1. **Holes in inference patterns**

- **Terms and identity**

  (1a) 31 is prime.

  \[
  \varphi[31] = P(31) \]

  The number of persons in this room is 31.

  \[ n = 31 \]

  \[ \therefore \text{The number of persons in this room is prime.} \]

  \[
  \varphi[n] = P(n) \]

  (b) It is fact of elementary arithmetic that 31 prime.

  The number of persons in this room is 31.

  \[ \therefore \text{It is fact of elementary arithmetic that the number of persons in this room is prime.} \]

  (2a) John’s salary is higher than Mary’s.

  \[
  \varphi[j, m] = s(j) > s(m) \]

  John is the dean.

  \[ j = d \]

  Mary is the vice dean.

  \[ m = v \]

  \[ \therefore \text{The dean’s salary is higher than the vice dean’s.} \]

  \[
  \varphi[d, v] \]

  (b) Bill knows that the dean’s salary is higher than the vice dean’s.

  John is the dean.

  Mary is the vice dean.

  \[ \therefore \text{Bill knows that John's salary is higher than Mary’s.} \]

- **Problems with existential quantification**

  (3a) Urs is a Swiss millionaire.

  \[
  \varphi[M] = S(u) \land M(u) \]

  All millionaires admire Scrooge McDuck.

  \[ (\forall x) \left[ M(x) \rightarrow A(x) \right] \]

  Only millionaires admire Scrooge McDuck.

  \[ (\forall x) \left[ A(x) \rightarrow M(x) \right] \]

  Urs is a Swiss admirer of Scrooge McDuck.

  \[
  \varphi[A] = S(u) \land A(u) \]

  Urs is an alleged millionaire.

  All millionaires admire Scrooge McDuck.

  Only millionaires admire Scrooge McDuck.

  \[ \therefore \text{Kim is an alleged admirer of Scrooge McDuck.} \]

  (4a) Paul is wearing a pink shirt with green sleeves.

  All pink shirts with green sleeves have striped collars and gold buttons.

  Only pink shirts with green sleeves have striped collars and gold buttons.

  \[ \therefore \text{Paul is wearing a shirt with striped collars and gold buttons.} \]

  (b) Paul is looking for a pink shirt with green sleeves.

  All pink shirts with green sleeves have striped collars and gold buttons.

  \[ \therefore \text{Paul is looking for a shirt with striped collars and gold buttons.} \]
(5a) Susan is entering a restaurant on Main Street.
   The only restaurants on Main Street are La Gourmande and Le Gourmet.
   \[ \therefore \text{Susan is entering } La \text{ Gourmande, or } \text{[Susan is entering]} \text{ Le Gourmet.} \]

(b) Susan is looking for a restaurant on Main Street.
   The only restaurants on Main Street are La Gourmande and Le Gourmet.
   \[ \therefore \text{Susan is looking for } La \text{ Gourmande, or } \text{[Susan is entering]} \text{ Le Gourmet.} \]

(6a) Paul is wearing a pink shirt with green sleeves.
    \[ \therefore \text{There are pink shirts with green sleeves.} \]

(b) Paul is looking for a pink shirt with green sleeves.
    \[ \therefore \text{There are pink shirts with green sleeves.} \]

(7a) There have never been any pictures of Lily.
    \[ \therefore \text{It is not true that Pete showed Roger a picture of Lily.} \]

(b) There have never been any pictures of Lily.
    \[ \therefore \text{It is not true that Pete owed Roger a picture of Lily.} \]

2. Extensions
   • Compositionality

Substitution Principle
If two non-sentential expressions of the same category have the same meaning, either may replace the other in all positions within any sentence without thereby affecting the truth conditions of that sentence.

Principle of Compositionality
The meaning of a complex expression functionally depends on the meanings of its immediate parts and the way in which they are combined:

(8)

- Meaning as communicative function
  - Extension: [contribution to] reference
  - Intension: [contribution to] informational content
  - ...
• **Basic extensions**

(9a) \[ \text{[Ljubljana]} = \text{Ljubljana} \quad [\text{proper name}] = \text{bearer} \]

(b) \[ \text{[the largest city in Slovenia]} = \text{Ljubljana} \quad [\text{definite description}] = \text{descriptee} \]

(c) \[ \text{[city]} = \{\text{London, Paris, Rome, Ljubljana, Frankfurt, …}\} = \{x \mid x \text{ is a city}\} \]

(9d) \[ \text{[snore]} = \{x \mid x \text{ snores}\} \quad [\text{intransitive verb}] = \text{set of satisfiers} \]

(e) \[ \text{[meet]} = \{(x,y) \mid x \text{ meets } y\} \quad [\text{transitive verb}] = \text{set of satisfier pairs} \]

(f) \[ \text{[show]} = \{(x,y,z) \mid x \text{ shows } y \text{ to } z\} \quad [\text{ditransitive verb}] = \text{set of satisfier triples} \]

(g) \[ \text{[shows Joe]} = \{(x,y) \mid x \text{ shows } y \text{ to Joe}\} \quad [\text{2-place predicate}] = \text{set of satisfier pairs} \]

(h) \[ \text{[shows Joe the Vatican]} = \{x \mid x \text{ shows the Vatican to Joe}\} \quad [\text{1-place predicate}] = \text{set of satisfiers} \]

\[\parallel\]

**Parallelism between valency and type of extension**

Frege (1891)

The extension of an \(n\)-place predicate is a set of \(n\)-tuples.

E.g. \[ \text{[Silvio shows Joe the Vatican]} = \{(\ ) \mid \text{Silvio shows the Vatican to Joe}\} \]

\[\parallel\]

\[\text{Silvio shows Joe the Vatican}\]

\[\parallel\]

\[\text{the set of objects of the form } (\ ) \text{ such that Silvio shows the Vatican to Joe, i.e.:}\]

\[\parallel\]

\[\text{[Silvio shows Joe the Vatican]} = \{(\ )\}, \text{ if Silvio does show the Vatican to Joe}\]

\[\parallel\]

\[\text{NB: } (\ ) = \emptyset = 0; \text{ hence } \{(\ )\} = \{\emptyset\} = \{0\} = 1! \]

\[\parallel\]

Frege’s Generalization

Frege (1892)

The extension of a sentence \(S\) is its truth value, i.e. 1 if \(S\) is true and 0 if \(S\) is false.

