1. Holes in inference patterns

2. Extensions

3. Frege-Carnap intensions

4. Intensional constructions

5. Attitude reports

6. Unspecific objects

7. General aspects

8. Representing intensionality [separate handout]
1. **Holes in inference patterns**

- **Terms and identity**

  (1a) 31 is prime.  

  \[
  \varphi[31] \equiv P(31) \\
  \text{The number of persons in this room is } 31. \\
  \therefore \text{The number of persons in this room is prime.} 
  \]

  \[
  \varphi[n] \equiv P(n) \\
  \]

  (b) According to elementary arithmetic, 31 prime.  

  \[
  \text{The number of persons in this room is } 31. \\
  \therefore \text{According to elementary arithmetic, the number of persons in this room is prime.} 
  \]

- **Problems with existential quantification**

  (2a) John’s salary is higher than Mary’s.  

  \[
  \varphi[j, m] \equiv s(j) > s(m) \\
  \text{John is the dean.} \\
  \text{Mary is the vice dean.} \\
  \therefore \text{The dean’s salary is higher than the vice dean’s.} 
  \]

  \[
  \varphi[d, v] 
  \]

  (b) Bill knows that the dean’s salary is higher than the vice dean’s.  

  \[
  \text{John is the dean.} \\
  \text{Mary is the vice dean.} \\
  \therefore \text{Bill knows that John’s salary is higher than Mary’s.} 
  \]
(5a) Susan is entering a restaurant on Main Street.
   The only restaurants on Main Street are La Gourmande and Le Gourmet. 
∴ Susan is entering La Gourmande, or [Susan is entering] Le Gourmet.

(b) Susan is looking for a restaurant on Main Street.
   The only restaurants on Main Street are La Gourmande and Le Gourmet. 
∴ Susan is looking for La Gourmande, or [Susan is entering] Le Gourmet.

(6a) Paul is wearing a pink shirt with green sleeves. 
∴ There are pink shirts with green sleeves.

(b) Paul is looking for a pink shirt with green sleeves. 
∴ There are pink shirts with green sleeves.

(7a) There have never been any pictures of Lily. 
∴ It is not true that Pete showed Roger a picture of Lily.

(b) There have never been any pictures of Lily. 
∴ It is not true that Pete owed Roger a picture of Lily.
2. Extensions

- **Compositionality**

**Substitution Principle**
If two non-sentential expressions of the same category have the same meaning, either may replace the other in all positions within any sentence without thereby affecting the truth conditions of that sentence.

**Principle of Compositionality**
The meaning of a complex expression functionally depends on the meanings of its immediate parts and the way in which they are combined:

(8)
\[
\begin{array}{c}
\text{Exp} \\
\text{LP} \\
\text{RP}
\end{array}
\] = \[
\begin{array}{c}
\text{LP} \\
\text{RP}
\end{array}
\] + \[
\begin{array}{c}
\text{LP} \\
\text{RP}
\end{array}
\]

- Meaning as communicative function
  - Extension: [contribution to] reference
  - Intension: [contribution to] informational content
  - ...

- **Basic Carnapian extensions**
  
  Carnap (1947)

  (a) \([\text{Vienna}]=\text{Vienna}\)  
      \([\text{proper name}]=\text{bearer}\)

  (b) \([\text{the largest city in Austria}]=\text{Vienna}\)  
      \([\text{definite description}]=\text{descriptee}\)

  (c) \([\text{city}]=\{\text{London, Paris, Rome, Vienna, Frankfurt,…}\}=\{x\mid x \text{ is a city}\}\)  
      \([\text{count noun}]=\text{set of representatives}\)

  (d) \([\text{snore}]=\{x\mid x \text{ snores}\}\)  
      \([\text{intransitive verb}]=\text{set of satisfiers}\)

  (e) \([\text{meet}]=\{(x,y)\mid x \text{ meets } y\}\)  
      \([\text{transitive verb}]=\text{set of satisfier pairs}\)

  (f) \([\text{show}]=\{(x,y,z)\mid x \text{ shows } y \text{ to } z\}\)  
      \([\text{ditransitive verb}]=\text{set of satisfier triples}\)

  (g) \([\text{shows Angie}]=\{(x,y)\mid x \text{ shows } y \text{ to Angie}\}\)  
      \([\text{2-place predicate}]=\text{set of satisfier pairs}\)

  (h) \([\text{shows Angie Disneyland}]=\{(x)\mid x \text{ shows Disneyland to Angie}\}\)  
      \([\text{1-place predicate}]=\text{set of satisfiers}\)

\[\text{Parallelism between valency and type of extension}\]

Frege (1891)

The extension of an \(n\)-place predicate is a set of \(n\)-tuples.

E.g. \([\text{Donald shows Angie Disneyland}]=\{(x)\mid \text{Donald shows Disneyland to Angie}\}\)

= the set of objects of the form ‘( )’ such that Donald shows Disneyland to Angie, i.e.:

\[
[\text{Donald shows Angie Disneyland}]=\begin{cases}
\{(\}\}, & \text{if Donald does show Disneyland to Angie} \\
\{\}\}, & \text{otherwise}
\end{cases}
\]
NB: ( ) = Ø = 0; hence ⟨()⟩ = {Ø} = {0} = 1!

Frege’s Generalization

The extension of a sentence $S$ is its truth value, i.e. 1 if $S$ is true and 0 if $S$ is false.

