Compositionality Problems and How to Solve Them

Thomas Ede Zimmermann, Frankfurt

1. Compositionality …

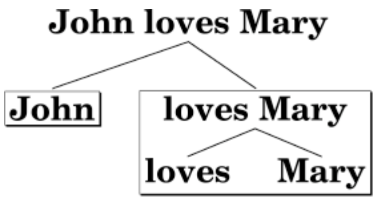
Generalised Principle of Compositionality
The *V* of a complex expression functionally depends on the *V*s of its immediate parts and the way in which they are combined.

Ordinary Principle of Compositionality
The meaning of a complex expression functionally depends on the meanings of its immediate parts and the way in which they are combined.

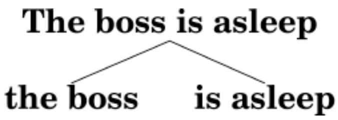
Extensional Principle of Compositionality
The extension of a complex expression functionally depends on the extensions of its immediate parts and the way in which they are combined.

Intensional Principle of Compositionality
The content of a complex expression functionally depends on the contents of its immediate parts and the way in which they are combined.

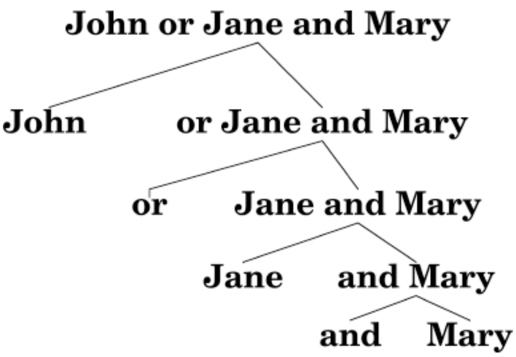
(1)



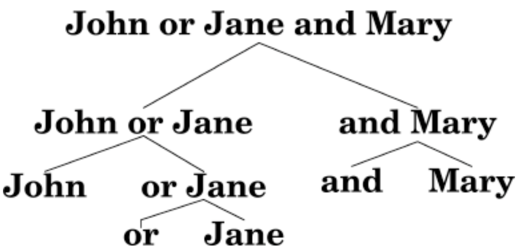
(2)



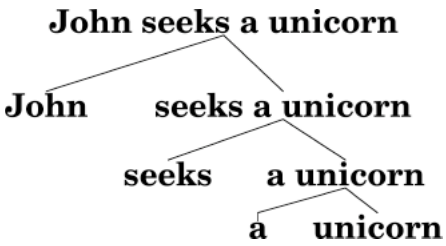
(3)



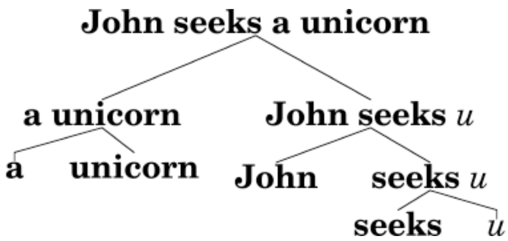
(4)



(5)



(6)

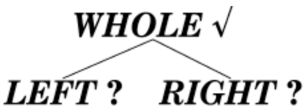


2. Problems …

(7)

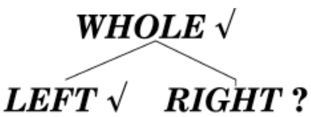
(8)

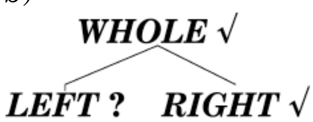
<u>*Type 0*</u>:



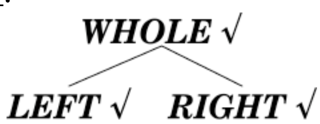
<u>*Type 1*</u>:

a) or: b)





<u>*Type 2*</u>:



