Quantification over alternative intensions

Thomas Ede Zimmermann*

1

(1) Only Mary is asleep.

(2) $(\forall p)\ [[\ {}^{\vee}p \wedge (\exists x)\ p =\ {}^{\wedge}\mathbf{S}(x)] \rightarrow p = {}^{\wedge}\mathbf{S}(\mathbf{m})]$

(3) $(\forall x)[\mathbf{S}(x) \rightarrow x = \mathbf{m}]$

(4) $(\forall x)[x \neq \mathbf{m} \rightarrow {}^{\wedge}\mathbf{S}(x) \neq {}^{\wedge}\mathbf{S}(\mathbf{m})]$

(5) $(\forall x)(\forall y)[x \neq y \rightarrow {}^{\wedge}\mathbf{S}(x) \neq {}^{\wedge}\mathbf{S}(y)]$

(6) a. John only meets MARY.
b. $(\forall p)[[\ {}^{\vee}p \wedge (\exists x)\ p = {}^{\wedge}\mathbf{M}(\mathbf{j},x)] \rightarrow p = {}^{\wedge}\mathbf{M(j,m)}]$
c. $(\forall x_1)(\forall x_2)(\forall y_1)(\forall y_2)[(x_1,x_2) \neq (y_1,y_2) \rightarrow {}^{\wedge}\mathbf{M}(x_1,x_2) \neq {}^{\wedge}\mathbf{M}(y_1,y_2)]$

(7) a. John only introduces MARY to Sue.
b. $(\forall p)[[\ {}^{\vee}p \wedge (\exists x)\ p = {}^{\wedge}\mathbf{I}(\mathbf{j},x,\mathbf{s})] \wedge \rightarrow p = {}^{\wedge}\mathbf{I(j,m,s)}]$
c. $(\forall x_1)(\forall x_2)(\forall x_3)(\forall y_1)(\forall y_2)(\forall y_3)[(x_1,x_2,x_3) \neq (y_1,y_2,y_3) \rightarrow$
$\quad {}^{\wedge}\mathbf{I}(x_1,x_2,x_3) \neq {}^{\wedge}\mathbf{I}(y_1,y_2,y_3)]$

(8) a. Only Mary is both drunk and asleep.
b. $(\forall p)[[\ {}^{\vee}p \wedge (\exists x)\ p = {}^{\wedge}[\mathbf{D}(x) \wedge \mathbf{S}(x)]] \rightarrow p = {}^{\wedge}[\mathbf{D}(\mathbf{m}) \wedge \mathbf{S}(\mathbf{m})]]$
c. $(\forall x)(\forall y)[x \neq y \rightarrow {}^{\wedge}[\mathbf{D}(x) \wedge \mathbf{S}(x)] \neq {}^{\wedge}[\mathbf{D}(y) \wedge \mathbf{S}(y)]]$

(9) a. John only knows that MARY knows that Harry introduces Bill to Sue.
b. $(\forall p)[[\ {}^{\vee}p \wedge (\exists x)\ p = {}^{\wedge}\mathbf{K}(\mathbf{j},{}^{\wedge}\mathbf{K}(x,{}^{\wedge}\mathbf{I}(\mathbf{h},\mathbf{b},\mathbf{s})))] \rightarrow$
$\quad p = {}^{\wedge}\mathbf{K}(\mathbf{j},{}^{\wedge}\mathbf{K}(\mathbf{m},{}^{\wedge}\mathbf{I}(\mathbf{h},\mathbf{b},\mathbf{s})))]$
c. $(\forall x_1)...(\forall x_5)(\forall y_1)...(\forall y_5)[(x_1,...,x_5) \neq (y_1,...,y_5) \rightarrow$
$\quad {}^{\wedge}\mathbf{K}(x_1,{}^{\wedge}\mathbf{K}(x_2,{}^{\wedge}\mathbf{I}(x_3,x_4,x_5))) \neq {}^{\wedge}\mathbf{K}(y_1,{}^{\wedge}\mathbf{K}(y_2,{}^{\wedge}\mathbf{I}(y3,y4,y5)))]$

(10) $\lambda x.[\mathbf{D}(x) \wedge \mathbf{S}(x)]$

(11) $\lambda x_5.\lambda x_4.\lambda x_3.\lambda x_2.\lambda x_1.\mathbf{K}(x_1,{}^{\wedge}\mathbf{K}(x_2,{}^{\wedge}\mathbf{I}(x_3,x_4,x_5)))$

(12) $(\forall \overrightarrow{x})(\forall \overrightarrow{y})[\overrightarrow{x} \neq \overrightarrow{y} \rightarrow [{}^{\wedge}R\overrightarrow{\{x\}}] \neq [{}^{\wedge}R\overrightarrow{\{y\}}]]$

* HAPPY BIRTHDAY, MANFRED!