- **Basic Fregean extensions**

(i) $[\text{Vienna}] = \text{Vienna}$

(j) $[\text{the largest city in Austria}] = \text{Vienna}$

(k) $[\text{Ludwig was born in Vienna}] \models \text{Wittgenstein was born in Vienna} \dashv 1$ *)

≠ $[\text{Rudolf was born in Vienna}] \models \text{Carnap was born in Vienna} \dashv 0$

*) Notation: $\models \ldots \dashv :$ the truth value that is 1 iff …

- **Derived extensions**

From:

<table>
<thead>
<tr>
<th>$[\text{Exp}]$</th>
<th>$[\text{Exp}]$</th>
<th>$[\text{Exp}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[\text{LP}]$</td>
<td>$[\text{RP}]$</td>
<td>$[\text{LP}]$</td>
</tr>
</tbody>
</table>

(b) $[\text{LP}][[\text{RP}]] = [\text{Exp}]$

(c) $[\text{LP}] = (\langle[\text{RP}],[\text{Exp}]\rangle \mid \text{Exp} = \text{LP} + \text{RP})$

- **Deriving (Carnapian) quantifier extensions**

(10a)

(b) $[\text{nobody}][[\text{sleeps}]] = [\text{nobody sleeps}] \Rightarrow [\text{nobody}](S) = 1$  
S: sleepers

(b) $[\text{nobody}][[\text{talks}]] = [\text{nobody talks}] \Rightarrow [\text{nobody}](T) = 0$  
T: talkers

(b) $[\text{nobody}][[\text{listens}]] = [\text{nobody listens}] \Rightarrow [\text{nobody}](L) = 1$  
L: hearers

(c) $[\text{nobody}] = (\langle S, 1 \rangle, \langle T, 0 \rangle, \langle L, 1 \rangle, \ldots)$

= $(Y, \vdash \langle \text{person} \rangle \cap Y = \emptyset \dashv \mid Y$ is a (possible) predicate extension)

= $\lambda Y. \vdash \langle \text{person} \rangle \cap Y = \emptyset \dashv$

(11a)

(b) $[\text{no}][[\text{boy}]] = \lambda Y. \vdash B \cap Y = \emptyset \dashv$  
B: boys

(b) $[\text{no}][[\text{girl}]] = \lambda Y. \vdash G \cap Y = \emptyset \dashv$  
G: girls

(b) $[\text{no}][[\text{city}]] = \lambda Y. \vdash C \cap Y = \emptyset \dashv$  
C: cities

(c) $[\text{no}] = \lambda X. \lambda Y. \vdash X \cap Y = \emptyset \dashv$
(12) \[ \text{[every]} = \lambda X. \lambda Y. \vdash X \subseteq Y \to \]
\[ \text{[some]} = \lambda X. \lambda Y. \vdash X \cap Y \neq \emptyset \to \]
\[ \text{[one]} = \lambda X. \lambda Y. \vdash |X \cap Y| = 1 \to \]
\[ \text{[most]} = \lambda X. \lambda Y. \vdash |X \cap Y| > |X \setminus Y| \to \]

- Deriving Fregean predicate extensions

(13a)

\[ [\text{Mary sleeps}] \checkmark \quad [\text{John sleeps}] \checkmark \quad [\text{Kim sleeps}] \checkmark \]

\[ [\text{Mary}] \checkmark \quad [\text{sleeps}]? \quad [\text{John}] \checkmark \quad [\text{sleeps}]? \quad [\text{Kim}] \checkmark \quad [\text{sleeps}]? \]

(b) \[ [\text{sleeps}](\text{[Mary]}) = [\text{Mary sleeps}] \Rightarrow [\text{sleeps}](\text{Mary}) = 1 \]
\[ [\text{sleeps}](\text{[John]}) = [\text{John sleeps}] \Rightarrow [\text{sleeps}](\text{John}) = 0 \]
\[ [\text{sleeps}](\text{[Kim]}) = [\text{Kim sleeps}] \Rightarrow [\text{sleeps}](\text{Kim}) = 0 \]

(c) \[ [\text{sleeps}] = \{(\text{Mary},1), (\text{John},0), (\text{Kim},1),\ldots\} = \{\text{Mary, John, Kim}\} \]
\[ = \{x, \vdash x \text{ sleeps} \to \} x \text{ is a (possible) name extension} \to \]
\[ = \lambda x. \vdash x \text{ sleeps} \to \]

- Montagovian term extensions

(14a) \[ \text{[Bill]}_M = \lambda X. \vdash \text{Bill} \in X \vdash = \text{Bill}^* \quad \text{cf. Montague (1970a)} \]

(b) \[ \text{[the]}_R = \lambda X. \lambda Y. \vdash |X| = 1 \& X \subseteq Y \to \quad \text{cf. Russell (1905)} \]

\[ \overset{\text{Extensional compositionality}}{=} \]

The extension of a complex expression functionally depends on the intensions of its immediate parts and the way in which they are combined:

\[ \begin{array}{c}
\text{ArbExp} \\
\text{LP} \\
\text{RP}
\end{array} = \begin{array}{c}
\text{LP} \\
\oplus \\
\text{RP}
\end{array} \]

(15)

\[ [\text{No girl likes Bill}] \]
\[ = [\text{no girl}] [\text{[likes Bill]}] \]
\[ = \lambda Y. \vdash G \cap Y = \emptyset \to (\{x, 1 | x \text{ likes Bill}\}) \]
\[ = \vdash G \cap (\{x, 1 | x \text{ likes Bill}\}) = \emptyset \to \]

\[ [\text{no girl}] [\text{[girl]}] \]
\[ = [\text{[no]} [\text{[girl]}]] \]
\[ = [\text{no}] [\text{[girl]}] \]
\[ = [\lambda X. \lambda Y. \vdash X \cap Y = \emptyset \to \} (G) \]
\[ = \lambda Y. \vdash G \cap Y = \emptyset \to \]
\[ [\lambda X. \lambda Y. \vdash X \cap Y = \emptyset \to \} (G) \]
\[ \lambda X. \lambda Y. \vdash X \cap Y = \emptyset \to \]
\[ [\text{no}] [\text{[girl]}] \]
\[ = G \]
\[ [\text{likes}] [\text{[likes Bill]}] \]
\[ = [\text{[likes]} [\text{[likes Bill]}]] \]
\[ = [\text{[likes]} [\text{[likes Bill]}]] \]
\[ = [\lambda Y. \vdash G \cap Y = \emptyset \to \] (G) \]
\[ = [\lambda Y. \vdash G \cap Y = \emptyset \to \]
\[ [\text{likes}] [\text{[likes Bill]}] \]
\[ = \lambda Y. \vdash G \cap Y = \emptyset \to \]
\[ [\text{likes}] [\text{[likes Bill]}] \]
\[ = \lambda X. \lambda Y. \vdash X \cap Y = \emptyset \to \]
\[ [\text{no}] [\text{[girl]}] \]
\[ = \text{Bill} \]
\[ [\text{likes}] [\text{[likes Bill]}] \]
\[ = \text{Bill} \]
\[ [\text{Bill}] \]
\[ = \text{Bill} \]
\[ = \text{Bill} \]
• Extensional types

