



ESSLI Summerschool 2014: Intro to Compositional Semantics

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First Lecture: Structural Ambiguity

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Our plan for this course:

- **Today:** Getting attuned: Structural Ambiguity (Wolfgang)
- **Tuesday:** Introducing Extensions (Ede)
- **Wednesday:** Composing Extensions (Wolfgang)
- **Thursday:** Quantifiers (Wolfgang and Ede)
- **Friday:** Propositions and Intensions (Ede)

Recall that this course is foundational . . .

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Reference:

Thomas Ede Zimmermann & Wolfgang Sternefeld (2013):

Introduction to Semantics: An Essential Guide to the Composition of
Meaning. De Gruyter Mouton. Berlin/Boston

Copies are available from the second author. Author's discount is 30%.
Please, have the exact amount of 21 Euro with you.



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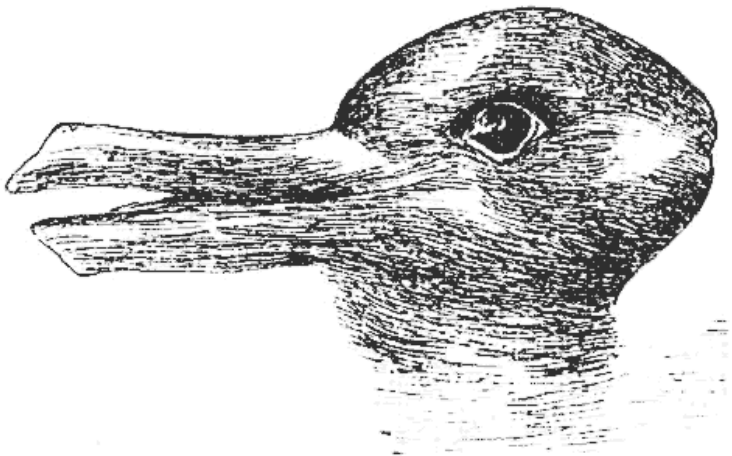
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Refers to either rabbit or duck but not both at a time

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Likewise, **ambiguity of words** arises by interpreting a string of sounds in two ways by referring to different things or concepts.

- (1) *bright*: shining or intelligent
to glare: to shine intensely or to stare angrily
deposit: minerals in the earth, or money in a bank, or a pledge, or ...

Similarities and differences:

- perception and understanding depend on context
- ambiguity resolution is unconscious and automatic
- ambiguity is not perceived as such
- Difference: the relation between a picture and its referent is more or less iconic (only partly conventional), whereas the relation between a word and its denotation is arbitrary and highly conventionalized

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Ambiguity of words also extends to **ambiguous sentences**:

- (2) They can fish
- They put fish into cans
 - They are able to fish

Different interpretations may arise from

- the meaning of lexical items
- their syntactic category
- the structure of the sentence

This last point is not obvious for (2), but there are more convincing examples...

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(3) John decided to marry on Tuesday

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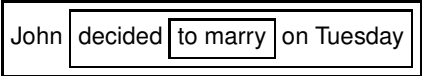
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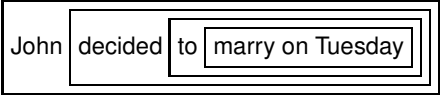
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- (3) John decided to marry on Tuesday
- a. John's decision to marry was taken on Tuesday
 - b. John decided that Tuesday be the day of his marriage

We say that a. and b. are different **paraphrases** of the ambiguous sentence.

No lexical ambiguity, but different structures (syntactic ambiguity):

(4) 

(5) 

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We use of boxes as a primitive kind of syntax:

- boxes provide partial tree structures
- the material inside a box is a constituent
- boxes are unlabelled
- boxes may not overlap

Syntactic Ambiguity

Two (partially) boxed structures of a sentence are incompatible if their joint structure contains overlapping boxes. Incompatibility is a test for **syntactic ambiguity**.



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Some basic principles of semantic analysis:

(6) The meaning of a sentence (or of complex constituents) is composed from the meaning of its parts.

Complex meanings are derived from simpler meanings in a recursive way, with lexical meanings as the basic building blocks.

(7) As shown by structural ambiguities, the composition of meaning also depends on the syntax.

Frege's Principle of Compositionality:

The meaning of a complex expression is a function of the meaning of its (immediate) constituents and the way they are combined.

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However, what is meaning? Today we do not specify the meaning of any expression whatsoever; rather. . .

