Semantics: The Theory of Extension and Intension

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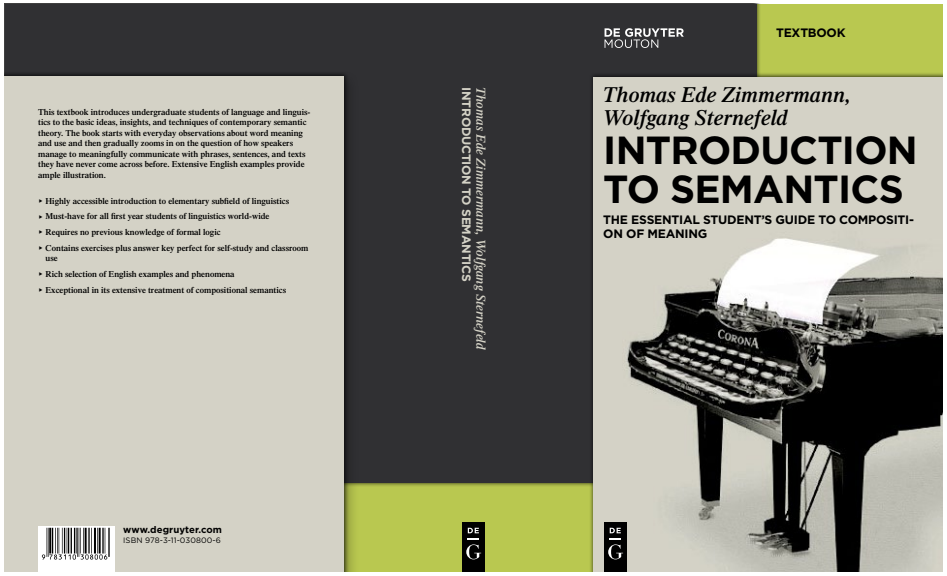
Guest Lecture in Greg Scontras's *Psychology of Language*
UCI, February 2018

Plan:

- Frege's Principle
- A Farewell to Psychologism
- Extensions and Truth Values
- Propositions and Intensions

. . . based on material from



T. E. Zimmermann, W. Sternefeld: *Introduction to Semantics.* Berlin/New York 2013

Frege's Principle

Two arrangements of unambiguous words can lead to different meanings:

(1) a. John's son introduced Mary's daughter to Bill and Jane
    b. Jane introduced John's daughter and Mary's son to Bill

. . . even if the word order is the same

(2) John's son introduced Mary's daughter to Bill and Jane or Harry

$\Rightarrow$ *Syntactic structure has an effect on interpretation*

Frege's Principle

. . . but how does syntactic structure affect interpretation?

