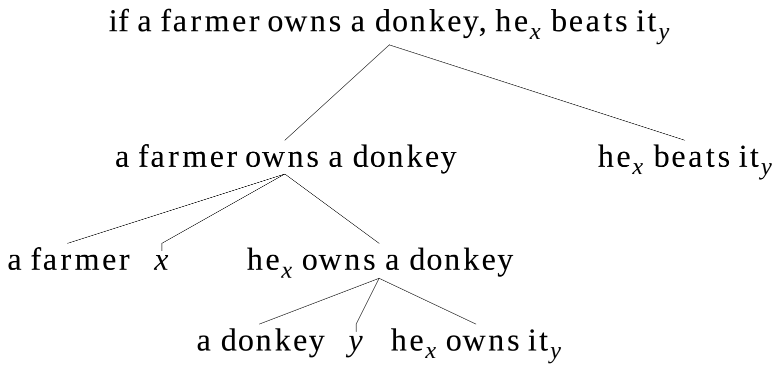
1. From a classical point of view

a. Conditional donkeys

(1) If a farmer owns a donkey, he beats it.

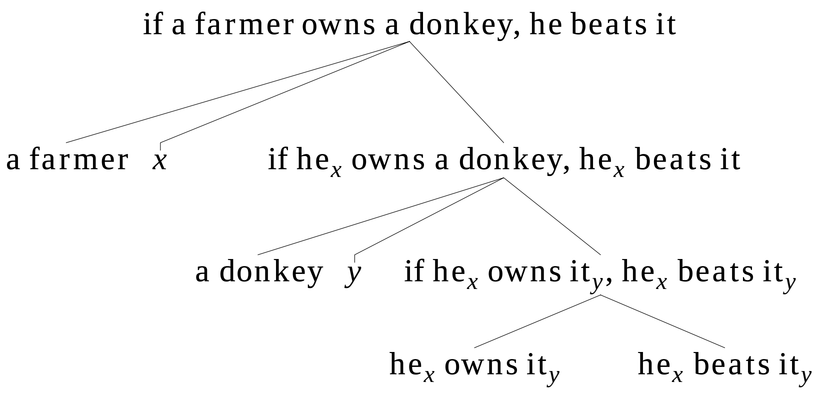
(2) $(\forall x)\ (\forall y)\ [\ [x \text{ is a farmer } \&\ y \text{ is a donkey } \&\ x \text{ owns } y] \rightarrow x \text{ beats } y\ ]$

(3)



(3') $(\ (\exists x)\ \underline{(\exists y)\ [\ \ \underline{\underline{\text{farmer}'(x)\ \&\ \text{donkey}'(y)\ \&\ \text{own}'(x,y)}}}\ ] \Rightarrow \text{beat}'(\boldsymbol{x},\boldsymbol{y})\ )$
['If a farmer owns a donkey, then *x* owns *y*'**]**

(4)



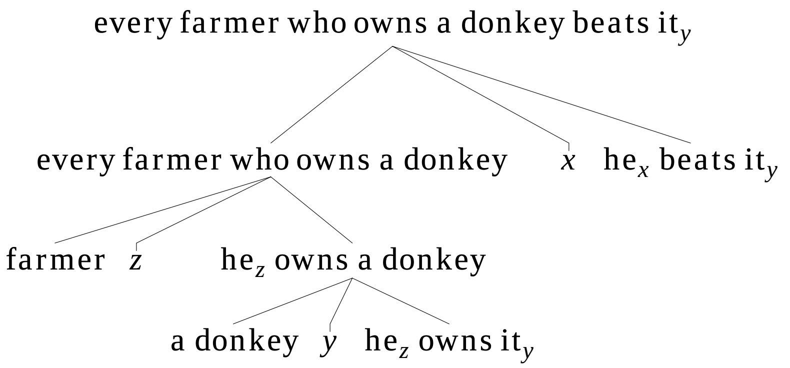
(4') $(\exists x)\ (\exists y)\ [\ \text{farmer}'(x)\ \&\ \text{donkey}'(y)\ \&\ (\text{own}'(x,y) \Rightarrow \text{beat}'(x,y)\ )\ ]$
['A certain farmer beats a certain donkey, if he owns it.'**]**

(5) $^{?}$If a farmer owns every/each donkey, he beats it.

