

Report on Massimo Poesio's *Semantic Ambiguity and Perceived Ambiguity*
[... SEE p. 3f.]

Here are some specific comments on the formal apparatus:

[...]

The ‘multiplication technique’ alluded to on p. 32 and exemplified by P’s approach is not new. It has, e.g., been used by Hamblin in his semantics for questions, and Rooth in his dissertation on focus. It consists in lifting operations Γ on meanings (to use a neutral term) to operations Γ^+ on sets of meanings, using the following straightforward recipe:

$$\Gamma^+(M_1, \dots, M_n) = \{\Gamma(m_1, \dots, m_n) \mid m_1 \in M_1, \dots, m_n \in M_n\},$$

where it is assumed that the Cartesian product of M_1, \dots, M_n is [a subset of] Γ ’s domain.

The above recipe can also be applied to λ -abstraction and then automatically solves P’s problem about generalizing it. In doing so, one only has to keep in mind that (ordinary) λ -abstraction, like all variable-binding operations, combines functions from variable assignments (in Ass) to unambiguous denotations:

$$\begin{aligned} & \|\lambda x \beta\| = \Lambda(\|x\|, \|\beta\|) \\ = & \text{that } f: \text{Ass} \rightarrow (\mathcal{S} \rightarrow (D_\tau \rightarrow D_\tau)) \text{ such that: for all } g \in \text{Ass}, s \in \mathcal{S}, a \in D_\tau: \\ & f(g)(s)(a) = \|\beta\|(s)(g\{x/a\}). \end{aligned}$$

(For this to work as a definition of Λ , one must show that the condition on f is independent of the choice of the variable x , which is standard and easy.) Lifting Λ according to the above recipe results in:

$$\begin{aligned} & \|\lambda x \beta\| \\ = & \Lambda^+(\|x\|, \|\beta\|) \\ = & \{\Lambda(m_1, m_2) \mid m_1 \in \|x\|, m_2 \in \|\beta\|\} \end{aligned} \quad \begin{array}{c} - \quad - \\ - \quad - \\ - \quad - \end{array}$$

$$\begin{aligned}
&= \{m \mid \text{for some } m_1 \in \|\mathbf{x}\|, m_2 \in \|\beta\|: m = \Lambda(m_1, m_2)\} \\
&= \{f: \text{Ass} \rightarrow (S \rightarrow (D_\tau \rightarrow D_\tau)) \mid \text{such that: for some } m_1 \in \|\mathbf{x}\|, m_2 \in \|\beta\|: \\
&\quad \text{for all } g \in \text{Ass}, \underline{s} \in S, \underline{a} \in D_\tau: f(g)(\underline{s})(\underline{a}) = m_2(\underline{s})(g\{\mathbf{x}/\underline{a}\}) \},
\end{aligned}$$

which is no longer subject to P's worries. (I am not saying that P's own repair is not working – I haven't checked this! – but I think that a more systematic and principled approach to set denotation would not have led to his problem in this first place.)

[...]