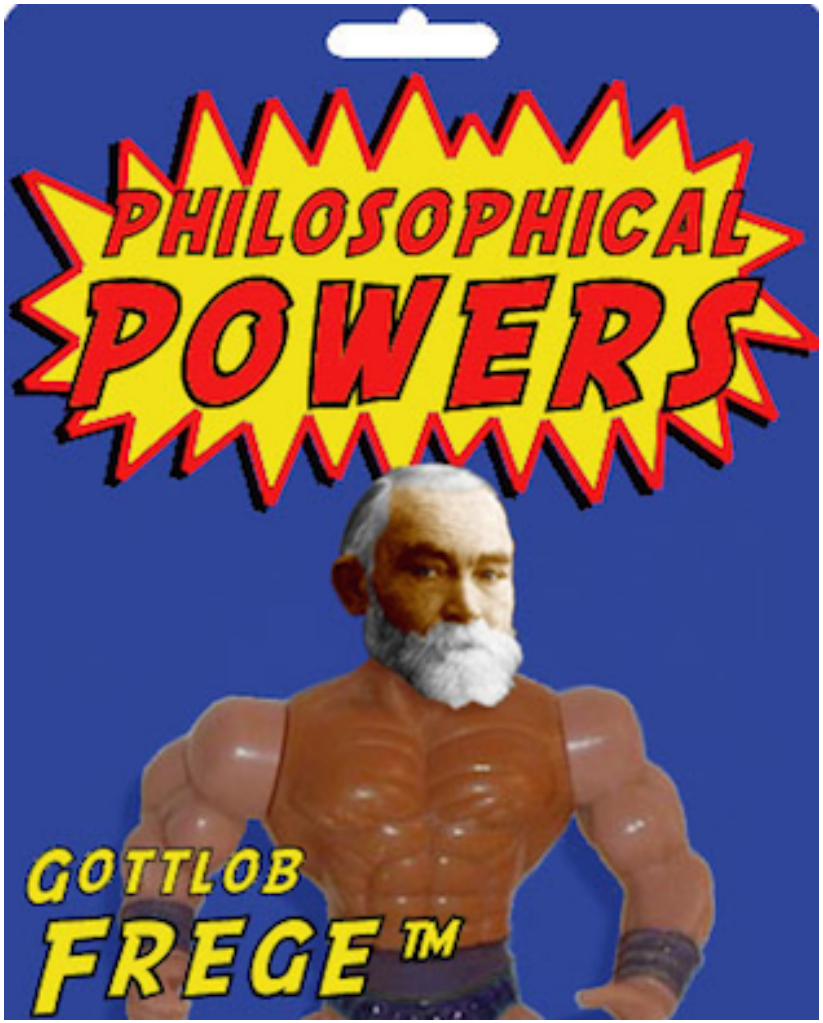
(3) *Frege's Principle of Compositionality*
The meaning of a composite expression is a function of the meanings of its immediate constituents and the way these constituents are put together.

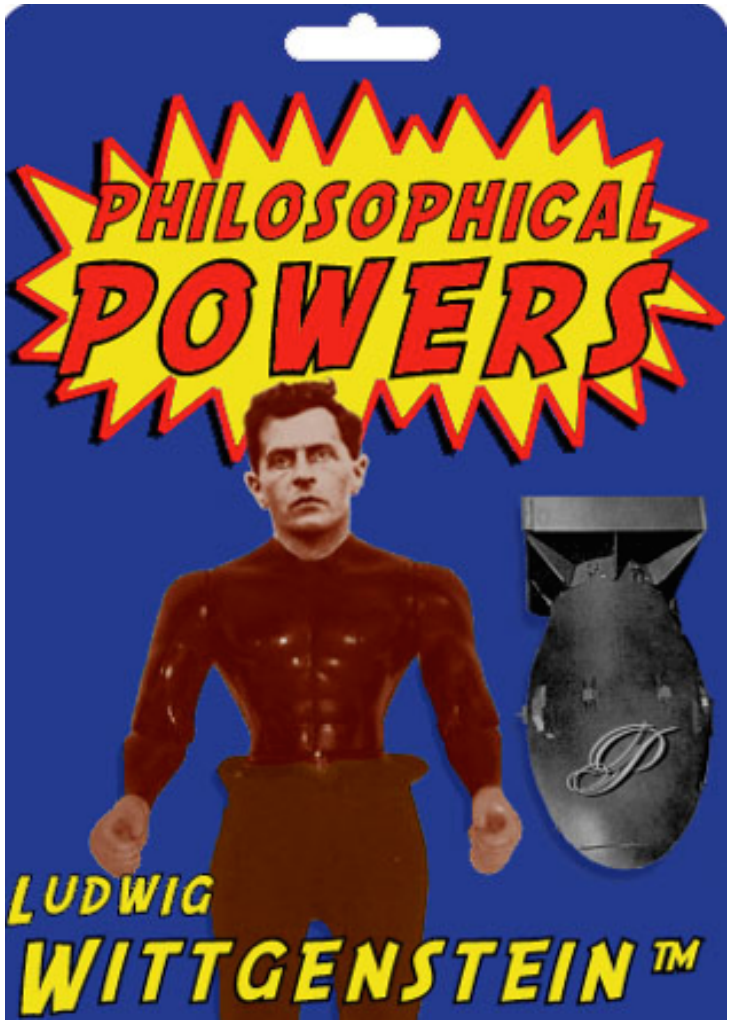
. . . Yes, but what (kind of objects) are all these meanings?