- we simply assume that lexical expressions do have meaning and leave it to our intuition that meanings can differ
- we concentrate on differences of meaning that derive from the way meanings are combined
- we compare different meanings by concentrating on ambiguous sentences
- we apply a simple criterion to differentiate between different meanings of sentences, namely:



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The Most Certain Principle of Semantics:

If a sentence A is true of a certain situation, and if a sentence B is false of the same situation in the same circumstances, then A and B differ in meaning.

In plain words: A and B differ iff they report different facts or state of affairs. Facts A and B differ iff one can hold (be true) without the other (being true).

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Cautionary notes:

The above criterion when applied to ambiguous sentences forces us to say that such sentences split up in two sentences A and B, one being true and the other being false in the same context of utterance.

Likewise, ambiguous words should rather be considered as two words, or two different lexemes.

However, we will not be strict and continue with every day use by saying:

- (8) If a (“)sentence(”) may be both true and false in the same circumstances, it is (semantically) ambiguous.

Nonetheless, we do insist that in order to describe the different state of affairs by using paraphrases, the paraphrases themselves must not be ambiguous. (Finding such unambiguous paraphrases with the same meaning as the sentence to be paraphrased may be quite a challenge!)

More examples:

(9) John told the girl that Bill liked the story

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More examples:

(9) John told the girl that Bill liked the story

(10) John told the girl that Bill liked the story

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More examples:

(9) John told the girl that Bill liked the story

(10) John told the girl that Bill liked the story

(11) John told the girl that Bill liked the story

Such ambiguities are **purely structural**.

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More examples:

(9) John told the girl that Bill liked the story

(10) John told the girl that Bill liked the story

(11) John told the girl that Bill liked the story

Such ambiguities are **purely structural**.

Likewise:

(12) John saw the man with the binoculars

More examples:

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(10) John told the girl that Bill liked the story

(11) John told the girl that Bill liked the story

Such ambiguities are **purely structural**.

Likewise:

(12) John saw the man with the binoculars

(13) John saw the man with the binoculars



More examples:

(9) John told the girl that Bill liked the story

(10) John told the girl that Bill liked the story

(11) John told the girl that Bill liked the story

Such ambiguities are **purely structural**.

Likewise:

(12) John saw the man with the binoculars

(13) John saw the man with the binoculars

(14) John saw the man with the binoculars

(15) a. He put the block in the box on the table

b. He put the block in the box on the table

(16) a. Er tat den Block in der Box auf den Tisch (= (15-a))

b. Er tat den Block in die Box auf dem Tisch (= (15-b))

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(15) a. He put the block in the box on the table

b. He put the block in the box on the table

(16) a. Er tat den Block in der Box auf den Tisch (= (15-a))

b. Er tat den Block in die Box auf dem Tisch (= (15-b))

Purely Structural?

Assumption: Both *in*+Dative and *in*+Accusative have the same meaning!
The directional “meaning” of *in*+Accusative then has to be contributed by the meaning of the verb.



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(17) a. John told the girl that Bill liked the story

b. John told the girl that Bill liked the story

Purely Structural?

Assumption: *that* is a complementizer in both structures.

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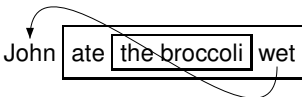
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(18) John ate the broccoli wet

(18) John ate the broccoli wet



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(19) John ate the broccoli wet

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Whether or not an ambiguity is purely structural depends on

- the analyses of critical words like prepositions
- additional theoretical constructs that do not meet the eye, like empty lexemes, e.g. relative pronouns
- the expressive power of the underlying grammatical theory, e.g. the question which kinds of grammatical relations are captured by the grammar (ie. phrase structure rules alone)
- assumptions about hidden syntactic operations like QR, as we will show in a minute

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Having introduced ambiguities by way of examples we now introduce some technical **terminology** used by linguists in analysing these ambiguities.

The basic semantic concept is the notion of **scope**. As this notion is notoriously difficult to define, we approach the problem by reference to the **syntactic** notion of a **domain**.

Let us first describe an ambiguity in terms of scope:

(20) ten minus three times two

a. $10 - (3 \times 2)$

b. $(10 - 3) \times 2$



Having introduced ambiguities by way of examples we now introduce some technical **terminology** used by linguists in analysing these ambiguities.

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Let us first describe an ambiguity in terms of scope:

(20) ten minus three times two

a. $10 - (3 \times 2)$

b. $(10 - 3) \times 2$

The brackets instruct us to apply subtraction and multiplication in different order, with different results. As for the notion of scope, we say that in (20-a) multiplication, being applied first, has **narrow scope** w.r.t. subtraction, being in the scope of subtraction. Conversely, subtraction has **wide scope** w.r.t. to multiplication, or **takes scope over** multiplication.