2

(13) $(\forall X)\Diamond[\mathbf{S} = X]$

(14) $(\forall R)[[\neg(\exists x)R(x,x)] \rightarrow \Diamond[\mathbf{M} = R]]$

(15) $(\forall S)[[\neg(\exists x)(\exists y)[S(x,y,x) \vee S(x,y,y)]] \rightarrow \Diamond[\mathbf{I} = S]]$

(16) $(\forall X)\ \Diamond\ [\lambda x.[\mathbf{D}(x) \wedge \mathbf{S}(x) = X]]$

(17) $(\forall T)\ \Diamond\ [[\lambda x_5.\lambda x_4.\lambda x_3.\lambda x_2.\lambda x_1.\ \mathbf{K}(x_1, {}^{\wedge}\mathbf{K}(x_2, {}^{\wedge}\mathbf{I}(x_3,x_4,x_5)))] = T]$

(18) $(\forall x)(\forall p)[\mathbf{K}(x,p) \rightarrow {}^{\vee}p]$

(19) $(\forall A)[[\neg(\exists x)(\exists p)[A(x,p) \wedge \neg \Diamond^{\vee}p]] \rightarrow \Diamond[\mathbf{K} = A]]$

3

(20) For any $X \subseteq D$, there is a $\mathscr{K}_0$ - model $\mathfrak{M}$ = $(D, W, F_{\mathfrak{M}})$ and a world $w \in W$ such that:
  •$F_{\mathfrak{M}}(\mathbf{S})(w) = X$

(21) a. For any $R \subseteq D^2$ there is a $\mathscr{K}_0$-model $\mathfrak{M} = (D,W,F_{\mathfrak{M}})$ and a world $w \in W$ such that:
  •$F_{\mathfrak{M}}(\mathbf{M})(w) = R$.
 b. For any $S \subseteq D^3$ there is a $\mathscr{K}_0$-model $\mathfrak{M} = (D,W,F_{\mathfrak{M}})$ and a world $w \in W$ such that:
  •$F_{\mathfrak{M}}(\mathbf{I})(w) = S$.

(22) a. $\neg(\exists x)\mathbf{M}(x,x)$
 b. $\neg(\exists x)(\exists y)[\mathbf{I}(x,y,x) \vee \mathbf{I}(x,y,y)]$

(23) a. For any irreflexive $R \subseteq D^2$ there's is a $\mathscr{K}_1$-model $\mathfrak{M}$ = $(D,W,F_{\mathfrak{M}})$ and a world $w \in W$ such that:
  •$F_{\mathfrak{M}}(\mathbf{M})(w) = R$.
 b. For any irreflexive$_3$ $S \subseteq D^3$ there's is a $\mathscr{K}_1$-model $\mathfrak{M}$ = $(D,W,F_{\mathfrak{M}})$ and a world $w \in W$ such that:
  •$F_{\mathfrak{M}}(\mathbf{I})(w) = S$.

(24) $\{\varphi|\mathfrak{M} \models_w \varphi\} = \{\varphi \big| \mathfrak{M}^* \models_{w*} \varphi\}$

(25) Mary is asleep.

(26) a. $\mathfrak{M}_0 \not\models_w \mathbf{S}(\mathbf{m})$, for any $\mathfrak{M}_0$-world w;
 b. $\mathfrak{M}_1 \models_{w'} \mathbf{S}(\mathbf{m})$, for some $\mathfrak{M}_1$-world w'.

