1. From a classical point of view

a. Conditional donkeys

(1) If a farmer owns a donkey, he beats it.

(2) (\forall x) (\forall y) \ [ \ [ x \text{ is a farmer} \& y \text{ is a donkey} \& x \text{ owns } y ] \rightarrow x \text{ beats } y ]

(3) if a farmer owns a donkey, he$_x$ beats it$_y$

   a farmer owns a donkey
   \quad he$_x$ beats it$_y$

   a farmer \quad x
   \quad \quad he$_x$ owns a donkey

   a donkey \quad y
   \quad \quad he$_x$ owns it$_y$

(3') (\exists x) (\exists y) \ [ \ \text{farmer}(x) \& \text{donkey}(y) \& \text{own}(x,y) \Rightarrow \text{beat}(x,y) \ ]

[If a farmer owns a donkey, then he owns it]

(4)

if a farmer owns a donkey, he beats it

   a farmer \quad x
   \quad \quad if he$_x$ owns a donkey, he$_x$ beats it

   a donkey \quad y
   \quad \quad if he$_x$ owns it$_y$, he$_x$ beats it$_y$

   he$_x$ owns it$_y$
   \quad he$_x$ beats it$_y$

(4') (\exists x) (\exists y) \ [ \ \text{farmer}(x) \& \text{donkey}(y) \& (\text{own}(x,y) \Rightarrow \text{beat}(x,y)) \ ]

[A certain farmer beats a certain donkey, if he owns it.]

(5) ?If a farmer owns every/each donkey, he beats it.
b. Relative donkeys

(6) Every farmer who owns a donkey beats it.

(7) every farmer who owns a donkey beats it$_y$

   every farmer who owns a donkey $x$ he$_x$ beats it$_y$

   farmer $z$ he$_z$ owns a donkey

   a donkey $y$ he$_z$ owns it$_y$

(7') $(\forall x) \left( \left[ \text{farmer}'(x) \& (\exists y) \left[ \text{donkey}'(y) \& \text{own}'(x,y) \right] \right] \rightarrow \text{beat}'(x,y) \right)$

   ["Every farmer who owns a donkey beats $y"]$

(8) every farmer who owns a donkey beats it

   a donkey $y$ every farmer who owns it$_y$ beats it$_y$

   every farmer who owns it$_y$ $x$ he$_x$ beats it$_y$

   farmer $x$ he $it$

(8') $(\exists y) \left[ \text{donkey}'(y) \& (\forall x) \left( \left[ \text{farmer}'(x) \& \text{own}'(x,y) \right] \rightarrow \text{beat}'(x,y) \right) \right]$

   ["Every farmer who owns a certain donkey beats it"]

(9) $\exists$ Every farmer who owns every/each donkey beats it.
2. Discourse Anaphora

(10) A farmer owns a donkey. He likes it.

(11) \[(\exists x)(\exists y) [\text{farmer}'(x) \& \text{donkey}'(y) \& \text{own}'(x,y) \& \text{like}'(x,y)]\]

(12) a farmer owns a donkey. He likes it.

(13) ?Every farmer owns a donkey. He likes it.

(14) The man who gave his paycheck to his wife was wiser than the one who gave it [i.e., his paycheck] to his mistress.

(15) (a) A boy owns a guinea-pig.
    (b) He [i.e., the boy who owns guinea-pig] likes it [i.e., the guinea-pig that he, the boy who owns guinea-pig, owns].

(a') \[(\exists x)(\exists y) [\text{boy}'(x) \& \text{guinea-pig}'(y) \& (\text{own}'(x,y))]\].

(b') \[\text{like}'(x) [\text{boy}'(x) \& (\exists y) [\text{guinea-pig}'(y) \& (\text{own}'(x,y))]], \\
        (y) [\text{guinea-pig}'(y) \& (\exists x) [\text{boy}'(x) \& (\text{own}'(x,y))]]\).
(16) (a) A farmer \( \{ \text{rides on a bicycle} \} \).
(b) It \( \{ \text{the bicycle that the farmer who rides on a bicycle rides on} \} \) does not belong to him \( \{ \text{the farmer who rides on a bicycle} \} \).