In (20-b), it's the other way round.



Turning next to the syntactic notion of a **domain**, scope taking depends on different syntactic structures that display different “domains”:

(21)

ten minus	three times two
-----------	-----------------

(22)

ten minus three	times two
-----------------	-----------

In syntactic terminology, we say that in (21), “times” is in the domain of “minus”, and conversely in (22).

(23) Let X and Y be constituents. Then X is in the **syntactic domain** of Y if and only if X is not contained in Y but is contained in the smallest box that contains Y .¹

Note: the notion “smallest box” requires a complete analyses.

¹ Readers with some background in syntax should notice the obvious similarity to the concept of c-command in Generative Syntax. Presupposing a customary definition of c-command, it follows that X is in the domain of Y if and only if Y c-commands X .

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Correlation between syntax and semantics:

The Scope Principle:

If α takes scope over β then β is in the syntactic domain of α .

What elements of NL play the role of subtraction and multiplication?

How do these operations comply with syntactic operations?

At this point we cannot fully answer these questions, but confine ourselves with examples that illustrate the concepts of scope and scope dependence.



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High attachment of PP:

(24)

Low attachment of PP:

(25)

Paraphrases?

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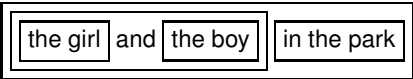
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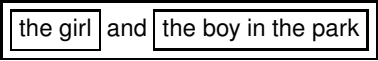
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High attachment of PP:

(24) 

Low attachment of PP:

(25) 

Paraphrases?

(26) the girl and the boy who are in the park

(27) the girl and the boy who is in the park



(28) a. The doctor

didn't leave

because he was angry

b. The doctor

didn't

leave

because he was angry

Cautionary note:

It follows from (28-b) that the doctor left! Hence, *leave* is not negated, though in the domain of *didn't*!

Therefore, the Scope Principle only goes one way. That is, if α is in the domain of β , β is not necessarily in its semantic scope of α .

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A problem for the Scope Principle:

(29) Beide Studenten kamen nicht
both students came not
'Both students didn't come'

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A problem for the Scope Principle:

- (29) Beide Studenten kamen nicht
both students came not
'Both students didn't come'
- (30) Reading *A*: neither of the two students came
- (31) Reading *B*: not both of the students came (one of them came)



Syntactic analyses:

- (32) a. (dass) beide Studenten **nicht** kamen
(that) both students not came
b. (dass) **nicht** beide Studenten kamen
(that) both students not came

Verb movement, leaving what is called a **trace**; traces are coindexed with the moved material (their antecedent):

- (33) a. kamen_x beide Studenten nicht t_x
b. kamen_x nicht beide Studenten t_x

Topicalization (leaving again a trace):

- (34) a. Beide Studenten_y kamen_x t_y nicht t_x
b. Beide Studenten_y kamen_x nicht t_y t_x

We can account for the ambiguity assuming that semantic interpretation refers to the position of the trace, either by undoing the movement or by assuming that the trace somehow retains the semantic material of the moved items.



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The general technical term for this is **reconstruction**.

Note: the same method could also be applied to the English version if it is assumed that the subject is generated inside the VP, as shown in (35):

(35) both students_y didn't t_y come

The choice would then be to reconstruct *both students*, or to interpret *both students* **in situ**, i.e. at the surface position.

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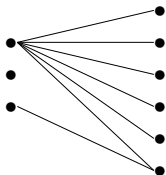
The following ambiguity pertains to German:

(36) jeden Schüler_{object} lobte genau ein Lehrer_{subject}
every pupil praised exactly one teacher

(37) a. Reading A: For every pupil there is exactly one teacher who praised him

b. Reading B: There is exactly one teacher who praised every pupil

(38) teachers pupils



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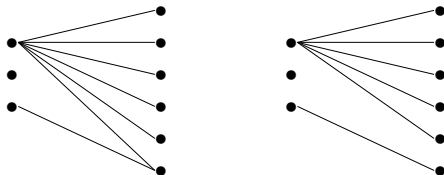
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(38) teachers pupils teachers pupils





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Here the choice is again to reconstruct or to interpret in situ:

(39) *jeden Schüler*_{object} lobte_x genau ein Lehrer_{subject} *t*_{object} *t*_x
every pupil praised exactly one teacher

If we assume backwards movement to the position of the trace, the structure that is interpreted semantically differs from what we see (and hear); in pre-minimalist terms the syntactic representation that serves as the input to semantics was called the **Logical Form** of a sentence.

Accordingly, (39) can have two different LFs, one with reconstruction of the object, and one without.