b. Relative donkeys

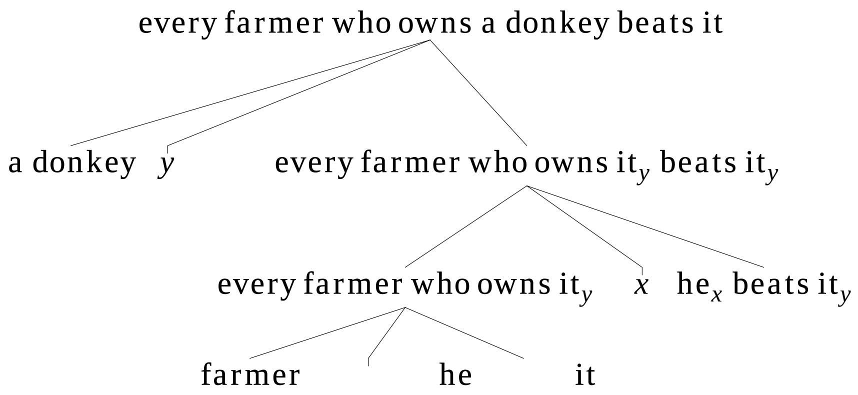
(6) Every farmer who owns a donkey beats it.

(7)



(7') $(\forall x)\ (\ [\ farmer'(x)\ \&\ (\exists y)\ [\ \underline{donkey'(y)\ \&\ own'(x,y)}\ ]\ ]\rightarrow beat'(x,\mathbf{y})\ )$
['Every farmer who owns a donkey beats *y*'**]**

(8)



(8') $(\exists y)\ [\ donkey'(y)\ \&\ (\forall x)\ (\ [\ farmer'(x)\ \&\ own'(x,y)\ ]\ \rightarrow beat'(x,y)\ )\ ]$
['Every farmer who owns a certain donkey beats it'**]**

(9) ?Every farmer who owns every/each donkey beats it.

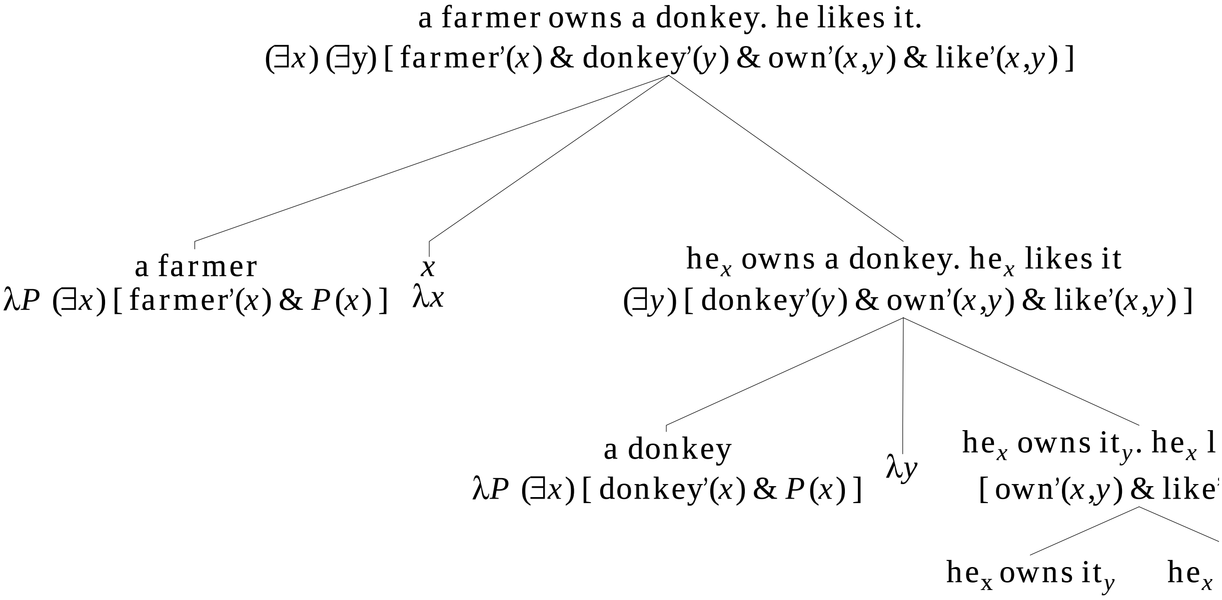
2. Discourse Anaphora

(10) A farmer owns a donkey. He likes it.

