Semantics:
The Theory of Extension and Intension

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Plan:

- Frege’s Principle
- A Farewell to Psychologism
- Extensions and Truth Values
- Propositions and Intensions
...based on material from

Two arrangements of unambiguous words can lead to different meanings:

(1) a. John’s son introduced Mary’s daughter to Bill and Jane
    b. Jane introduced John’s daughter and Mary’s son to Bill

... even if the word order is the same

(2) John’s son introduced Mary’s daughter to Bill and Jane or Harry

⇒ *Syntactic structure has an effect on interpretation*
... but how does syntactic structure affect interpretation?

(3) Frege’s Principle of Compositionality
The meaning of a composite expression is a function of the meanings of its immediate constituents and the way these constituents are put together.

... Yes, but what (kind of objects) are all these meanings?
When learning a new word, we learn how to combine a certain pronunciation, its phonetics and phonology, with its meaning. Thereby, a previously meaningless sequence of sounds becomes vivid, we associate with it an idea. In this case, one might be tempted to say that the meaning of an expression is the idea or conception a speaker associates with its utterance.
Fregean and Wittgensteinian objections . . .
... against such a “psychologistic” notion of meaning:

- **Subjectivity**: Different speakers may associate different things with a single word at different occasions: such “meanings,” however, cannot be objective, but will rather be influenced by personal experience, and one might wonder how these “subjective meanings” serve communication between different subjects.

- **Limited Coverage**: We can have mental images of nouns like *horse* or *table*, but what on earth could be associated with words like *and*, *most*, *only*, *then*, *of*, *if*, . . . ?

- **Irrelevance**: Due to different personal experiences, speakers can have all sorts of associations without this having any influence on the meaning of an expression.

- **Privacy**: The associations of an individual person are in principle inaccessible to other speakers. So, again, how can they be used for interpersonal communication?
On the other hand ...

**MEANING** SERV**E**S COMMUNICATION ... and so:

**MEANINGS** ought to be identified with

**COMMUNICATIVE FUNCTIONS** of expressions

... as in the tradition of ...
A Farewell to Psychologism

... or (more recently)

FORMAL SEMANTICS
LOGICAL [or FORMAL] SEMANTICS

Meanings $\approx$ (certain) communicative functions of expressions, viz.:

- **Content**: *Which information* is expressed ...
- **Reference**: ... and *what* this information is *about*
A Farewell to Psychologism

LOGICAL [or FORMAL] SEMANTICS

The meaning of any expressions has (at least) two components, viz. its:

- **intension** ≈ its contribution to the content of expressions in which it occurs
- **extension**: ≈ its contribution to the reference of expressions in which it occurs
- ... and maybe more (but not in this course)

In the simplest cases:

- Intension is content.
- Extension is reference.

We will start with the latter ...
Extensions for Words and Phrases

Some examples:

(4) — *Irvine, Noam Chomsky* (proper names)
— *the president of the US, the capital of Germany* (definite descriptions)
— *table, horse, book* (nouns)
— *bald, red, stupid, alleged* (adjectives)
— *nobody, nothing, no dog* (negative quantifiers)

- What do these expressions refer to?
- What is their contribution to reference?
Extensions for Words and Phrases

[What do these expressions refer to?]

**Referential** expressions like

- proper names (like *Vienna, Roman Polanski, ...*)
- definite descriptions (like *the capital of Austria, the director of ROSEMARY’S BABY...*)
- (some uses of) personal pronouns (like *she*)
- ...

(are used to) refer to persons, places, or other **individuals**.

The referent of a referential expression also forms its **extension**.
Extensions for Words and Phrases

[What do these expressions refer to?]

- **common (count) nouns** like *table*, *car*, ...

as well as some (‘intersective‘)

- **adjectives** like *blond*, *rectangular*, ...

do not refer to single individuals but show **multiple** reference.

The **set** of all its referents forms the **extension** of such a multiply extensional expression.
A set is an abstract collection of (possibly, but not necessarily concrete) objects, their elements.

Elemenhood is a relational concept: an object $x$ is or is not an element of a given set $y$.
Notation: $x \in y$ vs. $x \notin y$

A set $A$ is a subset of a (not necessarily distinct) set $B$ iff (= if and only if)
every element of $A$ is an element of $B$ and vice versa.
Notation: $A \subseteq B$

The identity criterion for sets $A$ and $B$ is sharing the same elements (‘extensionality’):
$A = B$ iff $A \subseteq B$ and $B \subseteq A$

Sets are defined by set abstraction:
$\{x : \ldots x \ldots \}$ is that set whose elements are precisely those objects $x$ such that the condition $\ldots x \ldots$ holds.
Notation: $\emptyset$ is $\{x: x \neq x\}$
Extensions for Words and Phrases

[What do these expressions refer to?]

