Concerning Philippe Schlenker's *Conditionals as Definite Descriptions*, submitted to the *Journal of Language and Computation* (special issue *Choice Functions in Semantics*)

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Details

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p. 15: Condition 1* is *not* just Condition 1 repeated in that it replaces elementhood by subsethood; also it should force the choice to be non-empty whenever the domain is: $\emptyset \neq f(d,E) \subseteq E$ if $E \neq \emptyset$. This is well motivated in terms of the intended applications and in terms of formalism (see my Condition 1^{+} below) but it is not covered by the other conditions.

p. 15: The motivation of Condition 2* is somewhat incomplete. In fact, together with (modified) Condition 1 it turns out to be equivalent to the choice being based on a universal (centered) salience/similarity ordering. (See the appendix below.) Makes me wonder why the whole story is not told directly in terms of orderings in the first place, especially given the fact that the restrictions (transitivity, well-foundedness) are more easily formulated and motivated than Condition 2*. (This also applies to von Heusinger's original approach.)

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Appendix

Let U and D be non-empty sets and f: $D \times \wp(U) \rightarrow U$. Consider the following conditions (depending on $d \in D$):

(C1+) $\emptyset \neq f(d,E) \subseteq E$, for each and non-empty $E \subseteq U$.

(C2)* $f(d,E') = f(d,E) \cap E'$ whenever $d \in D$, $E' \subseteq E \subseteq U$ and $f(d,E) \cap E' \neq \emptyset$.

Then (for any $d \in D$) the conjunction of *(C1+)* and *(C2*)* holds iff there is a transitive well-founded relation $\leq_d$ such that $f(d,E) = \{e \subseteq E \mid \text{for all } e' \in E\text{: } e \leq_d e'\}$.

Proof

Fix $d \in D$ and omit reference to it: $f(E) := f(d,E)$, $e \leq e' :\Leftrightarrow e \leq_d e'$.

"$\Leftarrow$":

ad (C1+): This is the well-foundedness of $\leq$.

ad (C2)*: (This is what the argument at the bottom of p. 15 is about.)

Assume $E' \subseteq E \subseteq U$, $f(E) \cap E' \neq \emptyset$ and $e \in E$. It must be shown that:

$e \in f(E')$ iff $e \in f(E) \cap E'$, i.e. that:

(!) [$e \in E'$ and for all $e' \in E'$: $e \leq e'$] iff [$e \in E'$and for all $e' \in E$: $e \leq e'$]

"$\Rightarrow$": Assume $e \in E'$ and $e \leq e'$, for all $e' \in E'$. Pick $e^* \in f(E) \cap E'$ ($\neq \emptyset$, by assumption). Then <u>$e \leq e^*$</u> (because $e^* \in E'$). But if $e' \in E$, then <u>$e^* \leq e'$</u>, because $e^* \in f(E)$. Hence, by the transitivity of $\leq$, $e \leq e'$.

"$\Leftarrow$": Obvious because $E' \subseteq E$.

"$\Rightarrow$":

Now assume *(C1+)* and *(C2*)* and, for given $d \in D$, put:

$$\leq_d := \{(e,e') \in E \mid e \in f(d,\{e,e'\})\}.$$

The following propositions must be shown:

a): $\leq$ is transitive;

b): $\leq$ is well-founded;

c): $f(E) = \{e \in E \mid \text{for all } e' \in E\text{: } e \leq e'\}$.

ad a): Assume $e \leq e' \leq e''$. It is to be shown that (*) $e \leq e''$. We first show that:

(#) $e \in f(E)$,

where $E = \{e, e', e''\}$. From this (*) follows: Since $e \in \underline{f(E) \cap \{e, e''\}}$, *(C2*)* implies: $e \in \underline{f(\{e, e''\})}$, i.e. $e \leq e''$.

Now for (#). There are two cases:

Case 1: $e' \in f(E)$. Hence $f(E) \cap \{e, e'\} \neq \emptyset$. So *(C2*)* applies and (since $e \leq e'$): $\underline{e \in} f(\{e, e'\}) = \underline{f(E)} \cap \{e, e'\}$. So $\underline{(\#)}$ holds.

Case 2: $e' \notin f(E)$. Then (#) must hold. For assume otherwise. Then $e'' \in f(E)$, in view of *(C1+)* – and (since $e' \leq e''$): $\underline{e' \in} f(\{e', e''\}) = \underline{f(E)} \cap \{e', e''\}$, by *(C2*)* and given that $e'' \in f(E) \cap \{e', e''\}$, in contradiction to the case assumption.

ad b): Assume $\emptyset \neq E \subseteq U$. It must be shown that there exists $e^* \in E$ such that $e^* \leq e$, for all $e \in E$. By *(C1+)*, $\emptyset \neq f(E) \subseteq E$ (given that $E \neq \emptyset$). Pick $e^* \in f(E)$ and arbitrary $e \in E$. By *(C2*)*, $f(\{e^*, e\}) = f(E) \cap \{e^*, e\}$ (given that $\{e^*, e\} \subseteq E$ and $e^* \in f(E) \cap \{e^*, e\}$). Hence – again given that $e^* \in f(E) \cap \{e^*, e\}$ – $e^* \in f(\{e^*, e\})$, i.e.: $e^* \leq e$.

ad c): $\subseteq$: If $e \in f(E)$, then $e \in E$, by *(C1+)*. If, moreover, $e' \in E$, then $\{e, e'\} \subseteq E$ and $f(E) \cap \{e, e'\} \neq \emptyset$. Hence, by *(C2*)*, $f(\{e, e'\}) = f(E) \cap \{e, e'\}$. However, $e \in f(E) \cap \{e, e'\}$, and so $e \in f(\{e, e'\})$.

$\supseteq$: Let $e \in E$ and assume that (+) $e \leq e'$, for all $e' \in E$. $E \neq \emptyset$ and so, by *(C1+)*, there is some $e^* \in f(E) \cap E$. By (+), $e \leq e^*$, i.e.: $e \in f(\{e, e^*\}) = f(E) \cap \{e, e^*\}$, by *(C2*)*. So $e \in f(E)$.