(11)

$$\underbrace{(\exists x)\,(\exists y)}_{?}\underbrace{[\text{farmer'}(x)\ \&\ \text{donkey'}(y)\ \&\ \text{own'}(x,y)}_{\textbf{(a)}}\ \&\ \underbrace{\text{like'}(x,y)]}_{\textbf{(b)}}$$

(12)



(13) ?Every farmer owns a donkey. He likes it.

(14) The man who gave his paycheck to his wife was wiser than the one who gave it [i.e., ***his*** pacheck] to his mistress.

(15) **(a)** A boy owns a guinea-pig.
(b) He [i.e., ***the*** boy who owns guinea-pig] likes it [i.e., ***the*** guinea-pig that he, the boy who owns guinea-pig, owns].

(a') $(\exists x)\ (\exists y)\ [\ \text{boy'}(x)\ \&\ \text{guinea-pig'}(y)\ \&\ (\text{own'}(x,y))\ ]$.

(b') $\text{like'}(\ (\iota x)\ [\ \text{boy'}(x)\ \&\ (\exists y)\ [\text{guinea-pig'}(y)\ \&\ (\text{own'}(x,y))]\ ],$
$(\iota y)\ [\text{guinea-pig'}(y)\ \&\ (\exists x)\ [\text{boy'}(x)\ \&\ (\text{own'}(x,y))]\ ]\ )$

(16) **(a)** A farmer {rides on a bicycle / cycles} .

(b) It {[the bicycle that the farmer who rides on a bicycle rides on] / [the bicycle that the cycling farmer rides on]} does not belong to him {[the farmer who rides on a bicycle] / [the cycling farmer]} .

(17) **(a)** A farmer rides on a bicycle.

(b) It does not belong to him.

(a') $\lambda R\ (\exists x)\ (\exists y)\ [\text{farmer}'(x)\ \&\ \text{bicycle}'(y)\ \&\ \text{ride-on}'(x,y)\ \&\ \underline{R(x,y)}]$

(b') $\hat{x}\ \hat{y}\ [\neg\ \text{belong}'(y,x)\ \&\ R(x,y)\ ]$

(18) **(a)** A farmer cycles.

(b) It does not belong to him.

(a') $(\exists x)\ (\exists y)\ [\text{farmer}'(x)\ \&\ \text{bicycle}'(y)\ \&\ \text{ride-on}'(x,y)\ \&\ \underline{R(x)}]$

(b') $\hat{x}\ \hat{\boldsymbol{y}}\ [\neg\ \text{belong}'(\boldsymbol{y},x)\ \&\ R(x)\ ]$

(17') **(a)** $\hat{x}\ \hat{y}\ [\text{farmer}'(x)\ \&\ \text{bicycle}'(y)\ \&\ \text{ride-on}'(x,y)\ ]$

(b) $\hat{x}\ \hat{y}\ \neg\ \text{belong}'(y,x)$

(c) $\hat{x}\ \hat{y}\ [\text{farmer}'(x)\ \&\ \text{bicycle}'(y)\ \&\ \text{ride-on}'(x,y)\ \&\ \neg\ \text{belong}'(y,x)]$

(18') **(a)** $\hat{x}\ (\exists y)\ [\text{farmer}'(x)\ \&\ \text{bicycle}'(y)\ \&\ \text{ride-on}'(x,y)]$

(b) $\hat{x}\ \hat{\boldsymbol{y}}\ \neg\ \text{belong}'(\boldsymbol{y},x)$

(c) $\hat{x}\ \hat{\boldsymbol{y}}\ [\underline{(\exists y)}\ [\text{farmer}'(x)\ \&\ \text{bicycle}'(\underline{y})\ \&\ \text{ride-on}'(x,\underline{y})\ ]\ \&\ \neg$ $\text{belong}'(\boldsymbol{y},x)$

(19) **(a)** A man loves a woman. He kisses her.

(b) A man loves a woman. A man kisses her.

3. Adverbs of Quantification

(20)

If a farmer owns a donkey, he {always / sometimes / never / …} beats it.