(17a) $A \subseteq U \models \lambda x. \models x \in A \downarrow$

(b) $R \subseteq U^2 \models \lambda x. \lambda y. \models (x,y) \in R \downarrow = \lambda y. \lambda x. \models (x,y) \in R \downarrow$

(c) $R \subseteq U^3 \models \lambda z. \lambda y. \lambda x. \models (x,y,z) \in R \downarrow \models \models \lambda x. \lambda y. \lambda z. \models x \in U \downarrow$

(18) $x$ is of type $e \Leftrightarrow x \in U$;

\hspace{1cm} $u$ is of type $t \Leftrightarrow u \in \{0,1\}$;

\hspace{1.5cm} $f$ is of type $(a,b) \Leftrightarrow f: \{x \mid x \text{ is of type } a\} \rightarrow \{y \mid y \text{ is of type } b\}$

(19) Extensions and their types

<table>
<thead>
<tr>
<th>Category</th>
<th>Example</th>
<th>Extension</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>Vienna</td>
<td>Vienna $[\in U]$</td>
<td>$e$</td>
</tr>
<tr>
<td>Description</td>
<td>the capital of Austria</td>
<td>Vienna $[\in U]$</td>
<td>$e$</td>
</tr>
<tr>
<td>Noun</td>
<td>city</td>
<td>$C \subseteq U$</td>
<td>$et$</td>
</tr>
<tr>
<td>1-place predicate</td>
<td>sleep</td>
<td>$S \subseteq U$</td>
<td>$et$</td>
</tr>
<tr>
<td>2-place predicate</td>
<td>eat</td>
<td>$\subseteq U \times U$</td>
<td>$et$</td>
</tr>
<tr>
<td>3-place predicate</td>
<td>give</td>
<td>$\subseteq U \times U \times U$</td>
<td>$e(et)$</td>
</tr>
<tr>
<td>Sentence</td>
<td>It's raining</td>
<td>$0 \in {0,1}$</td>
<td>$t$</td>
</tr>
<tr>
<td>Quantified NP</td>
<td>everybody</td>
<td>$\lambda Y. \models [\text{person}] \subseteq Y \downarrow$</td>
<td>$(et)t$</td>
</tr>
<tr>
<td>Determiner</td>
<td>no</td>
<td>$\lambda X. \lambda Y. \models X \cap Y = \emptyset \downarrow$</td>
<td>$(et)((et)t)$</td>
</tr>
</tbody>
</table>
3. Frege-Carnap intensions

- **Logical Space as a model of content**

  (20a) 4 fair coins are tossed.
  (b) At least one of the 4 tossed coins lands heads up.
  (c) At least one of the 4 tossed coins lands heads down.
  (d) Exactly 2 of the 4 tossed coins land heads up.
  (e) Exactly 2 of the 4 tossed coins land heads down.

☞ **Carnap’s Content**

Carnap (1947)

The proposition expressed by a sentence is the set of possible cases of which that sentence is true.

(21a) 4 coins were tossed when John coughed.
(b) 4 coins were tossed and no one coughed.

☞ **Wittgenstein’s Paradise**

Wittgenstein (1921)

All (and only the) maximally specific cases (possible worlds) are members of a set \( W \) (aka Logical Space).

- **From propositions to intensions**

  (22) \( p [\subseteq W] = \lambda w. \vdash w \in p \dashv \) characteristic function (of \( p \) rel. to \( W \))

  (23) The intension of an expression is its extension relative to Logical Space:

\[
[E] : W \rightarrow \{x \mid x \text{ is of the “appropriate” type}\}
\]

- **Intensional types**

☞ **Montagovian types**

Montague (1970a)

- \( x \) is of type \( e \) \( \Leftrightarrow x \in U \);
- \( u \) is of type \( t \) \( \Leftrightarrow u \in \{0,1\} \);
- \( f \) is of type \((a,b)\) \( \Leftrightarrow f : \{x \mid x \text{ is of type } a\} \rightarrow \{y \mid y \text{ is of type } b\}\);
- \( g \) is of type \((s,c)\) \( \Leftrightarrow g : W \rightarrow \{y \mid y \text{ is of type } c\}\)

☞ **Two-sorted types**

“Gallin (1975)”

- \( x \) is of type \( e \) \( \Leftrightarrow x \in U \);
- \( u \) is of type \( t \) \( \Leftrightarrow u \in \{0,1\} \);
- \( w \) is of type \( s \) \( \Leftrightarrow w \in W \);
- \( f \) is of type \((a,b)\) \( \Leftrightarrow f : \{x \mid x \text{ is of type } a\} \rightarrow \{y \mid y \text{ is of type } b\}\)

- **Notation**

\[
\| \text{Exp} \|_{w} = \| \text{Exp} \|(w)
\]
**Intensional compositionality**

The intension of a complex expression functionally depends on the intensions of its immediate parts and the way in which they are combined:

\[
\begin{array}{c}
\text{ArbExp} \\
\text{LP} \quad \text{RP}
\end{array} = [LP] \oplus [RP]
\]

**Pointwise calculation of intensions**

\[
\begin{align*}
[\text{John loves Mary}] & = \lambda w. [\text{John loves Mary}]^w \\
& = \lambda w. [\text{loves Mary}]^w (\langle \text{John} \rangle^w) \\
& = \lambda w. \lambda x^w. \vdash \text{in } w, \text{John loves Mary} \vdash \\
\end{align*}
\]

\[
\begin{align*}
\langle \text{loves Mary} \rangle^w & = \langle \text{loves} \rangle^w (\langle \text{Mary} \rangle^w) \\
& = \lambda x^w. \vdash \text{in } w, x \text{ loves Mary} \vdash \\
\end{align*}
\]

\[
\begin{align*}
\langle \text{loves} \rangle^w & = \langle y^w, \lambda x^w. \vdash \text{in } w, x \text{ loves } y \vdash \rangle \\
\langle \text{Mary} \rangle^w & = \langle \text{Mary} \rangle \\
\end{align*}
\]
4. Intensional constructions

- Substitution failure

(24) Fritz thinks that Hamburg is larger than Cologne.
    Hamburg is larger than Cologne.
    Pfäffingen is larger than Breitenholz.