(9a) ⟦**Mary is coughing**⟧ = {(w,t) | Mary is coughing in world w at time t}

(b) ⟦**The boss is laughing**⟧

= {(w,t) | whoever is the boss in w at t, is coughing in w at t}

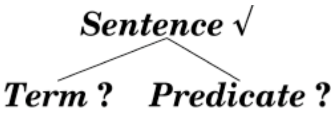
(c) ⟦***Sentence***⟧ ⊆ I

(10a) ⟦**Mary is coughing**⟧ = Γ(⟦**Mary**⟧,⟦**is coughing**⟧)

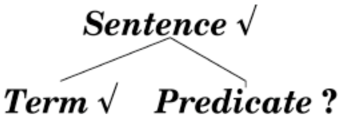
(b) ⟦**The boss is laughing**⟧ = Γ(⟦**the boss**⟧,⟦**is laughing**⟧)

(c) ⟦***Sentence***⟧ = Γ(⟦***Term***⟧,⟦***Predicate***⟧)

(11)



(12)



(13) $\llbracket \textbf{Mary} \rrbracket(w,t) = \text{Mary}$

(b) $\llbracket \textbf{the boss} \rrbracket(w,t) = \text{the boss in } w \text{ at } t$

(c) $\llbracket \textbf{\textit{Term}} \rrbracket: I \rightarrow \wp(I)$

(14a) $\llbracket \textbf{is coughing} \rrbracket(w,t) = \{x \mid x \text{ is coughing in } w \text{ at } t\}$

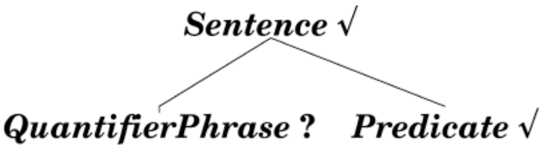
(b) $\llbracket \textbf{is laughing} \rrbracket(w,t) = \{x \mid x \text{ is laughing in } w \text{ at } t\}$

(c) $\llbracket \textbf{\textit{Predicate}} \rrbracket: I \rightarrow \wp(U)$

(15) $\Gamma(\llbracket \textbf{\textit{Term}} \rrbracket, \llbracket \textbf{\textit{Predicate}} \rrbracket) = \{(w,t) \mid \llbracket \textbf{\textit{Term}} \rrbracket(w,t) \in \llbracket \textbf{\textit{Predicate}} \rrbracket(w,t)\}$

(16) **Everyone is shouting.**

(17)



(18a) $\llbracket \textbf{kill} \rrbracket = \{(x,y) \in U^2 \mid x \text{ kills } y\}$

(b) $\llbracket \textbf{introduce} \rrbracket = \{(x,y,z) \in U^3 \mid x \text{ introduces } z \text{ to } y\}$

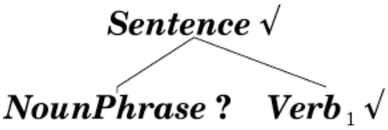
(c) $\llbracket \textbf{\textit{Verb}}_n \rrbracket \subseteq U^n$

(19a) $\llbracket \textbf{John loves Mary} \rrbracket = \Gamma(\llbracket \textbf{John} \rrbracket, \llbracket \textbf{loves Mary} \rrbracket) = 1$

(b) $\llbracket \textbf{Nobody loves Mary} \rrbracket = \Gamma(\llbracket \textbf{every boy} \rrbracket, \llbracket \textbf{loves Mary} \rrbracket) = 0$

(c) $\llbracket \textbf{\textit{Sentence}} \rrbracket = \Gamma(\llbracket \textbf{\textit{NounPhrase}} \rrbracket, \llbracket \textbf{\textit{Verb}}_1 \rrbracket) \subseteq \{\emptyset\}$

(20)



(21a) $\llbracket \textbf{John} \rrbracket = \{X \subseteq U \mid \text{John} \in X\}$

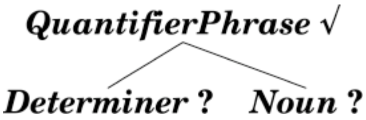
(b) $\llbracket \textbf{nothing} \rrbracket = \{\emptyset\}$

