertation.
– (1984): 'Individual Concepts are Useful'. In: F. Landman & F. Feldman (eds.), *Varieties of Formal Semantics*. Dordrecht: 171 – 192.
Kamp, Hans (1981): 'A Theory of Truth and Semantic Representation'. In: J. Groenendijk *et al.* (eds.), *Formal Methods in the Study of Language. Part 1*. Amsterdam: 277 – 322.
Kaplan, David (1968): 'Quantifying in'. *Synthèse* **19**, 178 – 214.
– (1975): 'How to Russell a Frege-Church'. *Journal of Philosophy* **72**, 716 – 729.
– (1989a): 'Demonstratives'. In: J. Almog *et al.* (eds.), *Themes from Kaplan*. Oxford: 481 – 563.
– (1989b): 'Afterthoughts'. In: J. Almog et al. (eds.), *Themes from Kaplan*. Oxford: 565 – 614.
Keenan, Edward L.; Stavi, Jonathan (1986): 'A Semantic Characterization of Natural Language Determiners'. *Linguistics and Philosophy* **9**, 253 – 326.
Klein, Ewan; Sag, Ivan (1985): 'Type-Driven Translation'. *Linguistics and Philosophy* **8**, 163 – 201.
Köpping, Jan; Zimmermann, Thomas Ede (to appear): 'Looking backwards in type logic'. *Inquiry*.
Kripke, Saul (1972): 'Naming and Necessity'. In: D. Davidson & G. Harman (eds.), *Semantics of Natural Language*. Dordrecht: 253 – 355.
Lambek, Joachim (1958): 'The Mathematics of Sentence Structure'. *American Mathematical Monthly* **65**, 154 – 170.
Larson, Richard (2002): 'The Grammar of Intensionality'. In: G. Preyer & G. Peter (eds.), *Logical Form and Language* Oxford. 228 – 262.
Lewis, David K. (1970): 'General Semantics'. *Synthèse* **22**, 18–67.
– (1975): 'Adverbs of Quantification'. In: E. L. Keenan (ed.), *Formal Semantics of Natural Language*. Cambridge: 3 – 15.
– (1979): 'Attitudes *de dicto* and *de se*'. *Philosophical Review* **88**, 513 – 543

– (1981): 'What Puzzling Pierre Does Not Believe'. *Australasian Journal of Philosophy* **59**, 283 – 289.
Liefke, Kristina (2015): 'Codability and Robustness in Formal Natural Language Semantics'. In: T. Murata *et al.* (eds.), *New Frontiers in Artificial Intelligence*. Berlin & Heidelberg: 6 – 22.
Link, Godehard (1979): *Montague-Grammatik*. Munich.
Mador-Heim, Sela; Winter, Yoad (2015): 'Far from obvious: the semantics of locative indefinites'. *Linguistics and Philosophy* **38**, 437 – 476.
Maier, Emar (2009): 'Iterated *de re*: A New Puzzle for the Relational Report Semantics'. In: A. Riester & T. Solstad (eds.), *Proceedings of Sinn und Bedeutung 13*. University of Stuttgart: 347 – 355.
May, Robert (1977): *The Grammar of Quantification.* Massachusetts Institute of Technology PhD dissertation.
Montague, Richard (1969): 'On the Nature of Certain Philosophical Entities'. *The Monist* **53**, 159 – 195. [Reprinted in Thomason 1974: 148 – 187]
– (1970a): 'English as a Formal Language'. In: B. Visentini (ed.), *Linguaggi nella società e nella tecnica*. Milan: 189 – 223. [Reprinted in Thomason 1974: 188 – 221]
– (1970b): 'Universal Grammar'. *Theoria* **36**, 373 – 398. [Reprinted in Thomason 1974: 222 – 246]
– (1970c): 'Pragmatics and Intensional Logic'. *Synthèse* **22**, 68 – 94. [Reprinted in Thomason 1974: 119 – 147]
– (1973) [Cited as *PTQ*]: 'The Proper Treatment of Quantification in Ordinary English'. In: J. Hintikka *et al.* (eds.), *Approaches to Natural Language*. Dordrecht: 221 – 242. [Reprinted in Thomason 1974: 247 – 270]
Muskens, Reinhard (1989): *Meaning and Partiality*. University of Amsterdam PhD dissertation.
Parsons, Terence (1990): *Events in the semantics of English. A study in subatomic semantics*. Cambridge, Mass.
Partee, Barbara (1975): 'Montague Grammar and Transformational Grammar'. *Linguistic Inquiry* **6**, 203 – 300.
– (1997): 'Montague Grammar' [with Herman Hendriks]. In: J. van Benthem & A. ter Meulen (eds.), *Handbook of Logic and Language*. Amsterdam: 5 – 91.