(17) (a) A farmer rides on a bicycle.
(b) It does not belong to him.
(a') \( \lambda R \exists x \exists y [ \text{farmer'}(x) & \text{bicycle'}(y) & \text{ride-on'}(x,y) & R(x,y)] \)
(b') \( x \ y \ [\neg \text{belong'}(y,x) & R(x,y) ] \)

(18) (a) A farmer cycles.
(b) It does not belong to him.
(a') \( \exists x \exists y [ \text{farmer'}(x) & \text{bicycle'}(y) & \text{ride-on'}(x,y) & R(x)] \)
(b') \( x \ y \ [\neg \text{belong'}(y,x) & R(x) ] \)

(17') (a) \( x \ y \ [\text{farmer'}(x) & \text{bicycle'}(y) & \text{ride-on'}(x,y) ] \)
(b) \( x \ y \ [\neg \text{belong'}(y,x) \]
(c) \( x \ y \ [\text{farmer'}(x) & \text{bicycle'}(y) & \text{ride-on'}(x,y) & \neg \text{belong'}(y,x)] \)

(18') (a) \( x \ (\exists y) [ \text{farmer'}(x) & \text{bicycle'}(y) & \text{ride-on'}(x,y)] \)
(b) \( x \ y \ [\neg \text{belong'}(y,x) \)
(c) \( x \ y \ [\ (\exists y) [ \text{farmer'}(x) & \text{bicycle'}(y) & \text{ride-on'}(x,y) ] & \neg \text{belong'}(y,x) ] \)

(19) (a) A man loves a woman. He kisses her.
(b) A man loves a woman. A man kisses her.

3. Adverbs of Quantification

(20) If a farmer owns a donkey, he \( \{ \text{always}, \text{sometimes}, \text{never} \} \) beats it.
\((\forall x)(\forall y)\)
\[[(x \text{ is a farmer } \& y \text{ is a donkey } \& x \text{ owns } y) \rightarrow (x \text{ beats } y)] \& (\ldots)\]

\((\exists x)(\exists y)\)
\[[(x \text{ is a farmer } \& y \text{ is a donkey } \& x \text{ owns } y) \rightarrow (x \text{ beats } y)] \& (\ldots)\]

\((\exists x)(\exists y)\)
\[[(x \text{ is a farmer } \& y \text{ is a donkey } \& x \text{ owns } y) \rightarrow (x \text{ beats } y)] \& (\ldots)\]

(21')

(21'')

(22) If a farmer owns a donkey, he usually beats it.

(23)

(24)

If a boy draws a picture of a girl, he 
\[
\begin{align*}
\{ & \text{always} \\
& \text{sometimes} \\
& \text{never} \\
& \text{usually} \\
& \ldots
\end{align*}
\]
gives it to her.

(25)

If \(\phi(a_{N_1}, \ldots, a_{N_n})\), [then] ADV \(\psi(it_{i_1}, \ldots, it_{i_n})\) →
ADV'('x_{i_1}, \ldots, x_{i_n} \mid x_{i_1} \text{ is a } N_1 \& \ldots \& x_{i_n} \text{ is a } N_n \& \phi'(x_{i_1}, \ldots, x_{i_n}) \) \(x_{i_1}, \ldots, x_{i_n} \psi'(x_{i_1}, \ldots, x_{i_n})\)
If a farmer owns a donkey, he
always
sometimes
never
usually
... 
beats it
always
sometimes
never
usually
... 

(26)

\[
\begin{align*}
\text{a farmer}_x \text{ owns a donkey}_y & \rightarrow [ \text{farmer}'(x) \& \text{donkey}'(y) \& \text{own}'(x,y) ] \\
\text{a farmer}_x & \rightarrow \text{farmer}'(x) \ ('\text{Indefinites as variables}') \\
\text{a farmer} \text{ owns a donkey} & \rightarrow [ (\text{farmer}' \times \text{donkey}' ) \cap \text{own}' ] \\
\text{a farmer}_x & \rightarrow \text{farmer}' \ ('\text{Indefinites as properties}') \\
\end{align*}
\]

(27)

\[
\begin{align*}
\text{every farmer who owns a donkey beats it} \\
\text{every} \\
\text{farmer}_x \text{ who owns a donkey}_y \\
\text{he}_x \text{ beats it}_y \\
\end{align*}
\]

(28)

\[
\begin{align*}
\text{every farmer who owns a donkey beats it} \\
\text{every} \\
\text{farmer}_x \text{ who owns a donkey}_y \\
\text{he}_x \text{ beats it}_y \\
\end{align*}
\]

4. Asymmetries

(29) Most farmers who own a donkey beat it.

(29') MOST( \( \exists \ y \ \text{farmer}'(x) \& \text{donkey}'(y) \& \text{own}'(x,y), \) \( \exists \ y \ \text{beat}'(x,y) ) \)
(30) Every person who has a dime will put it in the meter.