(c) $\llbracket \textbf{\textit{NounPhrase}} \rrbracket \subseteq \wp(U)$

(22) $$\Gamma(\llbracket \textbf{\textit{NounPhrase}} \rrbracket, \llbracket \textbf{\textit{Predicate}} \rrbracket) = \begin{cases} 1, \text{ if } \llbracket \textbf{\textit{NounPhrase}} \rrbracket \in \llbracket \textbf{\textit{Predicate}} \rrbracket \\ 0, \text{ if } \llbracket \textbf{\textit{NounPhrase}} \rrbracket \notin \llbracket \textbf{\textit{Predicate}} \rrbracket \end{cases}$$

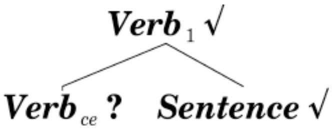
(23) $[\![\textbf{every semanticist}]\!] = \{X \subseteq U \mid S \subseteq X\} = \Gamma([\![\textbf{every}]\!],[\![\textbf{semanticist}]\!])$

(24)



(25a) **John says it is raining.**
(b) **Most experts believe Mary will win the election.**

(26)



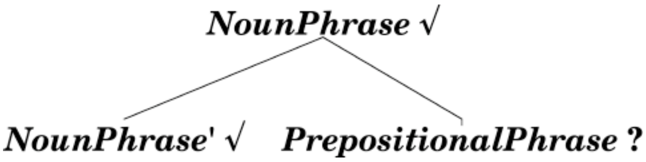
(27a) $[\![\textbf{says it is raining}]\!] = \Gamma([\![\textbf{says}]\!],[\![\textbf{says it is raining}]\!]$

(b) $[\![\textbf{believe Mary will win}]\!] = \Gamma([\![\textbf{believe}]\!],[\![\textbf{Mary will win}]\!])$

(c) $[\![\textit{\textbf{Verb}}_1]\!] = \Gamma([\![\textit{\textbf{Verb}}_{ce}]\!],[\![\textit{\textbf{Sentence}}]\!])$

(28) **no linguist from India**

(29)



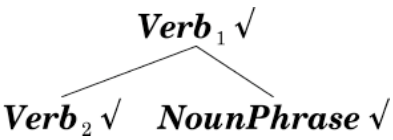
(30a) $[\![\textbf{no linguist from India}]\!] = \Gamma([\![\textbf{no linguist}]\!],[\![\textbf{from India}]\!])$

(b) $[\![\textbf{every pope from India}]\!] = \Gamma([\![\textbf{every pope}]\!],[\![\textbf{from India}]\!])$

(c) $[\![\textit{\textbf{NounPhrase}}]\!] = \Gamma([\![\textit{\textbf{NounPhrase}}']\!],[\![\textit{\textbf{PrepositionalPhrase}}]\!])$

(31) $[\![\textbf{every pope}]\!] = \{X \subseteq U \mid \{p\} \subseteq X\} = \{X \subseteq U \mid \{p\} \cap X \neq \emptyset\} = [\![\textbf{some pope}]\!]$

(32)



(33) $\Gamma([\![\textit{\textbf{Verb}}_2]\!],[\![\textit{\textbf{NounPhrase}}]\!])$

= $\{x \in U \mid \{y \in U \mid (x,y) \in [\![\textit{\textbf{Verb}}_2]\!]\} \in [\![\textit{\textbf{NounPhrase}}]\!]\}$

(34a) **Jones is looking for a sweater.**
(b) **Jones painted a unicorn.**

(34a) **If a farmer owns a donkey, he beats it.**
(b) **Every farmer who owns a donkey beats it.**

3. … and How to Solve Them

A compositionality problem is *solvable* just in case there is a way of replacing all **?** by √ without changing any √.

Observations cf. Zadrozny (1994), Hodges (2001)

Type 0 problems are always solvable.

A *Type 1* problem is solvable iff

$[\![\boldsymbol{RIGHT}_i]\!] = [\![\boldsymbol{RIGHT}_j]\!]$ implies: $[\![\boldsymbol{WHOLE}_i]\!] = [\![\boldsymbol{WHOLE}_j]\!]$,

[or: $[\![\boldsymbol{LEFT}_i]\!] = [\![\boldsymbol{LEFT}_j]\!]$ implies: $[\![\boldsymbol{WHOLE}_i]\!] = [\![\boldsymbol{WHOLE}_j]\!]$,]

for all i and j.