∴ Fritz thinks that Pfäffingen is larger than Breitenholz.

(25a)

\[
\begin{align*}
\text{thinks that Hamburg is larger than Cologne} & \equiv 1 \\
\text{thinks} & \equiv 1 \\
\text{Hamburg is larger than Cologne} & \equiv 1
\end{align*}
\]

(b)

\[
\begin{align*}
\text{thinks that Pfäffingen is larger than Breitenholz} & \equiv 1 \\
\text{thinks} & \equiv 1 \\
\text{Pfäffingen is larger than Breitenholz} & \equiv 1
\end{align*}
\]

- Ersatz extensions

(26a)

\[
\begin{align*}
\text{thinks that Hamburg is larger than Cologne} & \equiv \sqrt{p} \\
\text{thinks} & \equiv \sqrt{p} \\
\text{Hamburg is larger than Cologne} & \equiv \sqrt{p}
\end{align*}
\]

(b)

\[
\begin{align*}
\text{thinks that Pfäffingen is larger than Breitenholz} & \equiv \sqrt{q} \\
\text{thinks} & \equiv \sqrt{q} \\
\text{Pfäffingen is larger than Breitenholz} & \equiv \sqrt{q}
\end{align*}
\]
(27) $[\text{think}]^w(p) \neq [\text{think}]^w(q)$

$\Rightarrow$ Fregean Laziness

Substitution problems are solved by trading extensions for intensions.

(28a) *Jones thinks that Hesperus is Phosphorus.*

*word* type

think $t(et) (st)(et)$

(28b) *Jones is an alleged murderer.*

*alleged* $\lambda t (et)(et) (s(et))(et)$

(28c) *Jones is attentively eating every apple.*

*attentively* $\lambda t (et)(et) (s(et))(et)$

$\Rightarrow$ *Jones is eating every apple.*

(P1) $(\forall w)(\forall P) [\text{[attentively]}^w(P) \leq P_w(x)] \leq \approx \text{mat. impl.}$

$\Rightarrow$ *Every apple is such that Jones is attentively eating it.*

(P2) $(\forall w)(\forall R)(\forall Q)(\forall x) [\text{[attentively]}^w(R \odot Q) = (Q_w,y)[\text{[attentively]}^w(\lambda x. R_w(x,y))(x)]$,

$\Theta$: combination of intensions of transitive verb and its quantificational object

$\Rightarrow$ (Fregean) laziness does not (always) pay.

(d) *Jones seeks a unicorn.*

*seek* $e(et) (se)(et)$

Montague (1970a), only for verbs like *raise*

(29) *More expressions (of more types)*

<table>
<thead>
<tr>
<th>Category</th>
<th>Example</th>
<th>Extension</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attitude verb</td>
<td>think</td>
<td>$\subseteq U \times \wp W$</td>
<td>$(st)(et)$</td>
</tr>
<tr>
<td>Connective</td>
<td>or</td>
<td>$\lambda u^t \lambda v^t. u+v -(uv)$</td>
<td>$t(tt)$</td>
</tr>
</tbody>
</table>

$\Rightarrow$ Fregean Compositionality

The extension of a complex expression functionally depends on the extensions or intensions of its immediate parts and the way in which they are combined:

$$\begin{align*}
[\text{ExtExp} \begin{array}{c} LP \\ RP \end{array}]^w & = [LP]^w \oplus [RP]^w \\
\text{or:} \quad [\text{IntExp} \begin{array}{c} LP \\ RP \end{array}]^w & = [LP]^w \oplus [RP]^w
\end{align*}$$
5. Attitude reports

- Modelling cognitive states in Logical Space

(30a) Fritz in \( w^* \) ...

(31) \( S = \) Hamburg is larger than Cologne

(32) \([\text{Fritz thinks that Hamburg is larger than Cologne}] \)\(^{w^*} = 1\)

\[ \Leftrightarrow \neg (\exists w \in \diamond) [S](w) = 0 \]

\[ \Leftrightarrow (\forall w \in \diamond) [S](w) = 1 \]

\[ \Leftrightarrow \text{IV} = \emptyset \]

(33) \( \diamond \) depends on

- attitude subject (Fritz)
- world of evaluation: \( w^* \)
- lexical meaning of verb: \text{think}

\[ \Rightarrow \diamond = \text{Dox}(\text{Fritz}(w^*)) \subseteq W \]

\( \Rightarrow \text{Dox} \) is of type \( e(s(st)) \) (dependent) accessibility relation

(34a) \([\text{think}] = \lambda w^*. \lambda p^{st}. \lambda x^e. \vdash (\forall w) \text{Dox}(x)(w^*) \leq p(w) \vdash \leq \approx \text{mat. impl.}\)

(b) \([\text{know}] = \lambda w^*. \lambda p^{st}. \lambda x^e. \vdash (\forall w) \text{Epi}(x)(w^*) \leq p(w) \vdash \)

(c) \([\text{want}] = \lambda w^*. \lambda p^{st}. \lambda x^e. \vdash (\forall w) \text{Bou}(x)(w^*) \leq p(w) \vdash \)

...
(35) 

\[
[[ \text{Fritz thinks that Hamburg is larger than Cologne} ]]^{w*} = 
[[ \text{thinks that Hamburg is larger than Cologne} ]]^{w*}([[ \text{Fritz} ]]^{w*})
\]

\[
\vdash (\forall w) \text{Dox}(\text{Fritz})(w^*)(w) \leq p(w) \downarrow
\]

\[
[[ \text{thinks that Hamburg is larger than Cologne} ]]^{w*} = 
[[ \text{thinks} ]]^{w*}([[ \text{Hamburg is larger than Cologne} ]]^{w*})
\]

\[
\lambda p^{st}. \lambda x^e. \vdash (\forall w) \text{Dox}(x)(w^*)(w) \leq p(w) \downarrow
\]

\[
[[ \text{Hamburg is larger than Cologne} ]]^{w*} = p
\]

(36a) "Fritz knows that Breitenholz is larger than Pfäffingen."