A Farewell to Psychologism

When learning a new word, we learn how to combine a certain pronunciation, its phonetics and phonology, with its meaning. Thereby, a previously meaningless sequence of sounds becomes vivid, we associate with it an idea. In this case, one might be tempted to say that the meaning of an expression is the idea or conception a speaker associates with its utterance.

Fregean and Wittgensteinian objections . . .





A Farewell to Psychologism

... against such a "**psychologistic**" notion of meaning:

- **Subjectivity**: Different speakers may associate different things with a single word at different occasions: such "meanings," however, cannot be objective, but will rather be influenced by personal experience, and one might wonder how these "subjective meanings" serve communication between different subjects.
- **Limited Coverage**: We can have mental images of nouns like *horse* or *table*, but what on earth could be associated with words like *and, most, only, then, of, if*, . . . ?
- **Irrelevance**: Due to different personal experiences, speakers can have all sorts of associations without this having any influence on the meaning of an expression.
- **Privacy**: The associations of an individual person are in principle inaccessible to other speakers. So, again, how can they be used for interpersonal communication?

A Farewell to Psychologism

On the other hand ...

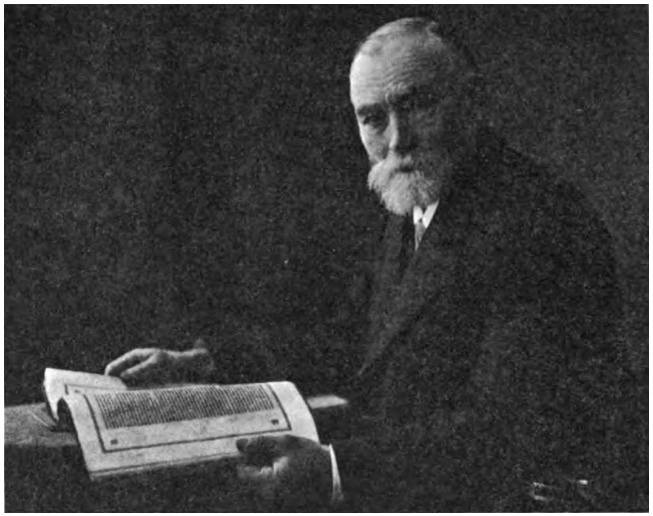
MEANING SERVES COMMUNICATION ... and so:

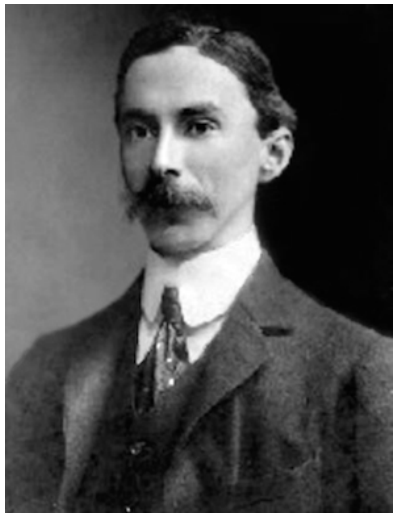
MEANINGS ought to be identified with

COMMUNICATIVE FUNCTIONS of expressions

... as in the tradition of ...

LOGICAL SEMANTICS





A Farewell to Psychologism

... or (more recently)
FORMAL SEMANTICS

A Farewell to Psychologism

LOGICAL [or FORMAL] SEMANTICS

Meanings $\approx$ (certain) **communicative functions** of expressions, viz.:

- **Content**: *Which information* is expressed ...
- **Reference**: ... and *what* this information is *about*

LOGICAL [or FORMAL] SEMANTICS

The **meaning** of any expressions has (at least) **two components**, viz. its:

- **intension** $\approx$ its contribution to the content of expressions in which it occurs
- **extension**: $\approx$ its contribution to the reference of expressions in which it occurs
- . . . and maybe more (but not in this course)

In the simplest cases:

- Intension is content.
- Extension is reference.

We will start with the latter ...

