1 Introduction

(1) What this talk is about:
\[ \text{Model-theoretic possible worlds semantics of natural language} \quad \rightarrow \text{Montague (1970)} \]
\[ \text{Alternative Semantics of focus} \quad \rightarrow \text{Rooth (1985)} \]

(2) What this talk is NOT about:
\[ \text{extensional models} \]
\[ \text{illustrative uses of models} \]
\[ \text{loose uses of ‘models’} \]
\[ \text{model-theoretic meta-semantics} \]
\[ \text{direct vs. indirect interpretation} \]
\[ \text{context dependence} \]
\[ \text{raw data of semantics} \]

Background:
- disambiguation, LF, compositionality
- types
- terminology: models vs. (‘Fregean’) interpretations

2 Model-theoretic ‘Semantics’

2.1 Relative Truth

My thesis is only that there are important differences between theories of relative, and of absolute, truth, and the differences make theories of the two sorts appropriate as answers to different questions.

(Davidson 1973: 79)

2.2 Models vs. ‘Small Worlds’

(3) \[ M = (D_M, W_M, F_M, \ldots) \quad \text{where } D_M \text{ and } W_M \text{ are arbitrary non-empty sets} \]
\[ [S]^{M,w} \subseteq \{0, 1\} \]
\[ \{w \in W_M | [S]^{M,w} = 1\} \]
\[ \lambda \cdot \{w \in W_M | [S]^{M,w} = 1\} \]
(4) Semantic (?) Levels

a. \( R(S_1, S_2) \) holds locally (in \( M \) at \( w \)) iff \( R_l([S_1]_M^w, [S_2]_M^w) \)
   - \( S_1 \) locally [materially] implies \( S_2 \) (in \( M \) at \( w \)) iff \( [S_1]_M^w \leq [S_2]_M^w \);
   - \( S_1 \) is locally distinct from \( S_2 \) (in \( M \) at \( w \)) iff \( [S_1]_M^w \neq [S_2]_M^w \).

b. \( R(S_1, S_2) \) holds regionally (in \( M \)) iff \( R_r([S_1]_M, [S_2]_M) \)
   - \( S_1 \) regionally [strictly] implies \( S_2 \) (in \( M \)) iff \( [S_1]_M \subseteq [S_2]_M \);
   - \( S_1 \) is regionally distinct from \( S_2 \) (in \( M \)) iff \( [S_1]_M \neq [S_2]_M \).

c. \( R(S_1, S_2) \) holds globally iff \( R_g([S_1]^M, [S_2]^M) \)
   - \( S_1 \) globally implies [logically entails] \( S_2 \) iff \( [S_1]^M \subseteq [S_2]^M \);
   - \( S_1 \) is globally distinct from \( S_2 \) iff \( [S_1]_M \neq [S_2]_M \).

(5) Model Space vs. Logical Space(s)

- Model Space is closed under (arbitrary) isomorphisms, . . .
- Logical Spaces are not – not even necessarily under automorphisms.
- Cross-model identities are (mostly) meaningless, . . . need for interpretation
- Cross-world identity is identity. \( \approx \) haecceitism
- Model Space ought to be small, . . . intended models
- Logical Spaces need to be large. see below

(6) Only John likes Mary.
\( \therefore \) Bill doesn’t like Mary.

(7) Rigidity

a. \( (\exists M)(\exists w \in W_M) [John]^M_w = [Bill]_M^w \)
   Oops!
b. \( (\forall M, M')(\forall w, w' \in W_M) [John]^M_w = [John]_M^{w'} (= John) \)
   \( \uplus \) (6-a)
c. \( (\forall M)(\forall w, w' \in W_M) [John]^M_w = [John]_M^{w'} (= 'John_M') \)
   cf. Fig. 1

(3) for all \( a \in \text{BST} \), the expressions

\[
\begin{align*}
  j & \{\text{entity}\} \equiv \lambda u [u = u], \\
  j & \{\text{be}\} \equiv \lambda Q \lambda P \lambda w [w = \{\{ u = v \} \}], \\
  \forall & u \{[a] \} \equiv \forall P [u]
\end{align*}
\]

are true sentences of \( L_a \) with respect to \( \langle \mathcal{B}', \langle i_0, j_0 \rangle \rangle \), where \( u, v, P, \mathcal{D}, \mathcal{Q} \) are \( v_0, v_0, v_1, v_0, v_0, v_1, v_1, v_1, v_1 \) respectively.