(b) \( (\forall w^*)(\forall p^{st})(\forall x^e)[\text{know}]^{w*}(p)(x) \leq p(w^*) \)

(c) \( (\forall w^*)(\forall x^e) \text{Epi}(x)(w^*)(w^*) = 1 \)

(37a) "Fritz knows that Rome is in Italy, but he doesn’t think so."

(b) \( (\forall w^*)(\forall p^{st})(\forall x^e)[\text{know}]^{w*}(p)(x) \leq [\text{think}]^{w*}(p)(x) \)

(c) \( (\forall w^*)(\forall w)(\forall x^e) \text{Dox}(x)(w^*)(w) \leq \text{Epi}(x)(w^*)(w) \)

(38a) * Fritz wants that Fritz meets Eike.

(b) Fritz wants to meet Eike.

(c) \( [\text{want}] = \lambda w^*. \lambda p^{s(\text{et})}. \lambda x^e. \vdash (\forall w) \text{Bou}(x)(w^*)(w) \leq P(w)(x) \downarrow \)
\[ \text{Fritz wants to meet Eike} \]
\[ \text{wants to meet Eike} \]
\[ \lambda x^e. \left[ \text{want} \right]^w \left( \lambda w. \left[ \text{meet Eike} \right]^w (w(x)) (x) \right) \]
\[ \lambda x^e. \left[ \text{want} \right]^w \left( \lambda w. \left[ \text{meet Eike} \right]^w (w(x)) (x) \right) \]
\[ \lambda x^e. \left[ \lambda y^x \right]^w \left( \lambda w. \left[ \text{meet Eike} \right]^w (w(x)) (x) \right) \]
\[ \lambda w. \left[ \text{meet Eike} \right]^w \]
\[ \lambda x^e. \left[ \lambda y^x \right]^w \left( \lambda w. \left[ \text{meet Eike} \right]^w (w(x)) (x) \right) \]
6. Unspecific Objects

- Paraphrases

(40a) John is looking for a sweater.
(b) John wants to find a sweater.

(41a) Mary owes me a horse.
(b) Mary is obliged to give me a horse.

(42a) This horse resembles a unicorn.
(b) This horse could (almost) be a unicorn.

- Relational analyses

(43a) Analysis of paraphrase

\[ \begin{array}{l}
\llbracket \text{John wants to find a sweater} \rrbracket^w = \\
\llbracket \text{wants to find a sweater} \rrbracket^w(\llbracket \text{John} \rrbracket^w) = \\
\llbracket \text{want} \rrbracket^w(\llbracket w \rrbracket^w. [\text{a sweater}]^w(\llbracket y (x) \rrbracket^w)) (x) (\text{John}) = \\
\llbracket \text{want} \rrbracket^w(\llbracket w \rrbracket^w. [\text{a sweater}]^w(\llbracket y (\text{John}) \rrbracket^w)) (x) \\
\end{array} \]

\[ \begin{array}{l}
\llbracket \text{wants to find a sweater} \rrbracket^w = \\
\lambda x. \llbracket \text{want} \rrbracket^w(\llbracket w \rrbracket^w. [\text{find a sweater}]^w(\llbracket w \rrbracket^w)(x)) (x) = \\
\lambda x. \llbracket \text{want} \rrbracket^w(\llbracket w \rrbracket^w. [\lambda y. [\text{find}]^w(\llbracket y (x) \rrbracket^w)]) (x) = \\
\lambda x. \llbracket \text{want} \rrbracket^w(\llbracket w \rrbracket^w. [\lambda y. [\text{find}]^w(\llbracket y (x) \rrbracket^w)]) (x) \\
\end{array} \]

\[ \begin{array}{l}
\llbracket \text{find a sweater} \rrbracket^w = \\
\lambda w. \lambda x. [\text{a sweater}]^w(\llbracket y (x) \rrbracket^w) = \\
\llbracket [\text{find}]^w \rrbracket^w [\text{a sweater}]^w \\
\end{array} \]

\[ \begin{array}{l}
\lambda x^e. [\text{want}]^w(\llbracket w \rrbracket^w. [\text{a sweater}]^w(\llbracket y^e. [\text{find}]^w(\llbracket y (x) \rrbracket^w))) (x) = \\
\lambda x^e. W(\llbracket w \rrbracket^w. S(\llbracket w \rrbracket^w. y^e. F(\llbracket y (x) \rrbracket^w))) (x) = \\
\lambda x^e. W(\llbracket w \rrbracket^w. Q(\llbracket w \rrbracket^w. y^e. F(\llbracket y (x) \rrbracket^w))) (x) (S) \]