(30') \( \forall (x \ y) \text{ person}'(x) \& \text{ dime}'(x) \& \text{ have}'(x,y) \& \text{ put-in-the-meter}'(x,y) \)

\[ = (\forall x) (\forall y) ( [\text{person}'(x) \& \text{ dime}'(x) \& \text{ have}'(x,y) \rightarrow \text{ put-in-the-meter}'(x,y) ] ) \]

(30'') \( \forall x \ (\exists y) [\text{person}'(x) \& \text{ dime}'(x) \& \text{ have}'(x,y) \& \text{ put-in-the-meter}'(x,y) ] \)

(31) \( \text{every}_{\text{universal}}' = \lambda R \lambda S (\forall x) (\forall y_2) \ldots (\forall y_n) \ [ R(x,y_2,\ldots,y_n) \rightarrow S(x,y_2,\ldots,y_n) ] \)

(32) \( \text{every}_{\text{existent}}' = \lambda R \lambda S (\forall x) [(\exists y_2) \ldots (\exists y_n) R(x,y_2,\ldots,y_n) \to (\exists y_2)\ldots(\exists y_n) [R(x,y_2,\ldots,y_n) \& S(x,y_2,\ldots,y_n) ] ] \)

(33) Most persons who have a dime will put it in the meter.

(33') MOST (\( x \ (\exists y) [\text{person}'(x) \& \text{ dime}'(x) \& \text{ have}'(x,y) ] , \)

\( x \ (\exists y) [\text{person}'(x) \& \text{ dime}'(x) \& \text{ have}'(x,y) \& \text{ will-put-in-the-meter}'(x,y) ] ) \)

(34) \( \text{EXISTENTIAL}(Q') = \lambda R \lambda S (Qx) [(\exists y_2) \ldots (\exists y_n) R(x,y_2,\ldots,y_n) , \)

(\( \exists y_2)\ldots(\exists y_n) [R(x,y_2,\ldots,y_n) \& S(x,y_1,\ldots,y_n) ] ] \)

(35) Every farmer who owns a donkey beats it.

(35') \( (\forall x) (\exists y) [\text{farmer}'(x) \& \text{ donkey}'(x) \& \text{ have}'(x,y) ] \rightarrow \ (\exists y) [\text{farmer}'(x) \& \text{ donkey}'(x) \& \text{ have}'(x,y) \& \text{ beat}'(x,y) ] ) \)

(36) Most people that owned a slave also owned his offspring.

(36') MOST (\( x \ (\exists y) [\text{person}'(x) \& \text{ slave}'(y) \& \text{ own}'(x,y) ] , \)

\( x \ (\exists y) [\text{person}'(x) \& \text{ slave}'(y) \& \text{ own}'(x,y)'s \text{ offspring} ] ) \)

(37) MOST (\( x \ (\exists y) [\text{person}'(x) \& \text{ slave}'(y) \& \text{ own}'(x,y) ] , \)

\( x \ (\forall y) [\text{person}'(x) \& \text{ slave}'(y) \& \text{ own}'(x,y) \rightarrow \text{own}'(x,y)'s \text{ offspring} ] ) \)
UNIVERSAL($Q^n$) =
\lambda R \lambda S (\exists y_2) \ldots \exists y_n R(x,y_2,\ldots,y_n),

(\forall y_2) \ldots (\forall y_n) [R(x,y_2,\ldots,y_n) \rightarrow S(x,y_1,\ldots,y_n)]

If a farmer owns a donkey he is usually rich.

(MOST $\tilde{x} \tilde{y}$) ([farmer($\tilde{x}$) & donkey($\tilde{y}$) & own($\tilde{x},\tilde{y}$)], rich($\tilde{x}$))

(MOST $\tilde{x}$) ( (\exists y) [farmer($\tilde{x}$) & donkey($y$) & own($\tilde{x},y$), rich($\tilde{x}$)]

= (EXISTENTIAL(MOST) $\tilde{x}$) ([farmer($\tilde{x}$) & donkey($\tilde{y}$) & own($\tilde{x},\tilde{y}$)], rich($\tilde{x}$))

If a DRUMMER lives in an apartment complex, it is usually half empty.

(MOST $\tilde{y}$) ( (\exists x) [apartment-complex($x$) & drummer($\tilde{y}$) & live-in($\tilde{y},x$)],

(\exists y) [apartment-complex($x$) & drummer($y$) & live-in($y,x$) & half-empty($y$)]

['The majority of apartment complexes with a drummer in them are half empty.]

If a drummer lives in an APARTMENT COMPLEX, it is usually half empty.

(MOST $\tilde{x}$) ( (\exists y) [apartment-complex($x$) & drummer($\tilde{y}$) & live-in($\tilde{y},x$)],

(\exists y) [apartment-complex($x$) & drummer($y$) & live-in($y,x$) & half-empty($y$)]

['The majority of drummers living in apartment complexes live in half empty apartment complexes.]