• **Constructing contributing extensions**

(10a) From: \[\parallel \]

\[\text{[Exp]} \\checkmark \parallel \text{[Exp]} \\checkmark \]

\[\parallel \text{[LP]} \checkmark \quad \text{[RP]} \checkmark \quad \text{[LP]} \checkmark \quad \text{[RP]} \checkmark \]

(b) \[\parallel \text{[LP]} \text{ ( [RP] ) } = \text{[Exp]} \]

(c) \[\parallel \text{[LP]} = \{(\text{[RP]}, \text{[Exp]}\) \mid \text{Exp} = \text{LP} + \text{RP}\} \]

(11a) \[\parallel\]

\[\text{[Nobody sleeps]} \checkmark \quad \text{[Nobody talks]} \checkmark \quad \text{[Nobody listens]} \checkmark \]

\[\parallel\]

\[\text{[nobody]} \checkmark \quad \text{[sleeps]} \checkmark \quad \text{[nobody]} \checkmark \quad \text{[talks]} \checkmark \quad \text{[nobody]} \checkmark \quad \text{[listens]} \checkmark \]

(b) \[\parallel \text{[nobody]} \text{ ( [sleeps] ) } = \text{[nobody sleeps]} \Rightarrow \text{[nobody]} (S) = 1 \quad S: \text{sleepers}\]

\[\parallel\]

\[\text{[nobody]} \text{ ( [talks] ) } = \text{[nobody talks]} \Rightarrow \text{[nobody]} (T) = 0 \quad T: \text{talkers}\]

\[\parallel\]

\[\text{[nobody]} \text{ ( [listens] ) } = \text{[nobody listens]} \Rightarrow \text{[nobody]} (L) = 1 \quad L: \text{hearers}\]

(c) \[\parallel \text{[nobody]} = \{(S,1), (T,0), (L,1),…\}\]

\[\parallel\]

\[\lambda Y. \, \neg \text{[person]} \cap Y = \emptyset \quad Y \text{ is a (possible) predicate extension}\]

\[\lambda Y. \, \neg \text{[person]} \cap Y = \emptyset \quad \lambda\]

\[\text{NB: } \lambda \quad \neg \quad := \text{the truth value that is } 1 \text{ iff } \ldots\]

3
(12a) \[
\begin{align*}
\text{[no boy]} & \vdash B \cap Y = \emptyset \dashv \quad B: \text{boys} \\
\text{[no girl]} & \vdash G \cap Y = \emptyset \dashv \quad G: \text{girls} \\
\text{[no chair]} & \vdash C \cap Y = \emptyset \dashv \quad C: \text{cities}
\end{align*}
\]

(b) \[
\begin{align*}
\text{[no]} (\text{[boy]:}) & = \lambda Y. \vdash B \cap Y = \emptyset \dashv \\
\text{[no]} (\text{[girl]:}) & = \lambda Y. \vdash G \cap Y = \emptyset \dashv \\
\text{[no]} (\text{[city]:}) & = \lambda Y. \vdash C \cap Y = \emptyset \dashv
\end{align*}
\]

(c) \[
\begin{align*}
\text{[no]} & = \lambda X. \lambda Y. \vdash X \cap Y = \emptyset \dashv
\end{align*}
\]

(13) \[
\begin{align*}
\text{[every]} & = \lambda X. \lambda Y. \vdash X \subseteq Y \dashv \\
\text{[some]} & = \lambda X. \lambda Y. \vdash X \cap Y \neq \emptyset \dashv \\
\text{[one]} & = \lambda X. \lambda Y. \vdash |X \cap Y| = 1 \dashv \quad |Z|: \# \text{ of elements of } Z \text{ (cardinality)} \\
\text{[most]} & = \lambda X. \lambda Y. \vdash |X \cap Y| > |X \setminus Y| \dashv
\end{align*}
\]

(14a) \[
\begin{align*}
\text{[Bill]}_{M} & = \lambda X. \vdash \text{Bill} \in X \dashv \quad \text{cf. Montague (1970)} \\
\text{[the]}_{R} & = \lambda X. \lambda Y. \vdash |X| = 1 \& X \subseteq Y \dashv \quad \text{cf. Russell (1905)}
\end{align*}
\]

- **Extensional constructions**

(15) \[
\begin{align*}
\begin{array}{c}
\text{[No girl likes Bill]} \\
\text{[no girl]}([\text{likes Bill}])
\end{array}
\end{align*}
\]

\[
\begin{align*}
\lambda Y. \vdash G \cap Y = \emptyset \dashv & \\
\vdash G \cap \{x \mid x \text{ likes Bill}\} = \emptyset \dashv
\end{align*}
\]

\[
\begin{align*}
\text{[no girl]} & = \lambda X. \lambda Y. \vdash X \cap Y = \emptyset \dashv \\
\text{[likes Bill]} & = \lambda X. \lambda Y. \vdash \{x \mid x \text{ likes Bill}\} \in \lambda Y. \vdash G \cap Y = \emptyset \dashv \\
\lambda X. \lambda Y. \vdash X \cap Y = \emptyset \dashv G & \\
\lambda X. \lambda Y. \vdash \{x \mid x \text{ likes Bill}\} \in \lambda Y. \vdash G \cap Y = \emptyset \dashv \{x, y \mid x \text{ likes } y\} &
\end{align*}
\]
• **Extensional types**

\[
\begin{align*}
\text{A} \subseteq U & \equiv \lambda x. \vdash x \in A \vdash \\
\text{R} \subseteq U^2 & \equiv \lambda x. \lambda y. \vdash (x,y) \in R \vdash \\
\text{R} \subseteq U^3 & \equiv \lambda x. \lambda y. \lambda z. \vdash (x,y,z) \in R \vdash \\
\end{align*}
\]

\[
\begin{align*}
\text{Ljubljana} \in U & \equiv x \in U \vdash \\
\text{city} & \in \{0,1\} \vdash \\
\text{everybody} & \in \{0,1\} \vdash \\
\end{align*}
\]