(b) Dissection

\[ \lambda x^e. [\text{want}]^w(\llbracket w \rrbracket^w. [\text{a sweater}]^w(\llbracket y^e. [\text{find}]^w(\llbracket y (x) \rrbracket^w))) (x) \]
(c) **Simplification**

\[\lambda Q^s((et)t) . \lambda x^e . W(\lambda w . Q(w) (\lambda y^e . F(y)(x))) (x)\]

\[\lambda Q^s((et)t) . \lambda x^e . [\text{look-for}]^w (\lambda w . Q(w) (\lambda y^e . [\text{find}]^w (y)(x))) (x)\]

\[\lambda Q^s((et)t) . \lambda x^e . [\lambda p^st . \lambda x^e . \uparrow (\forall w) \text{Bou}(x)(w^*)(w) \leq p(w) -]\]

\[\lambda w . Q(w) (\lambda y^e . \lambda w . \lambda y^e . \lambda x^e . \uparrow \text{in } w, x \text{ finds } y -] (w)(y)(x)) (x)\]

\[\lambda Q^s((et)t) . \lambda x^e . [\lambda p^st . \lambda x^e . \uparrow (\forall w) \text{Bou}(x)(w^*)(w) \leq p(w) -]\]

\[\lambda w . Q(w) (\lambda y^e . \uparrow \text{in } w, x \text{ finds } y -)) (x)\]

\[\lambda Q^s((et)t) . \lambda x^e . [\lambda p^st . \lambda x^e . \uparrow (\forall w) \text{Bou}(z)(w^*)(w) \leq p(w) -]\]

\[\lambda w . Q(w) (\lambda y^e . \uparrow \text{in } w, x \text{ finds } y -)) (x)\]

\[\lambda Q^s((et)t) . \lambda x^e . \uparrow (\forall w) \text{Bou}(x)(w^*)(w) \leq Q(w) (\lambda y^e . \uparrow \text{in } w, x \text{ finds } y -) \uparrow\]

(d) **Compositional analysis**

Montague (1969; 1970)

(44a) John is looking for most unicorns.

(b) \((\forall w) \text{Bou}(x)(w^*)(w) \leq \uparrow \text{ in } w, \#(\text{unicorns } x \text{ finds}) > \#(\text{unicorns } x \text{ doesn’t find}) \uparrow\)

(c) John wants to find most unicorns.

(45a) John is looking for each unicorn.

(b) \((\forall w) \text{Bou}(x)(w^*)(w) \leq \uparrow \text{ in } w, \text{John finds each unicorn} \uparrow\)

(c) John wants to find each unicorn.
(46a) **John is looking for no unicorn.**
(b) \((∀w)Bou(x)(w^*)(w) ≤ ⊬ in w, John doesn’t find a unicorn \(→\))
(c) **John wants to find no unicorn.**

(47a) An intension \(Q\) of type \(s((et)t)\) is *existential* iff
\[
Q = λw. λY^{et} . \vdash (∃x)[P(w)(x) = Y(x) = 1] \vdash
\]
for some intension \(P\) of (‘property’) type \(s(et)\).
(b) \(λP^{s(et)}. λw. λY^{et} . \vdash (∃x)[P(w)(x) = Y(x) = 1] \vdash\) Lerner & Zimmermann (1981: 148)
is a one-one mapping (called \(A\)) whose inverse (called \(BE\)) is: Partee (1987)
\(λQ^{s((et)t)} . λw. λx^{e} . Q(λy^{e}. \vdash x = y \vdash)\).

(48) \([\text{look-for}]^{(w^*)}\)  
= \(λP^{s(et)}. λx^{e}. \vdash (∀w)Bou(x)(w^*)(w) ≤ \vdash (∃y^{e})\) in \(w, P(y) = 1 \& x\) finds \(y \vdash\)

- **Relational readings**

(49) **I owe you a horse.** Buridanus (1350)
(50) **John is looking for Mary.**
Mary is an Austrian student.________________
∴ **John is looking for an Austrian student.**

(51a)
(a') \((\exists m^{s(e(e))}) [m \text{ is a mode of presentation } & m(w^*) (\text{John}) = \text{Mary} & (\forall w) \text{ Bou}(x)(w^*)(w) \leq \vdash \text{ in } w, \text{John finds } m(w)(\text{John})]\) Kaplan (1969)

(b)

More paraphrases

(52a) John is looking for a sweater.
(b) John wants to find a sweater.
(c) John is looking for an intentional sweater.

(53a) Mary owes me a horse.
(b) Mary is obliged to give me a horse.
(c) Mary owes me an arbitrary horse.

(54a) Jones hired an assistant.
(b) This horse could (almost) be a unicorn.
(c) This horse resembles a generic unicorn.

(55a) This horse resembles a unicorn.
(b) Jones saw to it that someone would become an/his assistant.
(c) Jones hired a would-be assistant.
Quantificational analyses

\[
[[\text{John is-looking-for a sweater}]]^w = \lambda x^e. \left[\exists u^e \right] \left[\text{sweater}^+\right]^w(u) = \left[\text{is-looking-for}^+\right]^w(u)(\text{John}) = 1 \Downarrow
\]

\[
[[\text{is-looking-for a sweater}]]^w = \lambda x^e. \left[\text{sweater}\right]^w(\lambda u^e. \left[\text{is-looking-for}^+\right]^w(u)(x))
\]

\[
\lambda x^e. \left[\exists u^e \right] \left[\text{sweater}^+\right]^w(u) = \left[\text{is-looking-for}^+\right]^w(u)(x) = 1 \Downarrow
\]

\[
\left[\text{a}^+ \text{sweater}^+\right]^w = \ldots
\]

\[
\lambda Y^{e'}. \lambda x^e. \left[\exists u^{e'} \right] \left[\text{sweater}^+\right]^w(u) = Y(u) = 1 \Downarrow
\]

\[
[[\text{sweater}^+]]^w = \lambda X^{e'}. \lambda Y^{e'}. \lambda x^e. \left[\exists u^{e'} \right] X(u) = Y(u) = 1 \Downarrow
\]

Notation: \(P \sqsubseteq Q : \equiv (\forall w) (\forall x^e) P(w)(x) \leq Q(w)(x)\)

Monotonicity

(57a) \(e^+ = s(et)\)

(58a) John is a looking for a red sweater.

\[
\therefore \quad \text{John is looking for a sweater.}
\]

(b) John is looking for a sweater.