5. Uniqueness

Every farmer who owns a donkey beats it.

(\forall x) ([farmer($x$) & (\exists y) [donkey($y$) & own($\tilde{x},\tilde{y}$)]],

beat($x,\tilde{y}$)[ donkey($\tilde{y}$) & own($\tilde{x},\tilde{y}$)]

['Every farmer beats the donkey he owns']

Every woman who bought a sage plant bought eight others along with it.

No parent with a teenage son lends him the car.

If a woman buys a sage plant here, she always buys eight others along with it.

If a woman has a teenage son, she never lends him the car.
1. Indirect interpretation

(1) \[ \mathcal{N} \rightarrow \mathcal{L} \Rightarrow M \supseteq \Gamma \]

natural translation formal inter- class of restricted by ‘good’
language algorithm language pretation models meaning models
(logic) postulates

\[ \Gamma = \{ m \in M \mid M \models \Pi \} = \{ m \in M \mid \forall \phi \in \Pi : M \models \phi \} \]

\[ m = (F, i, c, g) \]

2. Kinds of postulates

(2) \[ \phi = \phi(c_1, \ldots, c_n) \]

(3) \[ \text{be} = \hat{P} \check{x} \ P\check{y} \ x = y \]

(4) \[ \text{every} = \hat{P} \check{Q} (\forall x) [ P\check{x} \rightarrow Q\check{x}] \]

(5a) John is slowly eating a banana.
\[ \therefore \text{John is eating a banana.} \]

(5b) \[ (\forall P) (\forall x) [ \text{slowly}(x) \rightarrow P\check{x}] \]

(6a) John finds a banana.
\[ \therefore \text{There is a banana.} \]

(6b) Mary found a groundhog.
All woodchucks are groundhogs.
All groundhogs are woodchucks.
\[ \therefore \text{John found a woodchuck.} \]

(6c) \[ (\exists R) \text{find} = \hat{P} \check{x} \check{y} R\check{x}, \check{y} \]

(7a) Mary loves John.
\[ \therefore \text{Everyone loves John.} \]

(7b) \[ (\exists x) \text{Mary} = \hat{P} \check{x} P\check{x} \]

(8) \[ \text{kill} = \text{cause(die)} \]

(9) \[ \text{seek} = \hat{I} \check{x} \text{try}(x, P\check{y} \text{find}(x, y)) \]
(10a) $\text{Mary}' = \bar{P} P(m)$
(10b) $\text{Mary}' = \lambda i \lambda P P(m(i))$
(10c) $\text{Mary}' = \lambda i \lambda P P(m)$

(10) $(\forall x) [\text{bachelor}(x) \rightarrow \neg \text{married}(x)]$

(11) $\text{no}' = \bar{P} Q \text{every}(P)(\forall x \neg Q(x))$

(12) $(\forall x) [\text{pilot}(x) \rightarrow (\exists y) [\text{plane}(y) \& \text{fly}(x,y)]]$

(13) $(\forall y) [\text{plane}(y) \rightarrow (\exists x) [\text{pilot}(x) \& \text{fly}(x,y)]]$

3. Problems with postulates

(14a) **John saw nobody smile.**

   .

   .

   **There was nobody that John saw smile.**

(14b) **There was nobody that John saw smile.**

   .

   .

   **John saw nobody smile.**

(14c) $(\forall x) (\forall Q) (\forall P) [\text{see}(x,^Qy \ P \{y\}) \leftrightarrow Q(y \ \text{see}(x,^P\{y\}))]

   [= ... [\text{see}(x,^Qy \ P \{y\}) \leftrightarrow (Qy \ \text{see}(x,^P\{y\}))]]$

(15) **John opened the drawer. Mary closed it again.**

(16) It rained again.

(17) **John opened the drawer. Mary had closed it again.**

(18) $(\forall x) (\forall P) (\forall p)$

   $[\text{again2} (\text{CAUSE}(P \{x\}, \text{BECOME}(p))) \leftrightarrow \text{CAUSE}(P \{x\}, \text{again1}(\text{BECOME}(p)))]$

(19) $\Box (\forall x) (\forall P) (\forall p)$

   $[\text{CAUSE}(\text{press-the-button}(x)), \text{BECOME}(\text{water})) \leftrightarrow \text{CAUSE}(\text{press-the-button}(x), \text{BECOME}(\text{cold-water}))]$

(19') [\text{again2} (\text{CAUSE}(\text{press-the-button}(p)), \text{BECOME}(\text{water})) \leftrightarrow \text{CAUSE}(\text{press-the-button}(p), \text{again1}(\text{BECOME}(\text{cold-water})))$.