A *Type 2* problem is solvable iff

$[\![\boldsymbol{RIGHT}_i]\!] = [\![\boldsymbol{RIGHT}_j]\!]$ implies: $[\![\boldsymbol{WHOLE}_i]\!] = [\![\boldsymbol{WHOLE}_j]\!]$

and: $[\![\boldsymbol{LEFT}_i]\!] = [\![\boldsymbol{LEFT}_j]\!]$ implies: $[\![\boldsymbol{WHOLE}_i]\!] = [\![\boldsymbol{WHOLE}_j]\!]$,

for all i and j.

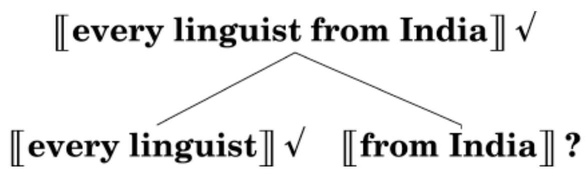
General Strategies for Unsolvable (and Solvable) Compositionality Problems

• <u>*Syntactic Solution*</u>: Redefine input.

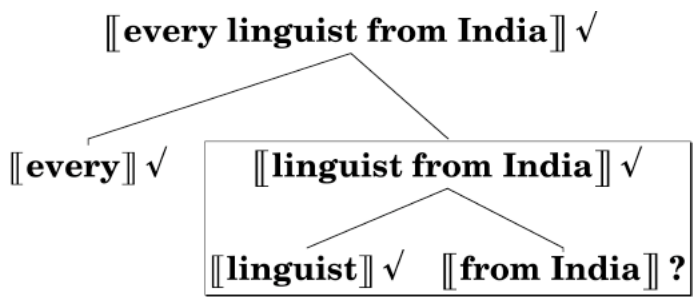
Applications:

– *Type 1* (unsolvable), creating another, *solvable Type 1* problem:

From:

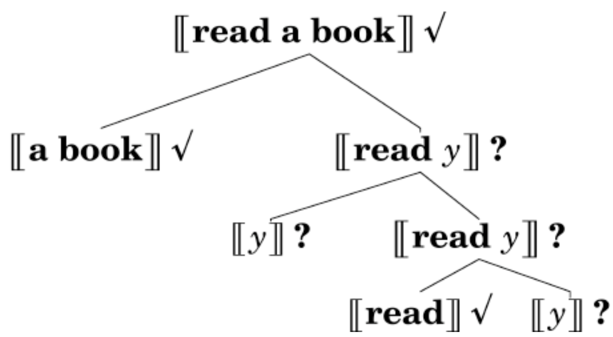


to:



– *Type 2* (solvable), but creating more *Type 0* and *Type 1* problems…

May (1985), Heim & Kratzer (1998)



• <u>*Ontological Solution*</u>: Replace semantic values by more fine-grained ones.
Applications:

\- *Type 1* (unsolvable): Frege (1892)
From:

⟦believes Smith is sick⟧ √

⟦believes⟧ ? **⟦Smith is sick⟧** √

(where ⟦***X***⟧ is ***X***'s extension) to the solvable *Type 1* problem:

⟦believes Smith is sick⟧ √

⟦believes⟧ ? **‖Smith is sick‖** √

(where ‖***X***‖ is suitably fine-grained: *sense, intension,…*).

\- *Type 1* (unsolvable): Lewis (1975), Kamp (1981), Heim (1982)
From:

⟦If a farmer owns a donkey, he beats it⟧ √

⟦a farmer owns a donkey⟧ √ **⟦he beats it⟧ ?**

(where ⟦***X***⟧ is ***X***'s extension) to the solvable *Type 2* problem:

‖If a farmer owns a donkey, he beats it‖ √

‖a farmer owns a donkey‖ √ **‖he beats it‖** √

(where ‖***X***‖ is suitably fine-grained: *relation, context change potential,…*).

