Extensions

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What they are and the role they play in compositional semantics

0. A Puzzle about Extensions

(SCP) How come speakers can (not only identify but also) grasp the meanings of indefinitely many linguistic expressions?

(C) The meaning of a compound expression derives from combining the meanings of its immediate parts.

(EC) The extension of a compound expression derives from combining the extensions of its immediate parts.

1. Truth and Reference

Rough characterisation: *Extension* generalises the notions of *reference* and *truth value* to arbitrary expressions.

Q: In what sense are truth values generalised referents?

A1: Truth values generalise the multiple referents of n-ary predicates P, which may be represented by sets of satisfiers; for sentences $n = 0$:

$[[P]]^i = \{(u_1,\ldots,u_n) \mid$ the (possibly open) formula $P(x_1,\ldots,x_n)$ is satisfied by $u_1,\ldots,u_n$ in $i\}$

$[[S]]^i = \{( ) \mid$ (closed) sentence S is satisfied in $i\}$

A2: A definite description D refers to the individual that satisfies it in the situation talked about (index); by (loose) analogy, a (declarative) sentence refers to the situation talked about just in case the latter satisfies the former:

$[[D]]^i = \iota u.$ D applies to u in i

$[[S]]^i = \iota j.$ S applies to j in i and $j = i$

2. Determining Extensions

(1st FP) The (unknown) extension of an expression X is that function that assigns to every extension of a (possible) sister constituent Y of X the extension of the mother constituent $X + Y$.

- Binarity is not essential: (1st FP) easily generalises to n-ary constructions (where $n \geq 1$).
- The daughters' extensions are their (local) *contributions* to the mother's extension; arguably, (1st FP) is a remnant of Frege's Context Principle.
- The same functional construction could also be used to define other semantic values – in particular Russellian (denotations).
- (1st FP) is merely a *heuristic* principle for determining extensions, which can be iterated *ad libitum*.
- It is *not deterministic* in that it requires the choice of a 'canonical' construction (syntactic environment) in which the extensions of X's sisters and their mothers have been previously determined – by (1st FP) or otherwise.
- (1st FP) presupposes *extensionality* (= extensional substitutivity) of the canonical constructions: if sisters Y_1 and Y_2 are extensionally equivalent, then so are their mothers $X + Y_1$ and $X + Y_2$.
- Substitutivity guarantees *extensional compositionality* in all canonical constructions.

(2nd FP) In the absence of extensionality, the (unknown) extension of X is that function that assigns to every intension of a (possible) sister constituent Y of X the extension of the mother constituent $X + Y$.

- The extension and intension of daughters X and Y are their respective (local) *contributions* to the mother's extension; again, (2nd FP) can be seen as a remnant of Frege's Context Principle.
- In single-layered semantics, the functional principle can be applied to construct (Russellian) denotations.

Montagovian Types

- *ur*-extensions receive types $\boldsymbol{e}$ and $\boldsymbol{t}$;
- extensions constructed according to (1st FP) receive type (a,b);
- extensions constructed according to (2nd FP) receive type $((\boldsymbol{s},a),b)$,

… where a and b are the respective types of the sister's and mother's extension.

Fregean Compositionaliy

(FC) The extension of a compound expression derives from combining the extensions or intensions of its immediate parts, depending on whether the construction admits extensional substitutivity.

> Frege's functionality principle […]: the extension of a formula is a function of the extensions […] of those of its parts not standing within indirect contexts […], together with the intensions […] of those parts that do stand within indirect contexts.
>
> [Montague (1970a: 75f.), where *extension* translates as used by Frege's (1892) *Bedeutung*]

Intensional Compositionaliy

(IC) The intension of a compound expression derives from combining the intensions of its immediate parts.

3. The Polysemy of *Extension*

- *Extension*$_2$ (*in L*) is a binary function assigning 'referents' to expressions and points in Logical Space.
- *Extension*$_1$ (*in L*) is that unary function that assigns to any expression its 'actual' extension$_2$.
- *Extension*$_{\text{indef}}$ is the unary predicate *extension*$_2$ *of some expression at some point in some possible language L*

[cf. *Extensional object* is the unary predicate *Extension*$_{\text{indef}}$ *of some expression at some point in some possible extensional language L*]

- *Extension*$_{\text{loc}}$ (*in L*) is a binary function assigning to any occurrence of an expression and its host expression the former's contribution to the latter's extension$_1$

4. Uniform Extensionality

(EC) The extension of a compound expression derives from combining the extensions of its immediate parts.