- **Extensions and their types**

<table>
<thead>
<tr>
<th>Category</th>
<th>Example</th>
<th>Extension</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>Ljubljana</td>
<td>Ljubljana [\in U]</td>
<td>e</td>
</tr>
<tr>
<td>Description</td>
<td>the capital of Slovenia</td>
<td>Ljubljana [\in U]</td>
<td>e</td>
</tr>
<tr>
<td>Noun</td>
<td>city</td>
<td>C [\subseteq U]</td>
<td>et</td>
</tr>
<tr>
<td>1-place predicate</td>
<td>sleep</td>
<td>S [\subseteq U]</td>
<td>et</td>
</tr>
<tr>
<td>2-place predicate</td>
<td>eat</td>
<td>\subseteq U \times U</td>
<td>et</td>
</tr>
<tr>
<td>3-place predicate</td>
<td>give</td>
<td>\subseteq U \times U \times U</td>
<td>e(et)</td>
</tr>
<tr>
<td>Sentence</td>
<td>It’s raining</td>
<td>0 [\in {0,1}]</td>
<td>t</td>
</tr>
<tr>
<td>Quantified NP</td>
<td>everybody</td>
<td>\lambda Y. [\text{person}] \in Y \vdash</td>
<td>(et)t</td>
</tr>
<tr>
<td>Determiner</td>
<td>no</td>
<td>\lambda X. \lambda Y. \vdash X \cap Y = \emptyset \vdash</td>
<td>(et)((et)t)</td>
</tr>
</tbody>
</table>
3. Intensions

- Logical Space as a model of content
  (20a) 4 fair coins are tossed.
  (b) At least one of the 4 tossed coins lands heads up.
  (c) At least one of the 4 tossed coins lands heads down.
  (d) Exactly 2 of the 4 tossed coins land heads up.
  (e) Exactly 2 of the 4 tossed coins land heads down.

☞ Carnap’s Content
  Carnap (1947)
  The proposition expressed by a sentence is the set of possible cases of which that sentence is true.

(21a) 4 coins were tossed when John coughed.
  (b) 4 coins were tossed and no one coughed.

☞ Wittgenstein’s Paradise
  Wittgenstein (1921)
  All (and only the) maximally specific cases (possible worlds) are members of a set \( W \) (aka Logical Space).

- From propositions to intensions
  (22) \( p \subseteq W = \lambda w. \vdash w \in p \rightarrow \) characteristic function (of \( p \) rel. to \( W \))

(23) The intension of an expression is its extension relative to Logical Space:
  \[ [E] : W \rightarrow \{ x \mid x \text{ is of the “appropriate” type} \} \]

- Intensional types
  ☞ Montagovian types
  Montague (1970)
  \( x \) is of type \( e \Leftrightarrow x \in U \);
  \( u \) is of type \( t \Leftrightarrow u \in (0,1) \);
  \( f \) is of type \( (a,b) \Leftrightarrow f : \{ x \mid x \text{ is of type } a \} \rightarrow \{ y \mid y \text{ is of type } b \} \)
  \( g \) is of type \( (s,c) \Leftrightarrow g : W \rightarrow \{ y \mid y \text{ is of type } c \} \)

☞ Two-sorted types
  “Gallin (1975)”
  \( x \) is of type \( e \Leftrightarrow x \in U \);
  \( u \) is of type \( t \Leftrightarrow u \in (0,1) \);
  \( w \) is of type \( s \Leftrightarrow w \in W \);
  \( f \) is of type \( (a,b) \Leftrightarrow f : \{ x \mid x \text{ is of type } a \} \rightarrow \{ y \mid y \text{ is of type } b \} \)

- Notation
  \[ [[Exp]]^u = [[Exp]](w) \]
4. Attitude reports

- Substitution failure

(24) Fritz thinks that Hamburg is larger than Cologne.
Hamburg is larger than Cologne.
Pfäffingen is larger than Breitenholz.

\[ \therefore \text{Fritz thinks that Pfäffingen is larger than Breitenholz.} \]

(25a)

\[
\text{[[thinks that Hamburg is larger than Cologne]}}_{\text{tr}}^{\text{tr}} =
\text{[[thinks]}}_{\text{tr}}^{\text{tr}} \oplus \text{[[Hamburg is larger than Cologne]}}_{\text{tr}}^{\text{tr}} =
\text{[[thinks]}}_{\text{tr}}^{\text{tr}} \oplus 1
\]

\[ \therefore \text{[[Hamburg is larger than Cologne]}}_{\text{tr}}^{\text{tr}} = 1 \]

(b)

\[
\text{[[thinks that Pfäffingen is larger than Breitenholz]}}_{\text{tr}}^{\text{tr}} =
\text{[[thinks]}}_{\text{tr}}^{\text{tr}} \oplus \text{[[Pfäffingen is larger than Breitenholz]}}_{\text{tr}}^{\text{tr}} =
\text{[[thinks]}}_{\text{tr}}^{\text{tr}} \oplus 1
\]

\[ \therefore \text{[[Pfäffingen is larger than Breitenholz]}}_{\text{tr}}^{\text{tr}} = 1 \]