General Strategies for Solvable Compositionality Problems

• <u>*Strategy 0:*</u> Frege (1884)
Find covariation between one part and some other entity, and take the latter to be the former's semantic value.

More precisely, given
(L)

$\textit{WHOLE}_1$ $\textit{WHOLE}_2$ $\textit{WHOLE}_3$ … $\textit{WHOLE}_i$ …

$\textit{LEFT}_1$ *RIGHT* $\textit{LEFT}_2$ *RIGHT* $\textit{LEFT}_3$ *RIGHT* $\textit{LEFT}_i$ *RIGHT*

[or:
(R)

$\textit{WHOLE}_1$ $\textit{WHOLE}_2$ $\textit{WHOLE}_3$,…

LEFT $\textit{RIGHT}_1$ *LEFT* $\textit{RIGHT}_2$ *LEFT* $\textit{RIGHT}_3$],

find objects x_i such that:

$[\![\boldsymbol{WHOLE}_i]\!] = [\![\boldsymbol{WHOLE}_j]\!]$ just in case $[\![\boldsymbol{LEFT}_i]\!] = [\![\boldsymbol{LEFT}_j]\!]$

[or $[\![\boldsymbol{WHOLE}_i]\!] = [\![\boldsymbol{WHOLE}_j]\!]$ just in case $[\![\boldsymbol{RIGHT}_i]\!] = [\![\boldsymbol{RIGHT}_j]\!]$]

Then put:

$[\![\boldsymbol{LEFT}_i]\!] := x_i$ [or $[\![\boldsymbol{RIGHT}_i]\!] := x_i$]

Applications:

$[\![\textbf{Mary coughed}]\!]$ √

$[\![\textbf{Mary}]\!]$ = Mary $[\![\textbf{coughed}]\!]$ **?**

or:

$[\![\textbf{Mary coughed}]\!]$ √

$[\![\textbf{Mary}]\!]$ **?** $[\![\textbf{coughed}]\!]$ = set of coughers

$[\![\textbf{every book}]\!]$ √

$[\![\textbf{every}]\!]$ **?** $[\![\textbf{book}]\!]$ = set of books

• <u>*Strategy 1:*</u> Frege (1892); cf. Kupffer (2008); Zimmermann (in prep.)

Determine primary occurrences of valueless expressions and construct their values as contributions in primary occurrences. More precisely, given $[\![\boldsymbol{RIGHT}_1]\!]$, $[\![\boldsymbol{RIGHT}_2]\!]$,… and $[\![\boldsymbol{WHOLE}_1]\!]$, $[\![\boldsymbol{WHOLE}_2]\!]$,… construct:

$[\![\boldsymbol{RIGHT}_1]\!]$	$[\![\boldsymbol{WHOLE}_1]\!]$
$[\![\boldsymbol{RIGHT}_2]\!]$	$[\![\boldsymbol{WHOLE}_2]\!]$
…	…
$[\![\boldsymbol{RIGHT}_i]\!]$	$[\![\boldsymbol{WHOLE}_i]\!]$
…	…

[or:

$[\![\boldsymbol{LEFT}_1]\!]$	$[\![\boldsymbol{WHOLE}_1]\!]$
$[\![\boldsymbol{LEFT}_2]\!]$	$[\![\boldsymbol{WHOLE}_2]\!]$
…	…
$[\![\boldsymbol{LEFT}_i]\!]$	$[\![\boldsymbol{WHOLE}_i]\!]$
…	…

]

and put $[\![\boldsymbol{LEFT}]\!] := f$ such that:

$f([\![\boldsymbol{RIGHT}_i]\!]) = [\![\boldsymbol{WHOLE}_i]\!]$ [or $f([\![\boldsymbol{LEFT}_i]\!]) = [\![\boldsymbol{WHOLE}_i]\!]$]

Application:

$[\![\textbf{a unicorn coughed}]\!]$ √

$[\![\textbf{a unicorn}]\!] = f$ $[\![\textbf{coughed}]\!]$ √

where:

$f([\![\textbf{coughed}]\!]) = [\![\textbf{a unicorn coughed}]\!]$,

$f([\![\textbf{neighed}]\!]) = [\![\textbf{a unicorn neighed}]\!]$, etc.