(21)

$$\left\{\begin{matrix} (\forall x)\,(\forall y) \\ (\exists x)\,(\exists y) \\ \neg\,(\exists x)\,(\exists y) \\ \dots \end{matrix}\right\} \;(\,[x \text{ is a farmer } \& \; y \text{ is a donkey } \& \; x \text{ owns } y] \quad \left\{\begin{matrix} \rightarrow \\ \& \\ \& \\ \dots \end{matrix}\right\} \quad x \text{ beats } y\,)$$

(21')

$$\left\{\begin{matrix} \forall xy \\ \exists x\text{y} \\ \neg\exists\, xy \\ \dots \end{matrix}\right\} \;(\,[x \text{ is a farmer } \& \; y \text{ is a donkey } \& \; x \text{ owns } y] \quad \left\{\begin{matrix} \rightarrow \\ \& \\ \& \\ \dots \end{matrix}\right\} \quad x \text{ beats } y\,)$$

(21'')

$$\left\{\begin{matrix} \forall \\ \exists \\ \neg\,\exists \\ \dots \end{matrix}\right\} \;(\hat{x}\,\hat{y} \quad ([x \text{ is a farmer } \& \; y \text{ is a donkey } \& \; x \text{ owns } y] \quad \left\{\begin{matrix} \rightarrow \\ \& \\ \& \\ \dots \end{matrix}\right\} \quad x \text{ beats } y))$$

(22) If a farmer owns a donkey, he usually beats it.

(23)

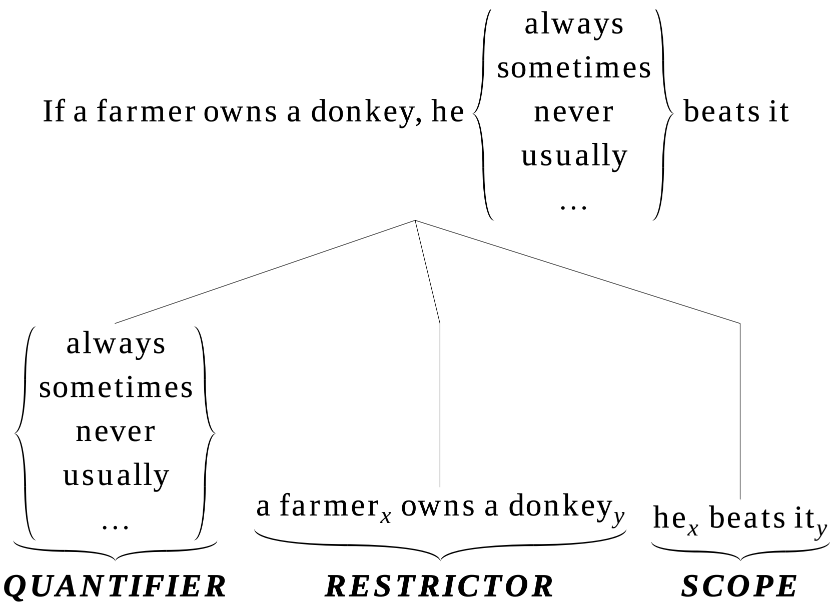
$$\left\{\begin{matrix} \forall \\ \exists \\ \neg\,\exists \\ \text{MOST} \\ \dots \end{matrix}\right\} \;(\hat{x}\,\hat{y} \quad x \text{ is a farmer } \& \; y \text{ is a donkey } \& \; x \text{ owns } y, \qquad \hat{x}\,\hat{y} \quad x \text{ beats } y)$$

(24)

$$\text{If a boy draws a picture of a girl, he} \quad \left\{\begin{matrix} \text{always} \\ \text{sometimes} \\ \text{never} \\ \text{usually} \\ \dots \end{matrix}\right\} \quad \text{gives it to her.}$$

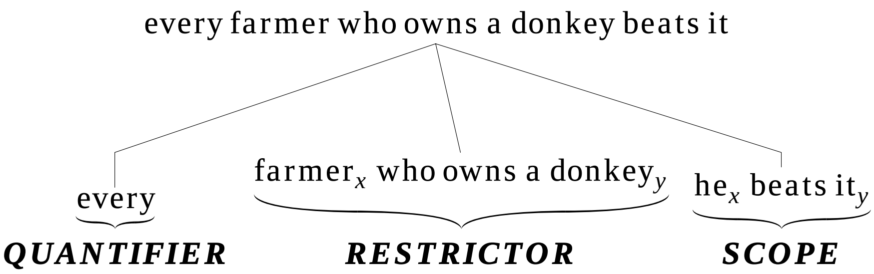
(25) If $\varphi(\text{a } N_1,\dots,\text{a } N_n)$, [then] ADV $\psi(\text{it}_1,\dots,\text{it}_n) \mapsto$
ADV'($\hat{x}_1,\dots,\hat{x}_n$ [x_1 is a N_1 & … & x_n is a N_n & $\varphi'(x_1,\dots,x_n)$], $\hat{x}_1,\dots,\hat{x}_n$
$\psi'(x_1,\dots,x_n)$)