- **common (count) nouns** like *table*, *car*, ...

as well as some (‘intersective‘)

- **adjectives** like *blond*, *rectangular*, ...

do not refer to single individuals but show **multiple** reference.

The **set** of all its referents forms the **extension** of such a multiply extensional expression.
**NB1**: The extension of

- *the current German chancellor*

is Angela Merkel

but this will change . . .

In four years from now the extension of *the current German chancellor* is going to be another person and it used to be 20 years ago . . .
SO:

- The extension of *the current German chancellor* is changing over time
  … and so are extensions in general.
NB2: The extension of

- *current German chancellor*

is the set of all current German chancellors – i.e., a set with one member.
However, the extension of

- **the current German chancellor**

is the current German chancellor, i.e., a person.
SO:

- *current German chancellor* (whose extension is \{A.M.\}),

and:

- *the current German chancellor*

do not have the same extension!\(^1\)

\(^1\)on standard set-theoretic assumptions
NB3: The (current) extension of

- *current French king*

is the set of all current French kings – i.e., the empty set.
However, the extension of

- the current king of France

would have to be the current French king

... but there is no such (existing) person!
SO: unlike

- current king of France (whose extension is $\emptyset$),

- the current king of France

appears to have no extension.

We will henceforth ignore such void descriptions. (Read chapter 9 for more on this ...)
Extensions for Words and Phrases

Not all nouns are count nouns — some are:

- **mass nouns**: milk, information,...
  Hallmark: no plural (without meaning shift)

- **relational nouns**: brother, copy,...
  Hallmark: possessives receive “special” meaning

- **functional nouns**: father, surface,...
  Hallmark: relational plus inherent uniqueness

Mass nouns will be ignored in the following.
The extensions of relational and functional nouns can be identified with sets of \textbf{ordered pairs} of individuals.

\textbf{Relational examples:}

(5)

\textit{brother}:

\{⟨Ethan, Joel⟩, ⟨Joel, Ethan⟩, ⟨Deborah, Joel⟩, ⟨Deborah, Ethan⟩, \ldots\}

\textit{arm}:

\{⟨Ludwig, Ludwig’s right arm⟩, ⟨Ludwig, Ludwig’s left arm⟩, ⟨Paul, Paul’s left arm⟩, \ldots\}

\textit{idea}:
Extensions for Words and Phrases

Functional examples:

(6)

*birthplace*:

\{\langle Adam, Paradise\rangle, \langle Eve, Paradise\rangle, \langle John, Liverpool\rangle, \langle Yoko, Tokyo\rangle, \ldots \}

*mother*:

\{\langle Cain, Eve\rangle, \langle Abel, Eve\rangle, \langle Stella, Linda\rangle, \langle Sean, Yoko\rangle, \ldots \}

*surface*:

\{\langle Mars, Mars’s surface\rangle, \langle Earth, Earth’s surface\rangle, \ldots \}

In addition to being relational, the extensions \( f \) of functional nouns in (6) are **functions**, i.e., they satisfy a **uniqueness** condition:
The extension of a referential expression is an individual.

The extension of a count noun (or intersective adjective) is a set of individuals.

The extension of a relational noun is a binary relation among [set of ordered pairs of] individuals.

The extension of a functional noun is a function mapping individuals to individuals.
Extensions of verbs and verb phrases

(8)

*sleep*: the set of sleepers

*kiss*: a relation between kissers and kissees, i.e., the set of pairs \( \langle x, y \rangle \) such that \( x \) kisses \( y \)

*donate*: a **three-place relation**, a set of triples
(9)

<table>
<thead>
<tr>
<th>type of expression</th>
<th>type of extension</th>
<th>example</th>
<th>extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>intransitive verb</td>
<td>set of individuals</td>
<td><em>sleep</em></td>
<td>the set of sleepers</td>
</tr>
<tr>
<td>transitive verb</td>
<td>set of pairs of individuals</td>
<td><em>eat</em></td>
<td>the set of pairs ⟨eater, eaten⟩</td>
</tr>
<tr>
<td>ditransitive verb</td>
<td>set of triples of individuals</td>
<td><em>donate</em></td>
<td>the set of triples ⟨donator, recipient, donation⟩</td>
</tr>
</tbody>
</table>
(10) **Parallelism between valency and type of extension:**
The extension of an $n$-place verb is always a set of $n$-tuples.
### Truth Values as Extensions of Sentences