(27) a. $\mathfrak{M}_0 \models_w \neg\Diamond\mathbf{S}(\mathbf{m})$
 b. $\mathfrak{M}_1 \models_{w'} \Diamond\mathbf{S}(\mathbf{m})$

(28) a. $\mathfrak{M}^* \models_{w_0} \neg\Diamond\mathbf{S}(\mathbf{m})$

b. $\mathfrak{M}^* \not\models_{w_1} \Diamond\mathbf{S}(\mathbf{m})$

(29) $\mathfrak{M}^* \models_{w^*} [[\neg\Diamond\mathbf{S}(\mathbf{m})] \wedge \Diamond\mathbf{S}(\mathbf{m})]$

4

(30) $(\forall\vec{x})(\forall\vec{y})[\vec{x} \neq \vec{y} \rightarrow [{}^\wedge R\{\vec{x}\}] \neq [{}^\wedge R\{\vec{y}\}]]$ [= (12)]

(31) $(\forall R)(\forall\vec{x})(\forall\vec{y})[\vec{x} \neq \vec{y} \rightarrow [{}^\wedge R\{\vec{x}\}] \neq [{}^\wedge R\{\vec{y}\}]]$

(32) $(\forall R)[\wp(R) \rightarrow (\forall\vec{x})(\forall\vec{y})[\vec{x} \neq \vec{y} \rightarrow {}^\wedge R\{\vec{x}\} \neq {}^\wedge R\{\vec{y}\}]]$

(33) Only three is an odd number .

(34) $(\forall x)[\mathbf{O}(x) \rightarrow x = \mathbf{3}]$

(35) $(\forall p)[[{}^\vee p \wedge (\exists x)p = {}^\wedge\mathbf{O}(x)] \rightarrow p = {}^\wedge\mathbf{O}(\mathbf{3})]$

(36) Only Mary is one of John and Mary and exactly as tall as either one.

(37) $(\forall x)[[x = \mathbf{j} \vee x = \mathbf{m}] \wedge (\forall y)[[[y = \mathbf{j} \vee y = \mathbf{m}] \rightarrow \mathbf{h}(x) = \mathbf{h}(y)] \rightarrow x = \mathbf{m}]]$

(38) $(\forall p)[[{}^\vee p \wedge (\exists x)p = {}^\wedge[[x = \mathbf{j} \vee x = \mathbf{m}] \wedge (\forall y)[[y = \mathbf{j} \vee y = \mathbf{m}] \rightarrow \mathbf{h}(x) = \mathbf{h}(y)]]] \rightarrow p = {}^\wedge[[\mathbf{m} = \mathbf{j} \vee \mathbf{m} = \mathbf{m}] \wedge (\forall y)[[y = \mathbf{j} \vee y = \mathbf{m}] \rightarrow \mathbf{h}(\mathbf{m}) = \mathbf{h}(y)]]]$

(39) $(\forall p)[[{}^\vee p \wedge p = p_{\mathbf{j}\approx\mathbf{m}}] \rightarrow p = p_{\mathbf{j}\approx\mathbf{m}}]$

(40) $[(\forall p)[[{}^\vee p \wedge (\exists x)p = {}^\wedge\mathbf{S}(x)] \rightarrow p = {}^\wedge\mathbf{S}(\mathbf{m})] \wedge \mathbf{S}(\mathbf{m})]$
$\equiv$ $[(\forall p)[[{}^\vee p \wedge (\exists x)p = {}^\wedge\mathbf{S}(x)] \leftrightarrow p = {}^\wedge\mathbf{S}(\mathbf{m})]$

(41) $[(\forall x)[\mathbf{S}(x) \rightarrow x = \mathbf{m}] \wedge \mathbf{S}(\mathbf{m})]$
$\equiv$ $(\forall x)[\mathbf{S}(x) \leftrightarrow x = \mathbf{m}]$

(42) $(\forall x)[\mathbf{O}(x) \leftrightarrow x = \mathbf{3}]$

(43) $(\forall p)[[{}^\vee p \wedge (\exists x)\ p = {}^\wedge\mathbf{O}(x)] \leftrightarrow p = {}^\wedge\mathbf{O}(\mathbf{3})]$

(44) $[(\forall p)[[{}^\vee p \wedge p = p_{\mathbf{j}\approx\mathbf{m}}] \rightarrow p = p_{\mathbf{j}\approx\mathbf{m}}] \wedge$
$[[\mathbf{m} = \mathbf{j} \vee \mathbf{m} = \mathbf{m}] \wedge (\forall y)[[y = \mathbf{j} \vee y = \mathbf{m}] \rightarrow \mathbf{h}(\mathbf{m}) = \mathbf{h}(y)]]]$
$\equiv$ $[(\forall p)[[{}^\vee p \wedge p = p_{\mathbf{j}\approx\mathbf{m}}] \rightarrow p = p_{\mathbf{j}\approx\mathbf{m}}] \wedge \mathbf{h}(\mathbf{m}) = \mathbf{h}(\mathbf{j})]$
$\equiv$ $\mathbf{h}(\mathbf{m}) = \mathbf{h}(\mathbf{j})$