– (2004): 'Reflections of a Formal Semanticist '. In: B. Partee (ed.), *Compositionality in Formal Semantics*. 2004. 1-25.
– (2011): 'Formal Semantics: Origins, Issues, Early Impact'. *Baltic International Yearbook of Cognition, Logic and Communication* **6**, 1 – 52.
Partee, Barbara; Rooth, Mats (1983): 'Generalized Conjunction and Type Ambiguity'. In: R. Bäuerle *et al.* (eds.), *Meaning, Use, and Interpretation of Language*. Berlin: 361–383.
Percus, Orin; Sauerland, Uli (2003): 'On the LFs of Attitude Reports'. In: M. Weisgerber (ed.), *Proceedings of Sinn und Bedeutung 7.* University of Konstanz: 228 – 242.
Putnam, Hilary (1975): 'The meaning of "meaning"'. In: K. Gunderson (ed.), *Language, Mind, and Knowledge*. Minneapolis: 131–193.
Quine, Willard Van Orman (1956): 'Quantifiers and Propositional Attitudes'. *Journal of Philosophy* **53**, 177 – 187.
– (1960): *Word and Object*. Cambridge, Mass. 1960.
Reinhart, Tanya (1983): *Anaphora and semantic interpretation*. Chicago.
Russell, Bertrand (1905): 'On Denoting'. *Mind* **14**, 479 – 493.
Saussure, Ferdinand de (1916): *Cours de linguistique générale*. Edited by C. Bally and A. Sechehaye. Lausanne & Paris. [Cited after W. Baskin's translation: *Course in General Linguistics*, New York 1959]
Schlenker, Philippe (2011): 'Indexicality and *De Se* Reports '. In: K. v. Heusinger *et al.* (eds.), *Semantics. An international Handbook of Natural Language Meaning*. *Volume 2*. Berlin: 1561 – 1604. [Revised paperback edition 2019]
Schwager, Magda (2007): 'Bodyguards Under Cover: The Status of Individual Concepts'. In: M. Gibson & T. Friedman (eds.), *Proceedings of Semantics and Linguistic Theory 17*. Ithaca, NY: 246–263.
Schwarz, Florian (to appear): 'Intensional Transitive Verbs: *I owe you a horse*'. In: D. Gutzmann *et al.* (eds.), *The Wiley Blackwell Companion to Semantics*. Oxford.
Stechow, Arnim von (1979): 'Deutsche Wortstellung und Montague-Grammatik'. In: J. M. Meisel & M. D. Pam (eds.), *Current Issues in Linguistic Theory, Vol. 7, Linear Order and Generative Theory*. Amsterdam: 327 – 490.
Tarski, Alfred (1986): 'What are logical notions?'. *History and Philosophy of Logic* **7**, 143 – 154.
Thomason, Richmond (ed.) (1974): *Formal Philosophy. Selected Papers of Richard Montague.* New Haven.
– (1980): 'A Model Theory for Propositional Attitudes'. *Linguistics and Philosophy* **4**, 47 – 70.
Thomason, Richmond; Stalnaker, Robert (1973): 'A Semantic Theory of Adverbs'. *Linguistic Inquiry* **4**, 195 – 220.
van Benthem, Johan (1988): 'Strategies of intensionalization'. In: I. Bodnár *et al.* (eds.), *Intensional Logic, History of Philosophy, and Methodology*. Department of Symbolic Logic, Eötvös University, Budapest: 41 – 59.
Wehmeier, Kai (2018): 'On deixis, pronouns, and variables'. Ms. University of California, Irvine.

Zimmermann, Thomas Ede (1983): 'Notes on a Recent Textbook in Semantics'. *Theoretical Linguistics* **10**, 65 – 79.
– (1985): 'A Note on Transparency Postulates'. *Journal of Semantics* **4**, 67–77.
– (1987): 'Transparent Adverbs and Scopeless Quantifiers'. In: J. Groenendijk *et al.* (eds.), *Foundations of Pragmatics and Lexical Semantics*. Dordrecht: 81 – 99.
– (1989): 'Intensional logic and two-sorted type theory'. *Journal of Symbolic Logic* **54**, 65 – 77.
– (1993): 'On the Proper Treatment of Opacity in Certain Verbs'. *Natural Language Semantics* **1**, 149 – 179.
– (2017): 'Quantification over alternative intensions'. *Semantics and Pragmatics* **10**.
– (to appear): 'Representing Intensionality: Variables vs. Parameters'. In: D. Gutzmann *et al.* (eds.), *The Wiley Blackwell Companion to Semantics*. Oxford.
Zimmermann, Thomas Ede; Sternefeld, Wolfgang (2013): *Introduction to Semantics*. Berlin.