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Another important case for LFs are the following ambiguous sentences:

(40) A student is robbed every day in Tübingen

(41) A carpet touched every wall

(42) A student read every book

Would-be pseudo structure:

- (43) a. 
- b. 

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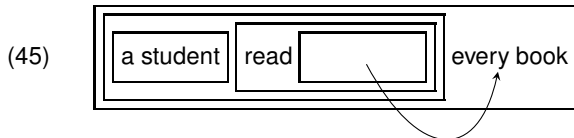
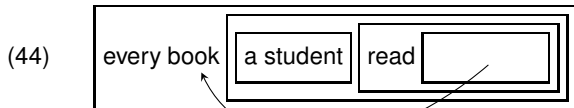
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Possible structures in accordance with the Scope Principle at LF:



The required LF-operation is called **Quantifier Raising (QR)**.

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- (46) Gertrude is looking for a book about Iceland
- There is a certain book about Iceland (the one Gertrude's sister requested as a Christmas present) that Gertrude is looking for
 - Gertrude is trying to find a present for her sister and it should be a book on Iceland (but she has no particular book in mind)

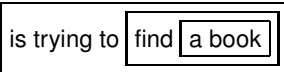
The reading of *a book* (paraphrased as “a certain book”) is often called **specific**, **referential**, or **transparent**. The reading in which the identity of the book does not matter is called the **unspecific**, **notional**, or **opaque reading**.

The ambiguity is often analysed as a matter of scope:

(47) Gertrude is trying to find a book

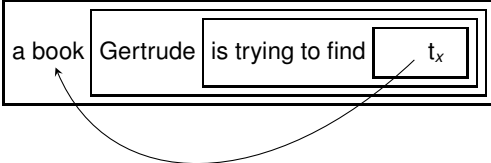
In situ interpretation (opaque):

(48) Gertrude is trying to find a book



QR-interpretation (transparent):

(49) a book Gertrude is trying to find t_x



The relevant scope-inducing item is the verb *try*.

Overview

Ambiguity:
Examples

Structural
Ambiguity

About
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Purely
Structural?

Scope and
Domains

Syntactic
Domains
and Recon-
struction

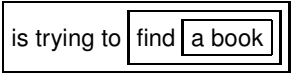
Logical
Form

Opaque
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Readings

(47) Gertrude is trying to find a book

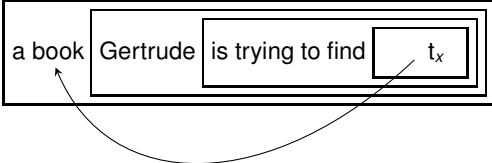
In situ interpretation (opaque):

(48) Gertrude is trying to find a book



QR-interpretation (transparent):

(49) a book Gertrude is trying to find t_x



The relevant scope-inducing item is the verb *try*. Compare also:

(50) a. John found a book b. John seeks a book

try and *seek* are called **opaque verbs**. *find* is transparent. Only opaque verbs can induce the observed ambiguity between opaque and transparent readings.

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A cautionary note:

QR was introduced to avoid a conflict with the Scope Principle. But the principle itself is not beyond doubt: it forces upon us a syntactic level of representation whose independent *syntactic* motivation is questionable (except for cases of reconstruction).

Alternatively, instead of introducing covert, invisible syntactic operations, it would also be possible to introduce covert invisible semantic operations. This requires advanced semantic techniques, as applied e.g. in categorial grammar.

The result would be a theory that derives the intended semantic results without movement but at the price of giving up the Scope Principle and complicating the semantics.



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ESSLI Summerschool 2014: Intro to Compositional Semantics

Thomas Ede Zimmermann, Goethe-Universität Frankfurt
Wolfgang Sternefeld, Universität Tübingen

Second Lecture: Introducing Extensions



Our plan for this course:

- Monday: Tuning in: Structural Ambiguity (Wolfgang)
- **Tuesday: Introducing Extensions (Ede)**
- Wednesday: Composing Extensions (Wolfgang)
- Thursday: Quantifiers (Wolfgang and Ede)
- Friday: Propositions and Intensions (Ede)



Two arrangements of unambiguous words can have different meanings:

- (1) a. Fritz kommt
 Fritz is-coming
 b. Kommt Fritz
 is-coming Fritz

Whereas the verb-second structure in (a) is normally interpreted as a declarative sentence, the verb-first structure in (b) is interpreted as a yes-no-question.



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(2) *Frege's Principle of Compositionality*

The **meaning** of a composite expression is a function of the **meaning** of its immediate constituents and the way these constituents are put together.

... Yes, but what (kind of objects) are all these **meanings**?