(26)



(27) a farmer$_x$ owns a donkey$_y$ $\mapsto$ [farmer'(x) & donkey'(y) & own'(x,y)]
a farmer$_x$ $\mapsto$ farmer'(x) ('Indefinites as variables')
a farmer owns a donkey $\mapsto$ [(farmer' $\times$ donkey') $\cap$ own']
a farmer$_x$ $\mapsto$ farmer' ('Indefinites as properties')

(28)



4. Asymmetries

(29) Most farmers who own a donkey beat it.
(29') MOST($\hat{x}\,\hat{y}$ farmer'(x) & donkey'(y) & own'(x,y), $\hat{x}\,\hat{y}$ beat'(x,y))

(30) Every person who has a dime will put it in the meter.

(30') $\forall\, (\hat{x}\,\hat{y}\ \ \text{person}'(x)\ \&\ \text{dime}'(x)\ \&\ \text{have}'(x,y)\ \&\ \text{put-in-the-meter}'(x,y))$

[= $(\forall x)\ (\forall y)\ (\ [\text{person}'(x)\ \&\ \text{dime}'(x)\ \&\ \text{have}'(x,y)]\ \rightarrow$
$\text{put-in-the-meter}'(x,y))$

$=$

$(\forall x)\ (\ [\text{person}'(x) \rightarrow$

$\underline{(\forall y)\ (\ [\ \text{dime}'(x)\ \&\ \text{have}'(x,y)]\ \ \rightarrow}\ \text{put-in-the-meter}'(x,y)\ ]\ )$ **]**

(30'') $(\forall x)\ (\ (\exists y)\ [\text{person}'(x)\ \&\ \text{dime}'(x)\ \&\ \text{have}'(x,y)] \rightarrow$
$\underline{(\exists y)\ [\text{person}'(x)\ \&\ \text{dime}'(x)\ \&\ \text{have}'(x,y)\ \&}\ \text{put-in-the-meter}'(x,y)\ ]\ )$

(31) $\text{every}^{n}_{\textbf{universal}}{}' =$
$\lambda R\ \lambda S\ (\forall x)\ (\forall y_2)\ \ldots\ (\forall y_n)\ [\ R(x,y_2,\ldots,y_n) \rightarrow S(x,y_2,\ldots,y_n)\ ]$

(32) $\text{every}^{n}_{\textbf{existential}}{}' =$
$\lambda R\ \lambda S\ (\forall x)\ [(\exists y_2)\ \ldots\ (\exists y_n)\ \text{R}(x,y_2,\ldots,y_n) \rightarrow$
$(\exists y_2)\ldots(\exists y_n)\ [R(x,y_2,\ldots,y_n)\ \&\ S(x,y_2,\ldots,y_n)\ ]\ ]$

(33) Most persons who have a dime will put it in the meter.

(33') $\text{MOST}\ (\ \hat{x}\ \ (\exists y)\ [\text{person}'(x)\ \&\ \text{dime}'(x)\ \&\ \text{have}'(x,y)]\ ,$
$\hat{x}\ \ (\exists y)\ [\text{person}'(x)\ \&\ \text{dime}'(x)\ \&\ \text{have}'(x,y)\ \&\ \text{will-put-in-the-meter}'(x,y))\ ]\ )$

(34) $\text{EXISTENTIAL}(Q^{n}) =$
$\lambda R\ \lambda S\ (Qx)\ ((\exists y_2)\ldots(\exists y_n)\ R(x,y_2,\ldots,y_n)\ ,$
$(\exists y_2)\ldots(\exists y_n)\ \ [R(x,y_2,\ldots,y_n)\ \&\ S(x,y_1,\ldots,y_n)\ ])$

[= $\lambda R\ \lambda S\ Q(\hat{x}\ \ (\exists y_2)\ldots(\exists y_n)\ R(x,y_2,\ldots,y_n)\ ,$
$\hat{x}\ \ (\exists y_2)\ldots(\exists y_n)\ [R(x,y_2,\ldots,y_n)\ \&\ S(x,y_1,\ldots,y_n)]\ )$**]**

(35) Every farmer who owns a donkey beats it.