• <u>*Strategy 2:*</u>

Define combination Γ by collecting all instances:

$\Gamma([\![\boldsymbol{LEFT}_i]\!], [\![\boldsymbol{RIGHT}_i]\!]) = [\![\boldsymbol{WHOLE}_i]\!]$

and find pattern.

Applications:
– quantified objects of transparent verbs:

$\Gamma([\![\boldsymbol{read}]\!], [\![\boldsymbol{a\ book}]\!]) = [\![\boldsymbol{read\ a\ book}]\!]$

$\Gamma([\![\boldsymbol{read}]\!], [\![\boldsymbol{every\ book}]\!]) = [\![\boldsymbol{read\ every\ book}]\!]$

$\Gamma([\![\boldsymbol{buy}]\!], [\![\boldsymbol{a\ book}]\!]) = [\![\boldsymbol{buy\ a\ book}]\!]$

etc.

If $[\![\boldsymbol{X}]\!]$ is $\boldsymbol{X}$'s extension, we have:

$\Gamma(\{(x,y) \mid x \text{ reads } y\}, \lambda P. \vdash P \cap \mathrm{B} \neq \varnothing \dashv) = \{x \mid \{y \mid x \text{ reads } y\} \cap \mathrm{B} \neq \varnothing \}$

$\Gamma(\{(x,y) \mid x \text{ reads } y\}, \lambda P. \vdash \mathrm{B} \subseteq \mathrm{P} \neq \varnothing \dashv) = \{x \mid \mathrm{B} \subseteq \{y \mid x \text{ reads } y\} \}$

$\Gamma(\{(x,y) \mid x \text{ buys } y\}, \lambda P. \vdash P \cap \mathrm{B} \neq \varnothing \dashv) = \{x \mid \{y \mid x \text{ buys } y\} \cap \mathrm{B} \neq \varnothing \}$

etc.

– the pattern being:

$\Gamma([\![\boldsymbol{LEFT}_i]\!], [\![\boldsymbol{RIGHT}_i]\!]) = [\![\boldsymbol{RIGHT}_i]\!](\{x \mid \{y \mid (x,y) \in [\![\boldsymbol{LEFT}_i]\!]\}\})$

– quantified objects of opaque verbs:

$[\![\textbf{seeks a unicorn}]\!]$ √

$[\![\textbf{seeks}]\!] = f$ $\quad [\![\textbf{a unicorn}]\!]$ √

where:

(*) $f([\![\textbf{a unicorn}]\!]) = [\![\textbf{seeks a unicorn}]\!]$

$f([\![\textbf{a horse}]\!]) = [\![\textbf{seeks a horse}]\!]$, etc.

If $[\![\boldsymbol{X}]\!]$ is $\boldsymbol{X}$'s extension, then:

$[\![\textbf{a unicorn}]\!] = [\![\textbf{a ghost}]\!]$

and so:

$f([\![\textbf{a unicorn}]\!]) = f([\![\textbf{a ghost}]\!])$

BUT: $f([\![\textbf{a unicorn}]\!]) = [\![\textbf{seeks a unicorn}]\!] = \{x \mid x \text{ seeks a unicorn}\}$

$\neq$ $\{x \mid \text{x seeks a ghost}\} = [\![\textbf{seeks a ghost}]\!] = f([\![\textbf{a ghost}]\!])$

=> NO EXTENSIONAL SOLUTION!

If $[\![\boldsymbol{X}]\!]$ is $\boldsymbol{X}$'s intension, then:

$f([\![\textbf{a unicorn}]\!])(i) = \{x \mid x \text{ seeks a unicorn at [index] } i\}$

$f([\![\textbf{a horse}]\!])(i) = \{x \mid x \text{ seeks a horse at } i\}$

$f([\![\textbf{a ghost}]\!])(i) = \{x \mid x \text{ seeks a horse at } i\}$

unclear how (and even: whether) value depends on argument
=> additional strategy needed:

Reduction by paraphrase
… also works for (34b)

References
(on request)