- Intensional compositionality

(26a)

\[
\text{[[thinks that Hamburg is larger than Cologne]}}_{\text{tr}}^{\text{\sqrt{v}}} =
\text{[[thinks]}}_{\text{tr}}^{\text{\sqrt{v}}} \oplus \text{[[Hamburg is larger than Cologne]}}_{\text{tr}}^{\text{\sqrt{v}}}
\]

\[ \therefore \text{[[Hamburg is larger than Cologne]}}_{\text{tr}}^{\text{\sqrt{v}}} = \text{p} \]

\[ \therefore \text{[[thinks]}}_{\text{tr}}^{\text{\sqrt{v}}} \]

\[ \therefore \text{[[Pfäffingen is larger than Breitenholz]}}_{\text{tr}}^{\text{\sqrt{v}}} = \text{p} \]
(b)

\[
\begin{align*}
&\text{[[thinks that Pfäffingen is larger than Breitenholz]]}^{w^*} = \\
&\text{[[thinks]]}^{w^*} \oplus \text{[[Pfäffingen is larger than Breitenholz]]} \\
&\text{[[think]]}^{w^*} \oplus q
\end{align*}
\]

(27) \[[\text{think}] \ (w^*)(p) \neq [\text{think}] \ (w^*)(q)\]

(28) More expressions (of more types)

<table>
<thead>
<tr>
<th>Category</th>
<th>Example</th>
<th>Extension</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attitude verb</td>
<td>think</td>
<td>\subseteq U \times \wp W</td>
<td>(st)(et)</td>
</tr>
<tr>
<td>Connective</td>
<td>or</td>
<td>\lambda u^t \cdot \lambda v^t \cdot u + v - (uv)</td>
<td>t(tt)</td>
</tr>
</tbody>
</table>

Fregean Compositionality

Frege (1892)

The extension of a complex expression functionally depends on the extensions or intensions of its immediate parts and the way in which they are combined:

\[
\begin{align*}
&\text{[[ExtExp \ (LP \overline{RP})]]}^{w} = \text{[[LP]]}^{w} \oplus \text{[[RP]]}^{w} \\
\text{or:} &\text{[[IntExp \ (LP \overline{RP})]]}^{w} = \text{[[LP]]}^{w} \oplus \text{[[RP]]}^{w}
\end{align*}
\]

... strengthens (by a uniformity condition):

Intensional compositionality

The intension of a complex expression functionally depends on the intensions of its immediate parts and the way in which they are combined:

\[
\begin{align*}
&\text{[[ArbExp \ (LP \overline{RP})]]} = \text{[[LP]]} \oplus \text{[[RP]]}
\end{align*}
\]

... and gives rise to the:

Distinction between extensional and intensional constructions

A (binary) construction Exp (understood as the family of expressions of the Form \(\text{Exp}_i = \varphi(\text{LP}_i, \text{RP}_i)\), for some syntactic operation \(\varphi\)) is extensional iff there is a (binary) function \(\oplus_\varphi\) such that, for any world \(w\) (and all \(i\)):

\[
\text{[[Exp}_i]^{w} = \text{[[LP}_i]^{w} \oplus_\varphi \text{[[RP}_i]^{w}}
\]
Pointwise calculation of intension

\[
\begin{align*}
\lambda w. [\text{John loves Mary}]^w & = \\
\lambda w. [\text{loves Mary}]^w ([\text{John}]^w) & = \\
\lambda x. \lambda x^w. \vdash w, \text{John loves Mary} & = \\
\lambda x^w. \vdash w, x \text{ loves Mary} & = \\
\lambda y^w. \lambda x^w. \vdash w, x \text{ loves } y & = \\
\lambda y^w. \lambda x^w. \vdash w, y & = \\
\lambda y^w. \lambda x^w. \vdash w, y & = \\
\end{align*}
\]

- Modelling cognitive states in Logical Space
(29a) Fritz in \(w^*\) ...

(30) \(S = \text{Hamburg is larger than Cologne}\)

(31) \(\text{[Fritz thinks that Hamburg is larger than Cologne]} \ (w^*) = 1\)

\[\neg (\exists w \in \Diamond) [S] (w) = 0\]

\[\forall w \in \Diamond \ [S] (w) = 1\] \(\Leftrightarrow IV = \emptyset\)
(32) ◊ depends on
... attitude subject (Fritz)
... world of evaluation: \( w^* \)
... lexical meaning of verb: think
\[ \Rightarrow \quad ◊ = \text{Dox}(\text{Fritz}(w^*)) \subseteq W \]
\[ = \quad \text{Dox} \text{ is of type } e(s(st)) \quad \text{(dependent) accessibility relation} \]

(33a) \[ \text{[think]} = \lambda w^*. \lambda p_{st}. \lambda x^e. \vdash (\forall w) \text{Dox}(x)(w^*)(w) \leq p(w) \vdash \leq \approx \text{mat. impl.} \]
(b) \[ \text{[know]} = \lambda w^*. \lambda p_{st}. \lambda x^e. \vdash (\forall w) \text{Epi}(x)(w^*)(w) \leq p(w) \vdash \]
(c) \[ \text{[want]} = \lambda w^*. \lambda p_{st}. \lambda x^e. \vdash (\forall w) \text{Bou}(x)(w^*)(w) \leq p(w) \vdash \]

(34)
\[ [\text{Fritz thinks that Hamburg is larger than Cologne}]^{w^*} = \]
\[ [\text{thinks that Hamburg is larger than Cologne}]^{w^*} ([\text{Fritz}]^{w^*}) \]
\[ \vdash (\forall w) \text{Dox}(\text{Fritz}(w^*)) (w) \leq p(w) \vdash \]

(35a) # Fritz knows that Breitenholz is larger than Pfäffingen.
(b) \[ (\forall w^*) (\forall p_{st}) (\forall x^e) \quad [\text{know}] \quad (w^*)(p)(x) \leq p(w^*) \]
(c) \[ (\forall w^*) (\forall x^e) \text{Epi}(x)(w^*)(w^*) = 1 \]

(36a) # Fritz knows that Rome is in Italy, but he doesn’t think so.
(b) \[ (\forall w^*) (\forall p_{st}) (\forall x^e) \quad [\text{know}] \quad (w^*)(p)(x) \leq [\text{think}] \quad (w^*)(p)(x) \]
(c) \[ (\forall w^*) (\forall w) (\forall x^e) \text{Dox}(x)(w^*)(w) \leq \text{Epi}(x)(w^*)(w) \]
(37a) Fritz wants that Fritz meets Eike.
(b) Fritz wants to meet Eike.
(c) \[ \text{[want]} = \lambda w^*. \lambda P^{st(\text{et})}. \lambda x^e. \vdash (\forall w) \text{Bou}(x)(w^*)(w) \leq P(w)(x) \rightarrow \]