(35') $(\forall x)\ (\ (\exists y)\ [\text{farmer}'(x)\ \&\ \text{donkey}'(x)\ \&\ \text{have}'(x,y)] \rightarrow$
$(\exists y)\ [\text{farmer}'(x)\ \&\ \text{donkey}'(x)\ \&\ \text{have}'(x,y)\ \&\ \text{beat}'(x,y)\ ]\ )$

(36) Most people that owned a slave also owned his offspring.

(36') $\text{MOST}(\ \hat{x}\ \hat{y}\ \ \ [\text{person}'(x)\ \&\ \text{slave}'(y)\ \&\ \text{own}'(x,y)],$
$\hat{x}\ \hat{y}\ \ \ [\text{person}'(x)\ \&\ \text{slave}'(y)\ \&\ \text{own}'(x,y\text{'s offspring})\ ]\ )$

(37) $\text{MOST}(\ \hat{x}\ \ (\exists y)\ [\text{person}'(x)\ \&\ \text{slave}'(y)\ \&\ \text{own}'(x,y)],$
$\hat{x}\ \ (\forall y)\ [\ [\text{person}'(x)\ \&\ \text{slave}'(y)\ \&\ \text{own}'(x,y)] \rightarrow \text{own}'(x,y\text{'s offspring})\ ]\ )$

(38) UNIVERSAL(Q^n) =
$\lambda R\ \lambda S\ (Qx)\ ((\exists y_2)\ldots(\exists y_n)\ R(x,y_2,\ldots,y_n)\ ,$
$(\forall y_2)\ldots(\forall y_n)\ [R(x,y_2,\ldots,y_n) \rightarrow S(x,y_1,\ldots,y_n)\ ])$

(39) If a farmer owns a donkey he is usually rich.
(39') (MOST $\hat{x}\ \hat{y}$) ([farmer'(x) & donkey'(y) & own'(x,y)] , rich'(x)])
(39") (MOST $\hat{x}$) (($\exists y$) [farmer'(x) & donkey'(y) & own'(x,y), rich'(x)]
(= (MOST $\hat{x}$) (($\exists y$) [farmer'(x) & donkey'(y) & own'(x,y),
($\exists y$) [farmer'(x) & donkey'(y) & own'(x,y) & rich'(x)])
=
(EXISTENTIAL(MOST) $\hat{x}$) ([farmer'(x) & donkey'(y) & own'(x,y)],rich'(x))**)**

(40) If a DRUMMER lives in an apartment complex, it is usually half empty.
(40') (MOST $\hat{y}$) (($\exists x$) [apartment-complex'(x) & drummer'(y) & live-in'(y,x)],
($\exists y$) [apartment-complex'(x) & drummer'(y) & live-in'(y,x) & half-empty'(y)])
['The majority of apartment complexes with a drummer in them are half empty.'**]**

(41) If a drummer lives in an APARTMENT COMPLEX, it is usually half empty.
(41') (MOST $\hat{x}$) (($\exists y$) [apartment-complex'(x) & drummer'(y) & live-in'(y,x)],
($\exists y$) [apartment-complex'(x) & drummer'(y) & live-in'(y,x) & half-empty'(y)])
['The majority of drummers living in apartment complexes live in half empty apartment complexes'**]**

5. Uniqueness

(42) Every farmer who owns a donkey beats it.
(42') ($\forall x$) ([farmer'(x) & ($\exists y$) [donkey'(y) & own'(x,y)]],
beat'(x,<u>(ιy)</u>[donkey'(y) & own'(x,y)])
['Every farmer beats ***the*** donkey he owns'**]**

(43) Every woman who bought a sage plant bought eight others along with it.
(44) No parent with a teenage son lends him the car.

(45) If a woman buys a sage plant here, she always buys eight others along with it.
(46) If a woman has a teenage son, she never lends him the car.