Extensions for Words and Phrases

Some examples:

(4) — *Irvine, Noam Chomsky* (**proper names**)
— *the president of the US, the capital of Germany* (**definite descriptions**)
— *table, horse, book* (**nouns**)
— *bald, red, stupid, alleged* (**adjectives**)
— *nobody, nothing, no dog* (**negative quantifiers**)

- What do these expressions refer to?
- What is their contribution to reference?

Extensions for Words and Phrases

[What do these expressions refer to?]

Referential expressions like

- proper names (like *Vienna*, *Roman Polanski*, ...)
- definite descriptions (like *the capital of Austria*, *the director of* ROSEMARY'S BABY...)
- (some uses of) personal pronouns (like *she*)
- ...

(are used to) refer to persons, places, or other **individuals**.

The referent of a referential expression also forms its **extension**.

Extensions for Words and Phrases

[What do these expressions refer to?]

- **common (count) nouns** like *table*, *car*, ...

as well as some ('intersective')

- **adjectives** like *blond*, *rectangular*, ...

do not refer to single individuals but show **multiple** reference.

The set of all its referents forms the **extension** of such a multiply extensional expression.

Set theory in 2 minutes (and without tears)

- A **set** is an abstract collection of (possibly, but not necessarily concrete) objects, their elements.
- Elementhood is a relational concept: an object x is or is not an **element of** a given set y.
  Notation: $x \in y$ vs. $x \notin y$
- A set A is a **subset** of a (not necessarily distinct) set B iff [= if and only if]
  every element of A is an element of B and *vice versa*.
  Notation: $A \subseteq B$
- The identity criterion for sets A and B is sharing the same elements ('extensionality'):
  $A = B$ iff $A \subseteq B$ and $B \subseteq A$
- Sets are defined by set abstraction:
  $\{x : \ldots x \ldots\}$ is that set whose elements are
  precisely those objects x such that the condition $\ldots x \ldots$ holds.
  Notation: $\emptyset$ is $\{x\text{: } x \neq x\}$

Extensions for Words and Phrases

[What do these expressions refer to?]

- **common (count) nouns** like *table*, *car*, ...

as well as some ('intersective')

- **adjectives** like *blond*, *rectangular*, ...

do not refer to single individuals but show **multiple** reference.

The set of all its referents forms the **extension** of such a multiply extensional expression.

Extensions for Words and Phrases

NB1: The extension of

- *the current German chancellor*

is Angela Merkel

but this will change . . .

In four years from now the extension of *the current German chancellor* is going to be another person and it used to be 20 years ago . . .

Extensions for Words and Phrases

SO:

- The extension of *the current German chancellor* is changing over time . . . and so are extensions in general.

NB2: The extension of

- *current German chancellor*

is the set of all current German chancellors – i.e., a set with one member.

Extensions for Words and Phrases

However, the extension of

- ***the*** *current German chancellor*

is the current German chancellor, i.e., a person.

SO:

- *current German chancellor* (whose extension is $\{A.M.\}$),

and:

- *the current German chancellor*

do not have the same extension![1]

[1] on standard set-theoretic assumptions

NB3: The (current) extension of

- *current French king*

is the set of all current French kings – i.e., the empty set.

Extensions for Words and Phrases

However, the extension of

- ***the*** *current king of France*

would have to be the current French king

... but there is no such (existing) person!

SO: unlike

- *current king of France* (whose extension is $\emptyset$),
- *the current king of France*

appears to have no extension.

We will henceforth ignore such **void** descriptions. (Read chapter 9 for more on this ...)

Extensions for Words and Phrases

Not all nouns are count nouns — some are:

- *mass nouns*: *milk*, *information*,...
  Hallmark: no plural (without meaning shift)
- **relational nouns**: *brother*, *copy*,...
  Hallmark: possessives receive "special" meaning
- **functional nouns**: *father*, *surface*,...
  Hallmark: relational plus inherent uniqueness

Mass nouns will be ignored in the following.

Extensions for Words and Phrases

The extensions of relational and functional nouns can be identified with sets of **(ordered pairs)** of individuals.