(38)
\[
[\text{Fritz wants to meet Eike}]^{w^*} \equiv [\text{wants to meet Eike}]^{w^*}([\text{Fritz}]^{w^*})
\]
\[
[\lambda x^e. \vdash (\forall w) \text{Bou}(x)(w^*)(w) \leq \vdash \text{in } w, x \text{ meets Eike } + \vdash (\text{Fritz})]
\]
\[
\vdash (\forall w) \text{Bou}(\text{Fritz})(w^*)(w) \leq \vdash \text{in } w, \text{ Fritz meets Eike } + \vdash
\]

5. Unspecific Objects
• Paraphrases Quine (1956)

(39a) John is looking for a sweater.
(b) John wants to find a sweater.

(40a) Mary owes me a horse.
(b) Mary is obliged to give me a horse.

(41a) This horse resembles a unicorn.
(b) This horse could (almost) be a unicorn.
Relational analyses

(42a) Analysis of paraphrase

\[
\begin{align*}
&\text{John wants to find a sweater}^w \\
&\quad = \text{[wants to find a sweater]}^w(\text{[John]}^w) \\
&\text{[want]}^w(\lambda x. \text{[a sweater]}^w(\lambda y. \text{[find]}^w(y)(x)) (x)) (\text{John}) \\
&\quad = \text{[want]}^w(\lambda w. \text{[a sweater]}^w(\lambda y. \text{[find]}^w(y)(\text{John})) (x)) (\text{John}) \\
&\quad = \lambda x^e. \text{[want]}^w(\lambda w. \text{[find a sweater]}^w(w(x))) (x) \\
&\quad = \lambda x^e. W(\lambda w. \text{S}(w)\lambda y^e. F(w)(y)(x))(x) \\
&\quad = [\lambda Q^s((et)t)]. \lambda x^e. W(\lambda w. Q(w) (\lambda y^e. F(y)(x)))(x) (S) \\
\end{align*}
\]

(b) Dissection

\[
\lambda x^e. \text{[want]}^w(\lambda w. \text{[a sweater]}^w(\lambda y^e. \text{[find]}^w(y)(x)) (x)) (x) \\
= \lambda x^e. W(\lambda w. \text{S}(w)\lambda y^e. F(w)(y)(x))(x) \\
= [\lambda Q^s((et)t)]. \lambda x^e. W(\lambda w. Q(w) (\lambda y^e. F(y)(x)))(x) (S) \\
\]

(c) Simplification

\[
\begin{align*}
&\text{look-for}^w (w^*) \\
&= \lambda Q^s((et)t). \lambda x^e. W(\lambda w. Q(w) (\lambda y^e. F(y)(x)))(x) \\
&= \lambda Q^s((et)t). \lambda x^e. \text{[want]}^w(\lambda w. Q(w) (\lambda y^e. \text{[find]}^w(y)(x)))(x) \\
&= \lambda Q^s((et)t). \lambda x^e. [\lambda p^{st}. \lambda x^e. \vdash (\forall w) \text{Bou}(x)(w^*)(w) \leq p(w) [-] \\
&\quad (\lambda w. Q(w) (\lambda y^e. \text{in } w, x \text{ finds } y \leftarrow) (w)(y)(x))(x) \\
&= \lambda Q^s((et)t). \lambda x^e. [\lambda p^{st}. \lambda x^e. \vdash (\forall w) \text{Bou}(x)(w^*)(w) \leq p(w) [-] \\
&\quad (\lambda w. Q(w) (\lambda y^e. \text{in } w, x \text{ finds } y \leftarrow)](x) \\
&= \lambda Q^s((et)t). \lambda x^e. [\lambda p^{st}. \lambda x^e. \vdash (\forall w) \text{Bou}(z)(w^*)(w) \leq p(w) [-] \\
&\quad (\lambda w. Q(w) (\lambda y^e. \text{in } w, x \text{ finds } y \leftarrow)](x) \\
&= \lambda Q^s((et)t). \lambda x^e. \vdash (\forall w) \text{Bou}(x)(w^*)(w) \leq Q(w) (\lambda y^e. \vdash \text{in } w, x \text{ finds } y \leftarrow) \vdash
\end{align*}
\]
(d) **Compositional analysis**

Montague (1969; 1970)

\[
\begin{align*}
\lambda Q^{s((et)\!t)} \cdot \lambda x. \; (\forall w) \; Bou(x)(w^*) & (w) \leq \vdash \exists y \; [\text{in } w, y \text{ is a sweater } \& \; x \text{ finds } y] \vdash \\
\lambda P^{s(et)} \cdot \lambda w. \; \lambda x^e. \; (\exists x) \; [P(w)(x) = Y(x) = 1] \vdash
\end{align*}
\]

(43a) **John is looking for most unicorns.**

(b) \( (\forall w) \; Bou(x)(w^*) (w) \leq \vdash \text{in } w, \#(\text{unicorns } x \text{ finds}) > \#(\text{unicorns } x \text{ doesn’t find}) \dashv \)

(c) **John wants to find most unicorns.**

(44a) **John is looking for each unicorn.**

(b) \( (\forall w) \; Bou(x)(w^*) (w) \leq \vdash \text{in } w, \text{John finds each unicorn} \dashv \)

(c) **John wants to find each unicorn.**

(45a) **John is looking for no unicorn.**

(b) \( (\forall w) \; Bou(x)(w^*) (w) \leq \vdash \text{in } w, \text{John doesn’t find a unicorn} \dashv \)

(c) **John wants to find no unicorn.**

(46a) An intension \( Q \) of type \( s((et)\!t) \) is existential iff

\[
Q = \lambda w. \; \lambda Y^e. \; \vdash (\exists x) \; [P(w)(x) = Y(x) = 1] \vdash
\]

for some intension \( P \) of (‘property’) type \( s(et) \).