(1a) The temperature can be read off from a thermometer.
⇔ The temperature-at-*i* can be read-off-at-*i* from a thermometer.

(2a) The temperature can be read off from www.timeanddate.com/weather/.
⇔ The temperature-at-*i* can be read-off-at-*j* from www.timeanddate.com/weather/.
⇔ The temperature-at-*i* can be read-off from www.timeanddate.com/weather/.

(3a) The extension$_2$-at-*i* of a compound expression is the result of combining-at-*i* the extensions$_2$-at-*i* of the relevant daughters:

$$[\![X\ Y]\!]^i = [\![X]\!]^i +_i [\![Y]\!]^i$$

(b) The extension$_2$-at-*i* of a compound expression is the result of combining-at-*j* the extensions$_2$-at-*i* of the relevant daughters (for whatever *j*).
⇔ The extension$_2$-at-*i* is the result of combining the extensions$_2$-at-*i* of the relevant daughters:

$$[\![X\ Y]\!]^i = [\![X]\!]^i + [\![Y]\!]^i$$

5. Compositionality à la Frege

(FEC$_1$) The extension$_1$ of a compound expression derives from combining the extensions$_{\text{loc}}$ of its immediate parts.

(FEC$_2$) The extension$_2$-at-*i* of a compound expression derives from combining-at-*j* the extensions$_{\text{loc}}$-at-*i* of its immediate parts.
⇔ The extension$_2$-at-*i* of a compound expression derives from combining the extensions$_{\text{loc}}$-at-*i* of its immediate parts.
⇔ The extension$_2$-at-*i* of a compound expression derives from combining the extensions or intensions of its immediate parts, depending on whether the construction is extensional:

$$[\![X\ Y]\!]_i = [\![X]\!]_i + [\![Y]\!]_i$$

(FEC$_+$) The intension of a compound expression derives from combining-at-*j* the extensions$_{\text{loc}}$ of its immediate parts.
⇔ The extension$_2$-at-*i* of a compound expression derives from combining the intensions or in-intensions of its immediate parts, depending on whether the construction is extensional.

6. Back to the Puzzle

Carnapian extensions satisfy:

- (EC) ⇒ (IC); (FC) ⇒ (IC) [(IC) ⇏ (FC)]

… and if intensions are meanings:

- (FC) ⇒ (C)

If meanings are characters (and no monsters are around):

- (FC) ⇒ (C)

HOWEVER, if intensions are Fregean senses:

- (EC) ⇏ (IC); (FC) ⇏ (IC)

7. Some Pertinent References

Carnap, Rudolf: *Meaning and Necessity*. Chicago/London 1947.
Frege, Gottlob: *Function und Begriff*. Jena 1891.
–: 'Über Sinn und Bedeutung'. *Zeitschrift für Philosophie und philosophische Kritik* **100** (1892), 25–50.
Heim, Irene; Kratzer, Angelika: *Semantics in Generative Grammar*. Oxford 1998.
Hodges, Wilfrid: 'Formal Features of Compositionality'. *Journal of Logic, Language and Information* **10** (2001), 7–28.
Janssen, Theo M.V.: *Foundations and Applications of Montague Grammar*. Dissertation, Unversiteit van Amsterdam 1983.
Kaplan, David: 'Deomonstratives'. In: J. Almog *et al. eds.), Themes from Kaplan*. Oxford, pp. 481–563.
Montague, Richard: 'Pragmatics and Intensional Logic'. *Synthèse* **22** (1970a), 68–94.
–: 'Universal Grammar'. *Theoria* **36** (1970b), 373–398.
Zimmermann, Thomas E.: 'Fregean Compositionality'. In: D. Ball & B. Rabern (eds.), *The Science of Meaning*. Oxford 2018, pp. 276–305.