Relational examples:

(5)

brother:

$\{\langle$Ethan, Joel$\rangle$, $\langle$Joel, Ethan$\rangle$, $\langle$Deborah,Joel$\rangle$, $\langle$Deborah, Ethan$\rangle$, . . . $\}$

arm:

$\{\langle$Ludwig, Ludwig's right arm$\rangle$, $\langle$Ludwig, Ludwig's left arm$\rangle$, $\langle$Paul, Paul's left arm$\rangle$, . . . $\}$

idea:

Extensions for Words and Phrases

Functional examples:

(6)

birthplace:

$\{\langle$Adam, Paradise$\rangle$, $\langle$Eve, Paradise$\rangle$, $\langle$John, Liverpool$\rangle$, $\langle$Yoko, Tokyo$\rangle$, $\ldots\}$

mother:

$\{\langle$Cain, Eve$\rangle$, $\langle$Abel, Eve$\rangle$, $\langle$Stella, Linda$\rangle$, $\langle$Sean, Yoko$\rangle$, $\ldots\}$

surface:

$\{\langle$Mars, Mars's surface$\rangle$, $\langle$Earth, Earth's surface$\rangle$, $\ldots\}$

In addition to being relational, the extensions *f* of functional nouns in (6) are **functions**, i.e., they satisfy a **uniqueness** condition:

Extensions for Words and Phrases: Taking stock

The extension of a referential expression is an **individual**.

The extension of a count noun (or intersective adjective) is a **set** of individuals.

The extension of a relational noun is a **binary relation** among [= set of ordered pairs of] individuals.

The extension of a functional noun is a **function** mapping individuals to individuals.

Extensions for Words and Phrases: verbal constituents

Extensions of verbs and verb phrases

(8)

sleep: the set of sleepers

kiss: a relation between kissers and kissees, i.e., the set of pairs $\langle x, y \rangle$ such that x kisses y

donate: a **three-place relation**, a set of triples

Extensions for Words and Phrases: verbal constituents

(9)

type of expression	**type of extension**	**example**	**extension**
intransitive verb	set of individuals	*sleep*	the set of sleepers
transitive verb	set of pairs of individuals	*eat*	the set of pairs ⟨eater, eaten⟩
ditransitive verb	set of triples of individuals	*donate*	the set of triples ⟨donator, recipient, donation⟩

Truth Values as Extensions of Sentences

(10) *Parallelism between valency and type of extension:*
The extension of an *n*-place verb is always a set of *n*-tuples.

Truth Values as Extensions of Sentences

(11) *The Pope shows the President the Vatican Palace*

(12)

verb or verb phrase	valency	extension
shows	3	the triples $\langle a, b, c \rangle$ where *a* shows *b* to *c*
shows the President	2	the pairs $\langle a, b \rangle$ where *a* shows *b* to the President
shows *the President the Vatican Palace*	1	the 1-tuples $\langle a \rangle$ where *a* shows the Vatican Palace to the President

(13)

sentence	valency	extension
The Pope shows the President the Vatican Palace	0	the 0-tuples $\langle\ \rangle$ where the Pope shows the Vatican Palace to the president

Truth Values as Extensions of Sentences

(14)	sentence	valency	extension
	The Pope shows the President the Vatican Palace	0	the 0-tuples $\langle\ \rangle$ where the Pope shows the Vatican Palace to the president

Standard Assumption 1

There is precisely one zero-tuple, viz., the empty set $\emptyset$.

Two cases:

- IF the Pope does NOT show the Vatican Palace to the president, then NO zero-tuple satisfies the condition that the Pope shows the Vatican Palace to the president
  and so the extension in (14) is empty, i.e.: $\emptyset$.

- IF the Pope DOES show the Vatican Palace to the president, then ANY zero-tuple satisfies the condition that the Pope shows the Vatican

Truth Values as Extensions of Sentences

Two cases:

- If the Pope does not show the Vatican Palace to the president, then the extension in (14) is: $\emptyset$.
- If the Pope does show the Vatican Palace to the president, then the extension in (14) is: $\{\emptyset\}$.

(Wildly) generalizing:

- If a (declarative) sentence is false, its extension is: $\emptyset$.
- If a (declarative) sentence is true, its extension is: $\{\emptyset\}$.