1. Indirect interpretation

(1)

N	$\longrightarrow$	**L**	$\Longrightarrow$	M	$\supseteq$	Γ
natural language	*translation algorithm*	formal language (logic)	*inter-pretation*	class of models	*restricted by meaning postulates*	'good' models

$\Gamma = \{m \in M \mid M \models \Pi\} = \{m \in M \mid \forall \varphi \in \Pi: M \models \varphi\}$

$m = (\mathbf{F}, i, c, g)$

2. Kinds of postulates

(2) $\varphi = \varphi(c_1, \ldots, c_n)$

(3) $\textbf{\textit{be}}' = \hat{P}\, \hat{x}\; P\{\hat{y}\;\; x = y\}$

(4) $\textbf{\textit{every}}' = \hat{P}\;\; \hat{Q}\;\; (\forall x)\,[\, P\{x\} \rightarrow Q\{x\}\,]$

(5a) <u>***John is slowly eating a banana.***</u>

∴ ***John is eating a banana.***

(5b) $(\forall P)\,(\forall x)\;\; [\, \textbf{\textit{slowly}}'(x) \rightarrow P\{x\}\,]$

(6a) <u>***John finds a banana.***</u>

∴ ***There is a banana.***

(6b) ***Mary found a groundhog.***

All woodchucks are groundhogs.

<u>***All groundhogs are woodchucks.***</u>

∴ ***John found a woodchuck.***

(6c) $(\exists R)\; \textbf{\textit{find}}' = \hat{P}\, \hat{x}\;\; P(\hat{y}\;\; R\{x,y\})$

(7a) <u>***Mary loves John.***</u> ∴ ***Someone loves John.***

<u>***Everyone loves John.***</u> ∴ ***Mary loves John.***

(7b) $(\exists x)\; \textbf{\textit{Mary}}' = \hat{P}\;\; P\{x\}$

(8) $\textbf{\textit{kill}}' = \textbf{\textit{cause}}'(\textbf{\textit{die}}')$

(9) $\textbf{\textit{seek}}' = \hat{P}\, \hat{x}\;\; \textbf{\textit{try}}'(\mathrm{x}, P\{\, \hat{y}\;\; \textbf{\textit{find}}'(x,y)\})$

(10a) ***Mary***' = $\hat{P}$ P(**m**)

(10b) ***Mary***' = $\lambda i\ \lambda P\ P$(**m**(i))

(10c) ***Mary***' = $\lambda i\ \lambda P\ P$(**m**)

(10) $(\forall x)$ [***bachelor***'(x) $\rightarrow \neg$ ***married***'(x)]

(11) ***no***' = $\hat{P}$ $\hat{Q}$ ***every***'(P) ($\hat{x}$ $\neg Q\{x\}$)

(12) $(\forall x)$ [***pilot***'(x) $\rightarrow (\exists y)$ [***plane***'(y) & ***fly***'(x,y)]]

(13) $(\forall y)$ [***plane***'(y) $\rightarrow (\exists x)$ [***pilot***'(x) & ***fly***'(x,y)]]

3. Problems with postulates

(14a) ***John saw nobody smile.***
∴ ***There was nobody that John saw smile.***

(14b) ***There was nobody that John saw smile.***
∴ ***John saw nobody smile.***

(14c) $(\forall x)\ (\forall Q)\ (\forall P)$ [***see***'(x, ^ Q($\hat{y}$ $P\{y\}$)) $\leftrightarrow$ Q($\hat{y}$ ***see***'(x,^ $P\{y\}$))]
[= … [***see***'(x, ^ (Qy) $P\{y\}$) $\leftrightarrow$ (Qy) ***see***'(x,^ $P\{y\}$)] **]**

(15) ***John opened the drawer. Mary closed it again$_2$.***

(16) ***It rained again$_1$.***

(17) ***John opened the drawer. Mary had closed it again$_2$.***

(18) $(\forall x)\ (\forall P)\ (\forall p)$
[***again$_2$***(^ CAUSE(^$P\{x\}$), ^ BECOME(p))
$\leftrightarrow$ CAUSE(^$P\{x\}$, ^ ***again$_1$*** (^ BECOME(p)))]

(19) □ $(\forall x)\ (\forall P)\ (\forall p)$
[CAUSE(^ **press-the-button**(x)), ^ BECOME(**water**))
$\leftrightarrow$ CAUSE(^ **press-the-button**(x), ^ BECOME(**cold-water**))]

(19') [***again$_2$***(^ CAUSE(^**press-the-button**(**p**)), ^ BECOME(**water**)) $\leftrightarrow$
CAUSE(^**press-the-button**(***p***), ^ ***again$_1$*** (^ BECOME(**cold-water**)))]