5

(45) a. John only meets MARY. [= (6-a)]
b. $(\forall p)[[{}^\vee p \wedge (\exists y)\ p = {}^\wedge\mathbf{M}(\mathbf{j}, y)] \rightarrow p = {}^\wedge\mathbf{M}(\mathbf{j,m})]$ [≈ (6-b)]
c. $(\forall P)[[P\{\mathbf{j}\} \wedge (\exists y)\ P = \hat{x}\,\mathbf{M}(x, y)] \rightarrow P = \hat{x}\,\mathbf{M}(x, \mathbf{m})]]$

(46) a. Harry only meets SUE.
b. $(\forall P)[[P\{\mathbf{h}\} \wedge (\exists y)\ P = \hat{x}\,\mathbf{M}(x, y)] \rightarrow P = \hat{x}\,\mathbf{M}(x, \mathbf{s})]$

(47) a. $\lambda y.\ \lambda Q.\ (\forall z)\ [Q\{z\} \rightarrow z = y]$

b. $\lambda p.\ \lambda\mathscr{A}.\ (\forall q)\ [[^{\vee}q \wedge \mathscr{A}(q)] \rightarrow q\ =\ p]$
c. $\lambda x.\ \lambda P.\ \lambda\mathscr{S}.\ (\forall S)\ [[S\{x\} \wedge \mathscr{S}(S)] \rightarrow S\ =\ P]$

6

(48)
A: I only advise co-housing FLUFFY with a distinct one of Fluffy and Buffy.
B: Really? I assume you also advise co-housing BUFFY with a distinct one of Fluffy and Buffy.
A: That's the same thing. To co-house Buffy with a distinct one of Fluffy and Buffy is to co-house Fluffy with a distinct one of Fluffy and Buffy.

(49) $\Phi(\mathbf{b})$
$\equiv$ $(\exists y)[\mathbf{C}(\mathbf{b},y)\ \wedge\ [y = \mathbf{f}\ \vee\ y = \mathbf{b}]]$
$\equiv$ $(\exists y)[\mathbf{C}(\mathbf{b},y)\ \wedge\ y = \mathbf{f}]$
$\equiv$ $\mathbf{C}(\mathbf{b},\mathbf{f})$
$\equiv$ $\mathbf{C}(\mathbf{f},\mathbf{b})$
$\equiv$ $(\exists y)[\mathbf{C}(\mathbf{f},y)\ \wedge\ y = \mathbf{b}]$
$\equiv$ $(\exists y)[\mathbf{C}(\mathbf{f},y)\ \wedge\ [y = \mathbf{f}\ \vee\ y = \mathbf{b}]]$
$\equiv$ $\Phi(\mathbf{f})$

(50) $(\forall x)[\mathbf{A}(\mathbf{a},^{\wedge}(\exists \mathbf{y})[\mathbf{C}(x,y)\ \wedge\ [y = \mathbf{b}\ \vee\ y = \mathbf{f}]]) \rightarrow x = \mathbf{f}]$
$\equiv$ $(\forall x)[\mathbf{A}(\mathbf{a},^{\wedge}\Phi(x))\ \rightarrow\ x = \mathbf{f}]$

(51) $(\forall p)[[^{\vee}p \wedge (\exists x)p = {}^{\wedge}\mathbf{A}(\mathbf{a},^{\wedge}\Phi(x))] \rightarrow p\ =\ {}^{\wedge}\mathbf{A}(\mathbf{a},^{\wedge}\Phi(\mathbf{f}))]$

(52) $(\forall q)[[\mathbf{A}(\mathbf{a},q) \wedge (\exists x)q = {}^{\wedge}\Phi(x)]\ \rightarrow\ q = {}^{\wedge}\Phi(\mathbf{f})]$

(53) $(\forall q : (\exists x)q =^{\wedge} \Phi(x))[A(a,q)\ \rightarrow\ q =^{\wedge} \Phi(\mathbf{f})]$

References

Krifka, Manfred. 2001. For a Structured Meaning Account of Questions and Answers. In Caroline Féry & W. Sternefeld (eds.), *Audiatur Vox Sapientiae. A Festschrift for Arnim von Stechow*, 287–319. Berlin: Akademie Verlag.

Rooth, Mats. 1985. *Association with Focus*: University of Massachusetts at Amherst at Amherst Thesis.

Rooth, Mats. 2016. Alternative semantics for 'only'. Email to author. March 21.