(b) \( \lambda P^{s(et)} \cdot \lambda w. \; \lambda Y^e. \; \vdash (\exists x) \; [P(w)(x) = Y(x) = 1] \vdash \)

is a one-one mapping (called \( A \)) whose inverse (called \( BE \)) is:

\[
\lambda Q^{s((et)\!t)} \cdot \lambda w. \; \lambda x^e. \; Q(\lambda y^e. \; \vdash x = y \dashv)
\]

(47) \( [\text{look-for}] \; (w^*) \)  

\[
= \lambda P^{s(et)} \cdot \lambda x^e. \; \vdash (\forall w) \; Bou(x)(w^*) (w) \leq \vdash (\exists y^e) \; \text{in } w, P(y) = 1 \& \; x \text{ finds } y \dashv
\]

Zimmermann (1993)
• Relational readings

(48) I owe you a horse. Buridanus (1350)

(49) John is looking for Mary.
Mary is a Slovenian student.

∴ John is looking for a Slovenian student.

(50a)

\[ \| \text{John is looking for Mary}_s \|^{w^*} = \| \text{is-looking for Mary}_s \|^{w^*} (\| \text{John} \|^{w^*}) \]

\[ (\forall w) \text{Boo}(x)(w^*)(w) \leq \top \text{ in w; John finds Mary -} \]

\[ \| \text{is-looking for Mary}_s \|^{w^*} = \| \text{is-looking for} \|^{w^*} (\| \text{Mary}_s \|^{w^*}) \]

\[ \| \text{John} \|^{w^*} = \| \text{John} \|^{w^*} \]

\[ (\forall w) \text{Boo}(x)(w^*)(w) \leq \top \text{ in w, x finds y} \]

\[ \lambda x^* \cdot (\forall w) \text{Boo}(x)(w^*)(w) \leq \top \text{ in w, x finds y} \]

\[ (\forall w) \text{Boo}(x)(w^*)(w) \leq \top \text{ in w, x finds y} \]

\[ (\lambda w \cdot \lambda Y^{w^*}. Y(\text{Mary}) = 1) \]

\[ (\exists m^s(e(e))) [m \text{ is a mode of presentation & } m(w^*)(\text{John}) = \text{Mary} \text{ & de re } (\forall w) \text{Boo}(x)(w^*)(w) \leq \top \text{ in w, John finds } m(w)(\text{John})] \]

Kaplan (1969)

(b)

\[ \| \text{John is looking for a student} \|^{w^*} = \| \text{is-looking for z} \|^{w^*} \]

\[ (\exists y^* \cdot \text{in w, y is a student & } (\forall w) \text{Boo}(\text{John})(w^*)(w) \leq \top \text{ in w, John finds y -} \]

\[ \lambda y^{w^*}. (\exists y^* \cdot \text{in w, y is a student & } Y(y) = 1) \]

\[ \| \text{John is looking for z} \|^{w^*} = \| \text{is-looking for z} \|^{w^*} (\| \text{John} \|^{w^*}) \]

\[ (\forall w) \text{Boo}(\text{John})(w^*)(w) \leq \top \text{ in w, John finds z +} \]

\[ \lambda z^{w^*}. (\forall w) \text{Boo}(z)(w^*)(w) \leq \top \text{ in w, x finds z} \]

\[ (\lambda w \cdot \lambda Y^{w^*}. Y(z) = 1) \]

\[ \| \text{John} \|^{w^*} = \| \text{John} \|^{w^*} \]

\[ \| \text{is-looking for z} \|^{w^*} = \| \text{is-looking for z} \|^{w^*} \]

\[ \lambda z^{w^*}. (\forall w) \text{Boo}(z)(w^*)(w) \leq \top \text{ in w, x finds z} \]

\[ (\lambda w \cdot \lambda Y^{w^*}. Y(z) = 1) \]
• **More paraphrases**

(51a) John is looking for a sweater.
(b) John wants to find a sweater.
(c) John is looking for an intentional sweater.

(52a) Mary owes me a horse.
(b) Mary is obliged to give me a horse.
(c) Mary owes me an arbitrary horse.

(53a) Jones hired an assistant.
(b) This horse could (almost) be a unicorn.
(c) This horse resembles a generic unicorn.

(53a) This horse resembles a unicorn.
(b) Jones saw to it that someone would become an/his assistant.
(c) Jones hired a would-be assistant.

• **Quantificational analyses**

(54)
(55a) \[ e^+ = s(et) \]

(b) \[ \text{[sweater]}(w^*) = \lambda P^{s(et)}. \vdash (\forall w)(\forall x^e) P \sqsubseteq \text{[sweater]} \quad \vdash \]

(c) \[ \text{[look-for]}(w^*) \quad \text{\footnotesize Zimmermann (2006): 'exact match'} \]

\[ = \lambda P^{s(et)}. \lambda x^e. \vdash (\forall w)\left[Bou(x)(w^*)(w) \leftrightarrow (\exists y^e) \right] \text{ in } w, P(y) = 1 & x \text{ finds } y \quad \vdash \]

\[ \text{Notation: } P \sqsubseteq Q \iff (\forall w)(\forall x^e) P(w)(x) \leq Q(w)(x) \quad \text{sub-concepthood} \]

- **Monotonicity**

(56a) **John is a looking for a red sweater.**

\[ \vdash \text{John is looking for a sweater.} \]

(b) **John is looking for a sweater.**

\[ \quad \text{Mary is looking for a book.} \]

\[ \quad \vdash \text{John is looking for something Mary is looking for.} \]

**Intersective construal (for simplicity):** \[ \text{[red} \text{sweater]} = [\text{sweater}] \cap [\text{red}] \]

\[ \text{Notation: } P \cap Q := sw, x^e. P(w)(x) = Q(w)(x) = 1 \]