Figure 1 (Montague 1970: 393)

\( \implies \) Rigidity is stability across worlds, not models.
3 Alternative Semantics: a Case Study

3.1 Quantification over propositions

(8) a. Only Mary is asleep.
b. \((\forall p)[[\forall p \land (\exists x) p = ^\wedge S(x)] \Rightarrow p = ^\wedge S(m)]\)
c. \((\forall x)[S(x) \Rightarrow x = m]\)

(9) \((\forall x)(\forall y)[x \neq y \Rightarrow ^\wedge S(x) \neq ^\wedge S(y)]\)

(10) a. John only met MARY.
b. \((\forall p)[[\forall p \land (\exists x) p = ^\wedge M(j,x)] \Rightarrow p = ^\wedge M(j,m)]\)
c. \((\forall x1)(\forall x2)(\forall y1)(\forall y2)[(x1,x2) \neq (y1,y2) \Rightarrow ^\wedge M(x1,x2) \neq ^\wedge M(y1,y2)]\)

(11) a. John only introduces MARY to Sue.
b. \((\forall p)[[\forall p \land (\exists x) p = ^\wedge I(j,x,s)] \land \Rightarrow p = ^\wedge I(j,m,s)]\)
c. \((\forall x1)(\forall x2)(\forall x3)(\forall y1)(\forall y2)(\forall y3)[(x1,x2,x3) \neq (y1,y2,y3) \Rightarrow ^\wedge I(x1,x2,x3) \neq ^\wedge I(y1,y2,y3)]\)

(12) a. Only Mary is both drunk and asleep.
b. \((\forall p)[[\forall p \land (\exists x) p = ^\wedge D(x) \land ^\wedge S(x)] \Rightarrow p = ^\wedge [D(m) \land S(m)]]\)
c. \((\forall x)(\forall y)[x \neq y \Rightarrow ^\wedge [D(x) \land S(x)] \neq ^\wedge [D(y) \land S(y)]]\)

(13) a. John only knows that MARY knows that Harry introduces Bill to Sue.
b. \((\forall p)[[\forall p \land (\exists x) p = ^\wedge K(j,^\wedge K(x,^\wedge I(h,b,s))) \Rightarrow p = ^\wedge K(j,^\wedge K(m,^\wedge I(h,b,s)))]]\)
c. \((\forall x1) \ldots (\forall x5)(\forall y1) \ldots (\forall y5)[(x1,\ldots,x5) \neq (y1,\ldots,y5) \Rightarrow ^\wedge K(x1,^\wedge K(x2,^\wedge I(x3,x4,x5))) \neq ^\wedge K(y1,^\wedge K(y2,^\wedge I(y3,y4,y5)))]\)

(14) \(\lambda x.[D(x) \land S(x)]\)

(15) \(\lambda x5.\lambda x4.\lambda x3.\lambda x2.\lambda x1.K(x1,^\wedge K(x2,^\wedge I(x3,x4,x5)))\)

(16) \((\forall \vec{\lambda})[\forall \vec{\wedge}][\vec{\lambda} \neq \vec{\wedge} \Rightarrow [^\wedge R\{\vec{x}\}] \neq [^\wedge R\{\vec{y}\}]]\)

3.2 Extensional Variation by Meaning Postulates

(17) \((\forall X)\Diamond [S = X]\)

(18) \((\forall R)[[\neg (\exists x)R(x,x)] \Rightarrow \Diamond [M = R]]\)

(19) \((\forall S)[[\neg (\exists x)(\exists y)[S(x,y,x) \lor S(x,y,y)]] \Rightarrow \Diamond [I = S]]\)

(20) \((\forall X) \Diamond [\lambda x.[D(x) \land S(x)] = X]\)

(21) \((\forall T) \Diamond [[\lambda x5.\lambda x4.\lambda x3.\lambda x2.\lambda x1.K(x1,^\wedge K(x2,^\wedge I(x3,x4,x5)))] = T]\)
3.3 Extensional variation by reflection principles

(22) a. For any $X \subseteq D$, there is a $\mathcal{K}_0$-model $M = (D, W, F_M)$ and a world $w \in W$ such that:
   \[ F_M(S)(w) = X. \]

b. For any $R \subseteq D^2$ there is a $\mathcal{K}_0$-model $M = (D, W, F_M)$ and a world $w \in W$ such that:
   \[ F_M(M)(w) = R. \]

c. For any $S \subseteq D^3$ there is a $\mathcal{K}_0$-model $M = (D, W, F_M)$ and a world $w \in W$ such that:
   \[ F_M(I)(w) = S. \]

(23) a. \(\neg (\exists x)M(x,x)\)

b. \(\neg (\exists x)(\exists y)[I(x,y,x) \lor I(x,y,y)]\)

(24) For any irreflexive $R \subseteq D^2$ there is a $\mathcal{K}_1$-model $M = (D, W, F_M)$ and a world $w \in W$ such that:
   \[ F_M(M)(w) = R. \]

(25) For any irreflexive $S \subseteq D^2$ there is a $\mathcal{K}_1$-model $M = (D, W, F_M)$ and a world $w \in W$ such that:
   \[ F_{M}(I)(w) = R. \]

(26) \(\{ \varphi | M \models w \varphi \} = \{ \varphi | M^* \models w^* \varphi \}\)

(27) Mary is asleep.

(28) a. \(M_0 \not| w S(m)\), for any $M_0$-world $w$;
     b. \(M_1 | w S(m)\), for some $M_1$-world $w'$.