Mary is looking for a book.

\[
\therefore \quad \text{John is looking for something Mary is looking for.}
\]

Intersective construal (for simplicity): \([[\text{red sweater}] = [[\text{sweater}]] \cap [[\text{red}]]\)

Notation: \(P \sqcap Q := \forall w. \exists x^e. P(w)(x) = Q(w)(x) = 1\)
(59) Relational analyses (with lexical decomposition)

(q) \((\forall w) [\text{Bou}(John)(w) (w)(w) \leq \vdash (\exists y^e) \text{ in } w, y \text{ is a sweater} & y \text{ is red} & \text{John finds } y \vdash] \Rightarrow (\forall w) [\text{Bou}(John)(w) (w)(w) \leq \vdash (\exists y^e) \text{ in } w, y \text{ is a sweater} & \text{John finds } y \vdash] \Rightarrow (\forall w) [\text{Bou}(Mary)(w) (w)(w) \leq \vdash (\exists y^e) \text{ in } w, y \text{ is a book} & \text{Mary finds } y \vdash] \Rightarrow [\text{John is looking for something}]_{w^*} = 1 \ldots\)

(p) \([(\forall w) [\text{Bou}(John)(w) (w)(w) \leq \vdash (\exists y^e) \text{ in } w, y \text{ is a sweater} & \text{John finds } y \vdash] \Rightarrow (\forall w) [\text{Bou}(Mary)(w)(w) (w) \leq \vdash (\exists y^e) \text{ in } w, y \text{ is a book} & \text{Mary finds } y \vdash] \Rightarrow [\text{John is looking for something}]_{w^*} = 1 \ldots\)

(60) Quantificational analysis (with exact match)

(a) \((\exists w^* P (e)) \subseteq \text{[sweater]} \cap \text{[red])}(\forall w)[\text{Bou}(j)(w)(w) (w) \leftarrow (\exists y^e) \text{ in } w, P(w)(y) = 1 & \text{John finds } y]\)

(b) \[(\exists w^* P (e)) \subseteq \text{[sweater]} \cap \text{[book]}(\forall w)[\text{Bou}(j)(w)(w) (w) \leftarrow (\exists y^e) \text{ in } w, P(w)(y) = 1 & \text{John finds } y]\)

& \[(\exists w^* P (e)) \subseteq \text{[book]} \cap \text{[sweater]}(\forall w)[\text{Bou}(j)(w)(w) (w) \leftarrow (\exists y^e) \text{ in } w, P(w)(y) = 1 & \text{John finds } y]\]

\[
\Rightarrow (\exists w^* P (e)) \subseteq \text{[book]} \cap \text{[sweater]}(\forall w)[\text{Bou}(j)(w)(w) (w) \leftarrow (\exists y^e) \text{ in } w, P(w)(y) = 1 & \text{John finds } y]
\]

\[= (\exists w^* P (e)) \subseteq \text{[book]} \cap \text{[sweater]}(\forall w)[\text{Bou}(j)(w)(w) (w) \leftarrow (\exists y^e) \text{ in } w, P(w)(y) = 1 & \text{John finds } y]
\]

Unspecificity \Rightarrow Intensionality?

(61) Arnim owns a bottle of 1981 Riesling-Sylvaner.

Riesling-Sylvaner is Müller-Thurgau.


(62) Arnim owns the bottle that Franzis does not own.

(a) \([\text{the]}_{w^*}(\text{bottle Franzis doesn't own})_{w^*}(\text{own})_{w^*}(\text{y}^e). (\text{own})_{w^*}(\text{lambda}^e). Y(y))(\text{Arnim})

\[\leq \dashv (\exists y^e) \subseteq [\text{bottle}]_{w^*}(y) = [\text{own}]_{w^*}(\text{lambda}^e). Y(y))(\text{Arnim}) = 1 \vdash\]

(b) \([\text{own}]_{w^*}((\text{Arnim}, \text{the})_{w^*}(\text{bottle Franzis doesn't own})_{w^*})\]

\[\leq [\text{own}]_{w^*}((\text{Arnim}, \text{the})_{w^*}([\text{unicorn}]_{w^*}))\]

(in given scenario)

(63a) Pfäffingen is near a river.

= (\exists y^e) \subseteq [\text{river}]_{w^*}(x) \cap [\text{near}]_{w^*}([\text{Pfäffingen}], x) \]

(b) Breitenholz is far from a river.

= (\forall x^e) \subseteq [\text{river}]_{w^*}(x) \cap [\text{far}]_{w^*}([\text{Pfäffingen}], x) \]

Zimmermann (1983; 2001)

Rooth (p.c., anno 1991)

(64) Landscape of intensional verbs

<table>
<thead>
<tr>
<th>VERBS OF...</th>
<th>EXAMPLES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absence</td>
<td>avoid, lack, omit</td>
</tr>
<tr>
<td>Anticipation</td>
<td>allow* (for), anticipate, expect, fear, foresee, plan, wait* (for)</td>
</tr>
<tr>
<td>Calculation</td>
<td>calculate, compute, derive</td>
</tr>
<tr>
<td>Creation</td>
<td>assemble, bake, build, construct, fabricate, make (these verbs in progressive aspect only)</td>
</tr>
<tr>
<td>Depiction</td>
<td>caricature, draw, imagine, portray, sculpt, show, visualize, write* (about)</td>
</tr>
<tr>
<td>Desire</td>
<td>hope* (for), hunger* (for), lust* (after), prefer, want</td>
</tr>
<tr>
<td>Evaluation</td>
<td>admire, disdain, fear, respect, scorn, worship (verbs whose corresponding noun can fill the gap in the evaluation ‘worthy of _’ or ‘merits _’)</td>
</tr>
<tr>
<td>Requirement</td>
<td>cry out* (for), demand, deserve, merit, need, require</td>
</tr>
<tr>
<td>Search</td>
<td>hunt* (for), look* (for), rummage about* (for), scan* (for), seek</td>
</tr>
<tr>
<td>Similarity</td>
<td>imitate, be reminiscent* (of), resemble, simulate</td>
</tr>
<tr>
<td>Transaction</td>
<td>buy, order, owe, own, reserve, sell, wager</td>
</tr>
</tbody>
</table>

Forbes (2006: 37)

(65a) Matt needed some change before the conference. Partee (1974); Schwarz (2006);
(b) Matt was looking for some change before the conference. Moulton (2013)

(66a) Matt needs most of the small bills that were in the cash-box.
(b) Matt is looking for most of the small bills that were in the cash-box.