(57) **Relational analyses (with lexical decomposition)**

(a) \[ (\forall w)Bou(\text{John})(w^*)(w) \leq \vdash (\exists y^e) \right] \text{ in } w, y \text{ is a sweater & } y \text{ is red & John finds } y \quad \vdash \]

\[ \Rightarrow (\forall w)Bou(\text{John})(w^*)(w) \leq \vdash (\exists y^e) \right] \text{ in } w, y \text{ is a sweater & John finds } y \quad \vdash \]

(b) \[ [(\forall w)Bou(\text{John})(w^*)(w) \leq \vdash (\exists y^e) \right] \text{ in } w, y \text{ is a sweater & John finds } y \quad \vdash \]

\& \[ (\forall w)Bou(\text{Mary})(w^*)(w) \leq \vdash (\exists y^e) \right] \text{ in } w, y \text{ is a book & Mary finds } y \quad \vdash \]

- quantifier analysis – e.g. \[ Q = \lambda w. \lambda P. P = P: \]

\[ \Rightarrow (\exists Q^{s(et)}) \quad [\text{[look-for]}(w^*)(Q)(\text{Mary}) \& [\text{look-for]}(w^*)(Q)(\text{John})] \]

- property analysis – e.g. \[ Q = \lambda w. \lambda P. P = P: \]

\[ \Rightarrow (\exists Q^{s(et)}) \quad [\text{[look-for]}(w^*)(P)(\text{Mary}) \& [\text{look-for]}(w^*)(P)(\text{John})] \]

(58) **Quantificational analysis (with exact match)**

(a) \[ (\exists P^{s(et)}) \sqsubseteq [\text{sweater}] \cap [\text{red}] \quad (\forall w)[\text{Bou}(j)(w^*)(w) \leftrightarrow (\exists y^e) \right] \text{ in } w, P(y)=1 & j \text{ finds } y \]

\[ \Rightarrow (\exists P^{s(et)}) \sqsubseteq [\text{sweater}] \quad (\forall w)[\text{Bou}(j)(w^*)(w) \leftrightarrow (\exists y^e) \right] \text{ in } w, P(y)=1 & j \text{ finds } y \]

(b) \[ [(\exists P^{s(et)}) \sqsubseteq [\text{sweater}] \quad (\forall w)[\text{Bou}(j)(w^*)(w) \leftrightarrow (\exists y^e) \right] \text{ in } w, P(y)=1 & j \text{ finds } y \]

\& \[ (\exists P^{s(et)}) \sqsubseteq [\text{book}] \quad (\forall w)[\text{Bou}(m)(w^*)(w) \leftrightarrow (\exists y^e) \right] \text{ in } w, P(y)=1 & m \text{ finds } y \]

\[ \Rightarrow (\exists P^{s(et)})(\forall w)[\text{Bou}(m)(w^*)(w) \leftrightarrow (\exists y^e) \right] \text{ in } w, P(y)=1 & m \text{ finds } y \]

\[ \quad \& [\text{Bou}(j)(w^*)(w) \leftrightarrow (\exists y^e) \right] \text{ in } w, P(y)=1 & j \text{ finds } y \]

\[ = (\exists P^{s(et)}) \quad [\text{[look-for]}(w^*)(P)(\text{Mary}) \& [\text{look-for]}(w^*)(P)(\text{John})] \]
• Unspecificity ⇒ Intensionality?

(59) Arnim owns a bottle of 1981 Riesling-Sylvaner.  
Riesling-Sylvaner is Müller-Thurgau.  

(60) Arnim owns the bottle that Franzis does not own.

(a) \[
\begin{align*}
\lambda y. & \, [\text{own}] (\omega^*) (\lambda y^e. \, Y(y)(\text{Arnim}) \\
\leq & \, \vdash (\exists y^e) \, [\text{bottle}] (\omega^*) (y) = [\text{own}] (\omega^*) (\lambda y^e. \, Y(y)(\text{Arnim}) = 1) \quad \vdash \\
\end{align*}
\]

(b) \[
\begin{align*}
\text{[own]} (\omega^*) (\lambda \text{the} (\omega^*) (\text{bottle Franzis doesn't own}) (\omega^*) (\text{Arnim}) \\
\leq & \, \vdash [\text{own}] (\omega^*) (\lambda \text{the} (\omega^*) (\text{unicorn}) (\omega^*) (\text{Arnim})
\]

\text{(in given scenario)}

• Landscape of intensional verbs

(61)

<table>
<thead>
<tr>
<th>VERBS OF ...</th>
<th>EXAMPLES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absence</td>
<td>avoid, lack, omit</td>
</tr>
<tr>
<td>Anticipation</td>
<td>allow* (for), anticipate, expect, fear, foresee, plan, wait* (for)</td>
</tr>
<tr>
<td>Calculation</td>
<td>calculate, compute, derive</td>
</tr>
<tr>
<td>Creation</td>
<td>assemble, bake, build, construct, fabricate, make (these verbs in progressive aspect only)</td>
</tr>
<tr>
<td>Depiction</td>
<td>caricature, draw, imagine, portray, sculpt, show, visualize, write* (about)</td>
</tr>
<tr>
<td>Desire</td>
<td>hope* (for), hunger* (for), lust* (after), prefer, want</td>
</tr>
<tr>
<td>Evaluation</td>
<td>admire, disdain, fear, respect, scorn, worship (verbs whose corresponding noun can fill the gap in the evaluation <code>worth of</code> or <code>merits</code>)</td>
</tr>
<tr>
<td>Requirement</td>
<td>cry out* (for), demand, deserve, merit, need, require</td>
</tr>
<tr>
<td>Search</td>
<td>hunt* (for), look* (for), rummage about* (for), scan* (for), seek</td>
</tr>
<tr>
<td>Similarity</td>
<td>imitate, be reminiscent* (of), resemble, simulate</td>
</tr>
<tr>
<td>Transaction</td>
<td>buy, order, owe, own, reserve, sell, wager</td>
</tr>
</tbody>
</table>

(62a) Matt needed some change before the conference.  
(b) Matt was looking for some change before the conference.