1. **The Pope shows the President the Vatican Palace**

2. **Verb or Verb Phrase**
   - **shows**
     - **Valency**: 3
     - **Extension**: triples \( \langle a, b, c \rangle \)
     - **Where**: \( a \) shows \( b \) to \( c \)

   - **shows**
     - **Valency**: 2
     - **Extension**: pairs \( \langle a, b \rangle \)
     - **Where**: \( a \) shows \( b \) to the President

   - **shows**
     - **Valency**: 1
     - **Extension**: 1-tuples \( \langle a \rangle \)
     - **Where**: \( a \) shows the Vatican Palace to the President

3. **Sentence**
   - **The Pope shows the President the Vatican Palace**
     - **Valency**: 0
     - **Extension**: 0-tuples \( \langle \rangle \)
     - **Where**: the Pope shows the Vatican Palace to the President
Truth Values as Extensions of Sentences

(14) sentence \[ \text{The Pope shows the President the Vatican Palace} \] valency 0 extension the 0-tuples \( \langle \rangle \) where the Pope shows the Vatican Palace to the president

Standard Assumption 1

There is precisely one zero-tuple, viz., the empty set \( \emptyset \).

Two cases:

- **IF** the Pope does **NOT** show the Vatican Palace to the president, then NO zero-tuple satisfies the condition that the Pope shows the Vatican Palace to the president and so the extension in (14) is empty, i.e.: \( \emptyset \).

- **IF** the Pope **DOES** show the Vatican Palace to the president, then ANY zero-tuple satisfies the condition that the Pope shows the Vatican Palace to the president.
Truth Values as Extensions of Sentences

Two cases:

- If the Pope does not show the Vatican Palace to the president, then the extension in (14) is: $\emptyset$.

- If the Pope does show the Vatican Palace to the president, then the extension in (14) is: $\{\emptyset\}$.

(Wildly) generalizing:

- If a (declarative) sentence is false, its extension is: $\emptyset$.

- If a (declarative) sentence is true, its extension is: $\{\emptyset\}$.
Truth Values as Extensions of Sentences

(Wildly) generalizing:

- If a (declarative) sentence is false, its extension is: $\emptyset$.
- If a (declarative) sentence is true, its extension is: $\{\emptyset\}$.

<table>
<thead>
<tr>
<th>Standard Assumption 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\emptyset = 0, {\emptyset} = 1$.</td>
</tr>
</tbody>
</table>

(15) **Frege’s Generalization**
The extension of a sentence $S$ is its truth value, i.e., 1 if $S$ is true and 0 if $S$ is false.
REMINDER

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Cases and Propositions

(3) Four fair coins are tossed
(4) At least one of the 4 tossed coins lands heads up
(5) At least one of the 4 tossed coins lands heads down
(6) Exactly 2 of the 4 tossed coins land heads up
(7) Exactly 2 of the 4 tossed coins land heads down
Cases and Propositions

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(7) Exactly 2 of the 4 tossed coins land heads down

*Definition [to be revised]*
The proposition expressed by a sentence is the set of possible cases of which that sentence is true.
Cases and Propositions

(16) possible cases | $c_1$ | $c_2$ | $c_3$ | $c_4$
--- | --- | --- | --- | ---
1 | 1 | 1 | 1 | 1
2 | 1 | 1 | 1 | 0
3 | 1 | 1 | 0 | 1
... | ... | ... | ... | ...
14 | 0 | 0 | 1 | 0
15 | 0 | 0 | 0 | 1
16 | 0 | 0 | 0 | 0
(17) a. Four coins were tossed when John coughed  
b. Four coins were tossed when John coughed and it started to rain  
c. . . .  

(18) **[Revised] Definition**  
   a. The proposition $||S||$ expressed by a sentence $S$ is the set of possible worlds of which that sentence is true.  
   
   A sentence $S$ is **true** of a possible world $w$ if and only if $w \in ||S||$. 
From Propositions to Intensions

<table>
<thead>
<tr>
<th>world</th>
<th>truth value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>1</td>
</tr>
<tr>
<td>$w_2$</td>
<td>0</td>
</tr>
<tr>
<td>$w_3$</td>
<td>1</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$w_n$</td>
<td>0</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

(19) 

(20) **Definition**

The **intension** of an expression $\alpha$ is that function $f$ such that for every possible world $w$, $f(w) = \alpha$’s extension at $w$. 

\[\begin{array}{c|c}
\text{world} & \text{truth value} \\
\hline
w_1 & 1 \\
\hline
w_2 & 0 \\
\hline
w_3 & 1 \\
\hline
\vdots & \vdots \\
\hline
w_n & 0 \\
\hline
\vdots & \vdots \\
\end{array}\]
If you want to know more about this, read . . .