(67) Zimmermann (2001: 526)

Existential Impact
From \( x \text{ Rs } an \ N \) infer: There is at least one \( N \).

Extensionality
From \( x \text{ Rs } an \ N \), Every \( N \) is an \( M \), and Every \( M \) is an \( N \) infer: \( x \text{ Rs } an \ M \).

Specificity
From \( x \text{ Rs } an \ N \) infer: Some (specific) individual is Red by \( x \).
7. General aspects

- **Propositionalism**

  (P) All (linguistic, mental, perceptual, pictorial, … ) content is propositional.
  
  (Q) All intensional contexts are parts of embedded clauses.

- **Russellian analysis**

  (69) Denotations and their types

<table>
<thead>
<tr>
<th>Category</th>
<th>Example</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>Vienna</td>
<td>e</td>
</tr>
<tr>
<td>Description</td>
<td>the capital of Austria</td>
<td>(e(st))(st)</td>
</tr>
<tr>
<td>Noun</td>
<td>city</td>
<td>e(st)</td>
</tr>
<tr>
<td>1-place predicate</td>
<td>sleep</td>
<td>e(st)</td>
</tr>
<tr>
<td>2-place predicate</td>
<td>eat</td>
<td>e(e(st))</td>
</tr>
<tr>
<td>3-place predicate</td>
<td>give</td>
<td>e(e(e(st)))</td>
</tr>
<tr>
<td>Sentence</td>
<td>It’s raining</td>
<td>st</td>
</tr>
<tr>
<td>Quantified NP</td>
<td>everybody</td>
<td>(e(st))(st)</td>
</tr>
<tr>
<td>Determiner</td>
<td>no</td>
<td>(e(st))((e(st))(st))</td>
</tr>
<tr>
<td>Attitude verb</td>
<td>think</td>
<td>(st)(et)</td>
</tr>
<tr>
<td>Connective</td>
<td>or</td>
<td>(st)((st)(st))</td>
</tr>
</tbody>
</table>

(70) How to Russell a Frege-Church

(a) \( R(\text{[the capital of Slovenia is larger than Breitenholz]}) = R(\text{[is larger than]})(R(\text{[Breitenholz]}))(R(\text{[the capital of Slovenia]})) \)

(b) \( R(\text{[the capital of Slovenia]}) = \lambda x. \lambda w. x = \text{[the capital of Slovenia]}(w) \)

(c) \( R(\text{[Breitenholz]} = \lambda x. \lambda w. x = \text{[Breitenholz]}(w) \)

(d) \( R(\text{[is larger than]})(\lambda e. \lambda Qe(\lambda s). \lambda x. \lambda y. (\forall x) (\forall y) P(x)(w) \times Q(x)(w) \leq [\text{is larger than}](w)(x)(y) \)

\( \lambda e. \lambda Qe(\lambda s). \lambda x. \lambda y. (\forall x) (\forall y) P(x)(w) \times Q(x)(w) \leq [\text{is larger than}](w)(x)(y) \)
• **Relativity of Reference**

(71a) \[|A| = \lambda w. [A], \text{ for lexical } A\]  

Lewis (1974)

(b) \[|A B| = \lambda w. |A|(w) \oplus |B|(w), \text{ if } [A B] = [A] \oplus [B]\]

(72a)[John thinks it's raining]

= \text{APP}^{\text{ext}}(\text{APP}^{\text{int}}(|\text{thinks}|,|\text{it's raining}|),|\text{John}|)

NB: \text{APP}^{\text{ext}}(A, B) = \lambda w. A(w)(B(w)); \text{APP}^{\text{int}}(A, B) = \lambda w. A(w)(B)

(b) \[|\text{John thinks it's raining}|(w)\]

= \text{APP}^{\text{ext}}(|\text{thinks it's raining}|(w),|\text{John}|(w))

= \text{APP}^{\text{ext}}(\text{APP}^{\text{int}}(|\text{thinks}|,(|\text{it's raining}|(w)),|\text{John}|(w)))

= \text{APP}^{\text{ext}}(\text{APP}^{\text{int}}(|\text{thinks}|,|\text{it's raining}|),|\text{John}|)

= [\text{John thinks it's raining}]

(73a) \[|A| = \pi(|A|), \text{ for lexical } A\]  

Putnam (1980)

(b) \[|A B| = |A| \oplus |B|, \text{ if } [A B] = [A] \oplus [B]\]

(c) \[\pi^e: U \rightarrow U \text{ is a (non-trivial) bijection; } \pi^s \text{ and } \pi^t \text{ are identities on } W \text{ and } \{0,1\}; \]

\[\pi^a b \text{ maps any } f \text{ of type } ab \text{ to } \{(\pi x, \pi y) \mid f(x) = y\}\]

(d) \[|S| = [S], \text{ for any sentence } S\]

... provided that all compositions \(\oplus\) are invariant

NB: \(\oplus\) is invariant iff \(\pi(\oplus) = \oplus\) for all permutations \(\pi\)  

Tarski (1986); van Benthem (1989)

• **Further topics**

– Externalism  
Putnam (1975); Burge (1979); Haas-Spohn (1995)

– Lexical meanings and intensions 
Zimmermann & Sternefeld (2013: sec. 8.4); Zimmermann (2014: Kap. 5)

– Fregean vs. intensional compositionality  
Zimmermann & Sternefeld (2013: sec. 8.6); Zimmermann (t.a.)

– De re attitude reports  
Kaplan (1968); Aloni (2001)

– Generalised de re  
Cresswell & von Stechow (1982); Bäuerle (1983); Zimmermann (t.a.)

– De se attitudes  
Lewis (1979); Schlenker (2011)

– Granularity  
Cresswell (1985); Stalnaker (1991; 1999)

– Hierarchy of senses  
Parsons (1981); Zimmermann (t.a.)

– Worlds and models  
Zimmermann (1999; 2011)

– Verbs of Depiction  
Zimmermann (2016)

References


– (ms.): ‘Representing Intensionality: Variables vs. Parameters’. Submitted. [Ms. available on request]