Truth Values as Extensions of Sentences

(Wildly) generalizing:

- If a (declarative) sentence is false, its extension is: $\emptyset$.
- If a (declarative) sentence is true, its extension is: $\{\emptyset\}$.

Standard Assumption 2

$\emptyset = 0, \{\emptyset\} = 1$.

(15) *Frege's Generalization*
The extension of a sentence S is its truth value, i.e., 1 if S is true and 0 if S is false.

REMINDER

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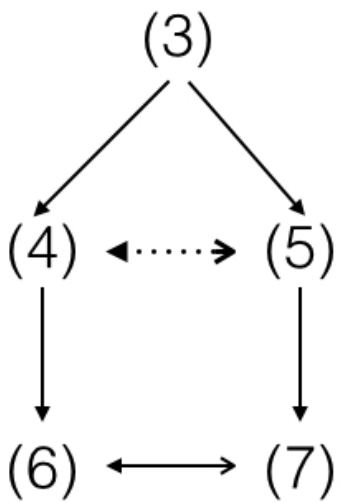
Cases and Propositions

(3) Four fair coins are tossed
(4) At least one of the 4 tossed coins lands heads up
(5) At least one of the 4 tossed coins lands heads down
(6) Exactly 2 of the 4 tossed coins land heads up
(7) Exactly 2 of the 4 tossed coins land heads down



Cases and Propositions

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Definition [to be revised]
The **proposition** expressed by a sentence is the set of possible cases of which that sentence is true.

Cases and Propositions

(16)

possible cases	c_1	c_2	c_3	c_4
1	1	1	1	1
2	1	1	1	0
3	1	1	0	1
...	...	...	...	...
14	0	0	1	0
15	0	0	0	1
16	0	0	0	0

Logical Space

(17) a. Four coins were tossed when John coughed
b. Four coins were tossed when John coughed and it started to rain
c. . . .

(18) *[Revised] Definition*
a. The **proposition** $||S||$ expressed by a sentence S is the set of possible worlds of which that sentence is true.

A sentence S is **true** of a possible world w if and only if $w \in ||S||$.

From Propositions to Intensions

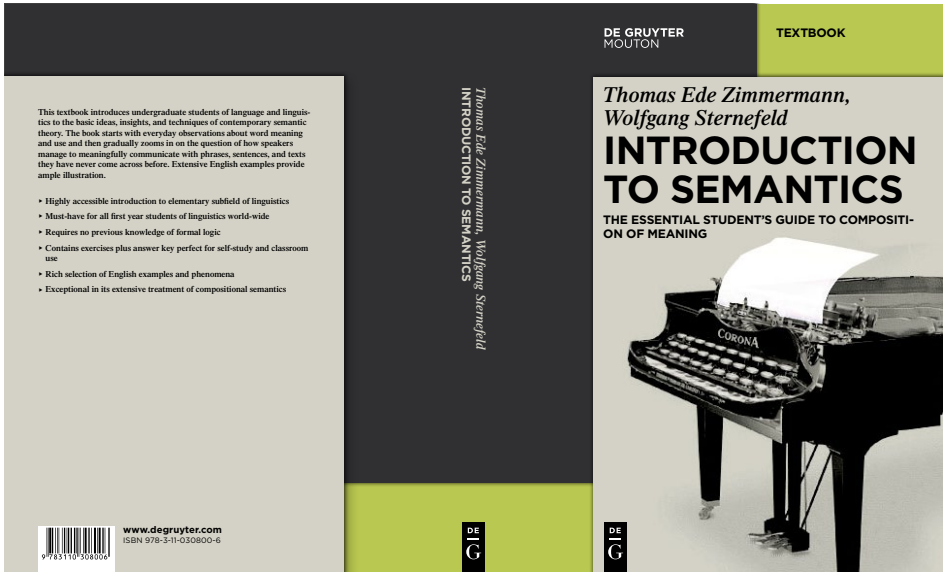
(19)

world	truth value
w_1	1
w_2	0
w_3	1
...	...
w_n	0
...	...

(20) *Definition*

The **intension** of an expression α is that function f such that for every possible world w, $f(w) = \alpha$'s extension at w.

If you want to know more about this, read . . .



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