(29) a. \(M_0 | w \neg \diamond S(m)\)
    b. \(M_1 | w^* \diamond S(m)\)

(30) a. \(M^* | w_0 \neg \diamond S(m)\)
    b. \(M^* \not| w_1 \diamond S(m)\)

(31) \(M^* | w^* [\neg \diamond S(m) \land \diamond S(m)]\)

3.4 Limits of propositional quantification

(32) \((\forall \overline{x})(\forall \overline{y})[\overline{x} \neq \overline{y} \rightarrow [^\wedge R(\overline{x})] \neq [^\wedge R(\overline{y})]]\]

(33) \((\forall R)(\forall \overline{x})(\forall \overline{y})[\overline{x} \neq \overline{y} \rightarrow [^\wedge R(\overline{x})] \neq [^\wedge R(\overline{y})]]\]

(34) \((\forall R)[\varphi(R) \rightarrow (\forall \overline{x})(\forall \overline{y})[\overline{x} \neq \overline{y} \rightarrow ^\wedge R(\overline{x}) \neq ^\wedge R(\overline{y})]]\]

(35) Only three is an odd number.
3.5 Quantification over alternative properties

(36) \( (\forall x)[O(x) \to x = 3] \)

(37) \( (\forall p)[[\forall p \land (\exists x)p = ^\land O(x)] \to p = ^\land O(3)] \)

(38) Only Mary is one of John and Mary and exactly as tall as either one.

(39) \( (\forall x)[[x = j \lor x = m] \land (\forall y)[[y = j \lor y = m] \to h(x) = h(y)] \to x = m] \)
\[ \Rightarrow \ j = m \]

(40) \( (\forall p)[[\forall p \land (\exists x)p = ^\land S(x)] \to p = ^\land S(m)] \)
\[ \equiv \ (\forall x)[S(x) \leftrightarrow x = m] \]

(41) \( [([\forall p][S(x) \to x = m] \land S(m)] \)
\[ \equiv \ (\forall x)[S(x) \leftrightarrow x = m] \]

(42) \( [([\forall p][S(x) \to x = m] \land S(m)] \)
\[ \equiv \ (\forall x)[S(x) \leftrightarrow x = m] \]

(43) \( [([\forall p][S(x) \to x = m] \land S(m)] \)
\[ \equiv \ (\forall x)[S(x) \leftrightarrow x = m] \]

(44) \( [([\forall p][S(x) \to x = m] \land S(m)] \)
\[ \equiv \ (\forall x)[S(x) \leftrightarrow x = m] \]

(45) \( [([\forall p][S(x) \to x = m] \land S(m)] \)
\[ \equiv \ (\forall x)[S(x) \leftrightarrow x = m] \]

(46) \( [([\forall p][S(x) \to x = m] \land S(m)] \)
\[ \equiv \ (\forall x)[S(x) \leftrightarrow x = m] \]

3.5 Quantification over alternative properties

(47) Bill knows only that MARY is one of John and Mary and exactly as tall as either one.

(48) Bill knows that MARY is one of John and Mary and exactly as tall as either one.

(49) Bill knows only that x is one of John and Mary and exactly as tall as either one.

(50) a. John only met MARY. \( [= (10-a)] \)
 b. \( (\forall p)[[\forall p \land (\exists y)p = ^\land M(j,y)] \to p = ^\land M(j,m)] \)
 \[ \approx (10-b) \]
 c. \( (\forall p)[[P(j)] \land (\exists y)P = ^\land M(x,y)] \to P = ^\land M(x,m)] \)

(51) a. Harry only met SUE.
 b. \( (\forall P)[[P(h)] \land (\exists y)P = ^\land M(x,y)] \to P = ^\land M(x,s)] \)

(52) a. \( \lambda y. \lambda Q. (\forall z)[Q\{z\} \to z = y] \)
 b. \( \lambda p. \lambda \mathcal{A}. (\forall q)[[\forall q \land \mathcal{A}(q)] \to q = p] \)
 c. \( \lambda x. \lambda P. \lambda \mathcal{S}. (\forall S)[[S\{x\} \land \mathcal{S}(S)] \to S = P] \)
3.6 Prospects of Alternative Semantics

(53) \( \lambda E. \lambda x. (\forall y)[E(y) \to y = x] \)  

(54) \( O^- (\forall \{x\}, \lambda p. (\exists y) p = ^\forall \{y\}) \equiv O_{\text{only}}(x, ^\forall P) \)

(55) \( \lambda P. \lambda x. (\forall y) \mu(P\{x\}) \leq \mu(P\{y\}) \)  


(56) \( O^- (\forall \{x\}, \lambda p. (\exists y) p = ^\forall \{y\}) \equiv O_{\text{even}}(x, P) \)

(8-a) Only Mary is asleep

(8-b) \((\forall p)[(\forall p \land (\exists x) p = ^S(x)] \to p = ^S(m)]\)

(8-c) \((\forall x)[S(x) \to x = m]\)

References