(63a) Matt needs most of the small bills that were in the cash-box.  
(b) Matt is looking for most of the small bills that were in the cash-box.
5. General topics

- **Propositionalism**
  
  (P) All (linguistic, mental, perceptual, pictorial,...) content is propositional.
  
  (Q) All intensional contexts are parts of embedded clauses.

- **Russellian analysis**
  
  (R) All (linguistic, mental, perceptual, pictorial,...) content is propositional.

- **Existential Impact**
  
  From $x \text{ Rs an } N$ infer: There is at least one $N$.

- **Extensionality**
  
  From $x \text{ Rs an } N$, Every $N$ is an $M$, and Every $M$ is an $N$ infer: $x \text{ Rs an } M$.

- **Specificity**
  
  From $x \text{ Rs an } N$ infer: Some (specific) individual is $\text{ Red by x}$.

(65a) $\text{ [Hesperus is a planet] } \neq \text{ [Phosphorus is a planet] }$

(65b) $\text{ The thirsty man wants beer.}$

(65c) $\text{ Jones worships a Greek goddess.}$

(65d) $\text{ Lex Luthor fears Superman (but not Clark Kent).}$

(65e) $\text{ Horatio believes that things Horatio doesn't believe in exist.}$

(65f) $\text{ John likes chocolate.}$

  John wants to have chocolate.

(66) **Denotations and their types**

<table>
<thead>
<tr>
<th>Category</th>
<th>Example</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>Ljubljana</td>
<td>$e$</td>
</tr>
<tr>
<td>Description</td>
<td>the capital of Slovenia</td>
<td>$(e(st))(st)$</td>
</tr>
<tr>
<td>Noun</td>
<td>city</td>
<td>$e(st)$</td>
</tr>
<tr>
<td>1-place predicate</td>
<td>sleep</td>
<td>$e(st)$</td>
</tr>
<tr>
<td>2-place predicate</td>
<td>eat</td>
<td>$e(e(st))$</td>
</tr>
<tr>
<td>3-place predicate</td>
<td>give</td>
<td>$e(e(e(st)))$</td>
</tr>
<tr>
<td>Sentence</td>
<td>It's raining</td>
<td>$st$</td>
</tr>
<tr>
<td>Quantified NP</td>
<td>everybody</td>
<td>$(e(st))(st)$</td>
</tr>
<tr>
<td>Determiner</td>
<td>no</td>
<td>$(e(st))(e(st))(st))$</td>
</tr>
<tr>
<td>Attitude verb</td>
<td>think</td>
<td>$(st)(et)$</td>
</tr>
<tr>
<td>Connective</td>
<td>or</td>
<td>$(st)((st)(st))$</td>
</tr>
</tbody>
</table>

(67) **How to Russell a Frege-Church**

(a) $\mathcal{R}(\text{the capital of Slovenia is larger than Breitenholz})$

(b) $\mathcal{R}(\text{is larger than}) \mathcal{R}(\text{Breitenholz}) \mathcal{R}(\text{the capital of Slovenia})$
(b) $\mathcal{R}(\text{the capital of Slovenia}) = \lambda x. \lambda w. x = \text{the capital of Slovenia} (w)$

(c) $\mathcal{R}(\text{Breitenholz}) = \lambda x. \lambda w. x = \text{Breitenholz} (w)$

(d) $\mathcal{R}(\text{is larger than})$

= $\lambda P^e. \lambda Q^e. \lambda w. \vdash (\forall x) (\forall y) P(x)(w) \times Q(x)(w) \leq \text{is larger than} (w)(x)(y)$

- *Relativity of Reference*

$$\text{(68a)} \parallel A \parallel = \lambda w. \parallel A \parallel , \text{ for lexical A}$$

(b) $\parallel A B \parallel = \lambda w. \parallel A \parallel (w) \oplus \parallel B \parallel (w)$, if $\parallel A \parallel = \parallel A \parallel \oplus \parallel B \parallel$

(69a) \[ \text{John thinks it's raining} \]

= $\text{APP}^{\text{ext}}(\text{APP}^{\text{int}}(\text{thinks}, \text{it's raining}), \text{John})$

NB: $\text{APP}^{\text{ext}}(A,B) = \lambda w. A(w)(B(w)); \text{APP}^{\text{int}}(A,B) = \lambda w. A(w)(B)$

(b) $\parallel \text{John thinks it's raining} \parallel (w)$

= $\text{APP}^{\text{ext}}(\parallel \text{thinks it's raining} \parallel (w), \parallel \text{John} \parallel (w))$

= $\text{APP}^{\text{ext}}(\text{APP}^{\text{int}}(\parallel \text{thinks} \parallel (w), \parallel \text{it's raining} \parallel (w)), \parallel \text{John} \parallel (w))$

= $\text{APP}^{\text{ext}}(\text{APP}^{\text{int}}(\text{thinks}, \text{it's raining}), \text{John})$

= $\text{John thinks it's raining}$

(70) $\parallel A \parallel = \pi(\parallel A \parallel ), \text{ for lexical A}$

(b) $\parallel A B \parallel = \parallel A \parallel \oplus \parallel B \parallel$, if $\parallel A \parallel = \parallel A \parallel \oplus \parallel B \parallel$

(c) $\pi_e: U \rightarrow U$ is a (non-trivial) bijection; $\pi_s$ and $\pi_t$ are identities on $W$ and \{0,1\};

$\pi_{ab}$ maps any $f$ of type $ab$ to $\{(\pi x, \pi y) \mid f(x) = y\}$

(d) $\parallel S \parallel = \parallel S \parallel$, for any expression $S$

\quad ... provided that all compositions $\oplus$ are invariant

NB: $\oplus$ is invariant iff $\pi(\oplus) = \oplus$ for all permutations $\pi$

- *Further topics*
  – Externalism
  – Attitudes de se
  – Granularity
References