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When learning a new word, we learn how to combine a certain pronunciation, its phonetics and phonology, with its meaning. Thereby, a previously meaningless sequence of sounds like *schmöll* becomes vivid, we associate with it the idea of someone who isn't thirsty any more. In this case, one might be tempted to say that the **meaning** of an expression is the idea or conception (*Vorstellung*) a speaker associates with its utterance.



Schreiben, die bleiben

Höhepunkte abendländischer Briefkultur,
ausgewählt von Kaplan Klappstuhl,
Folge 27.

An die Dudenredaktion, Abt. Neue Worte.

Betr. Anregung

Sehr geehrte Herren !

Mir ist aufgefallen, daß die deutsche Sprache ein Wort zu wenig hat. Wenn man nicht mehr "hungrig" ist, ist man "satt". Was ist man jedoch, wenn man nicht mehr "durstig" ist ? Na ? Naa ? Na bitte ! Dann "hat man seinen Durst gestillt" oder "man ist nicht mehr durstig" und was dergleichen unschöne Satzbandwürmer mehr sind. Ein knappe s einsilbiges Wort für besagten Zustand fehlt jedoch, ich würde vorschlagen, dafür die Bezeichnung "schmöll" einzuführen und in Ihre Lexika aufzunehmen.

Mit vorzüglicher Hoachtung

Werner Schmöll



To the data editors of the *Duden* publishers, dept. new words

re: suggestion

Dear Sirs,

I have noticed that the German language lacks a word. If you are no longer hungry, you are full. But what are you if you are no longer thirsty? Eh? Then you have 'sated your thirst' or you are 'no longer thirsty' or some similarly inelegant circumlocution. But we have no short monosyllabic word for this condition. I would suggest that you introduce the term 'schmöll' and include it in your reference works.

Yours faithfully,
Werner Schmöll



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(Fregean and Wittgensteinian) ...



(oops)

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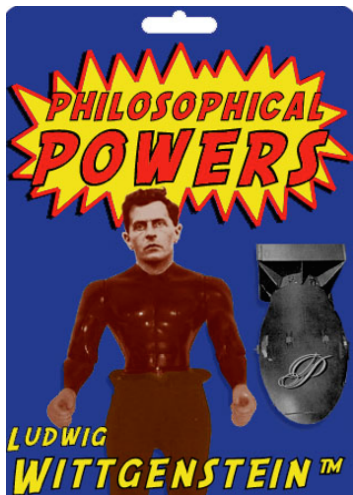
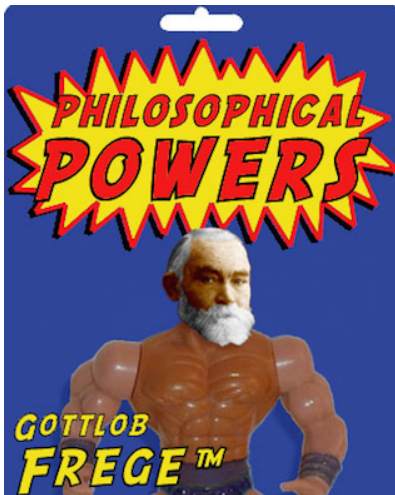
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... objections ...



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... against such a “**psychologistic**” notion of meaning:

- **Subjectiveness:** Different speakers may associate different things with a single word at different occasions: such “meanings,” however, cannot be objective, but will rather be influenced by personal experience, and one might wonder how these “subjective meanings” serve communication between different subjects.
- **Limited Coverage:** We can have mental images of nouns like *horse* or *table*, but what on earth could be associated with words like *and*, *most*, *only*, *then*, *of*, *if*, ... ?
- **Irrelevance:** Due to different personal experiences, speakers can have all sorts of associations without this having any influence on the meaning of an expression.
- **Privacy:** The associations of an individual person are in principle inaccessible to other speakers. So, again, how can they be used for interpersonal communication?



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On the other hand ...

MEANING SERVES COMMUNICATION ... and so:

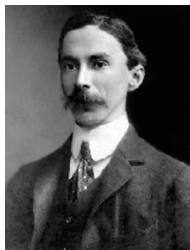
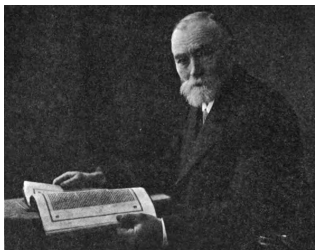
MEANINGS ought to be identified with

COMMUNICATIVE FUNCTIONS of expressions

... as in the tradition of ...



LOGICAL SEMANTICS



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... or (more recently)

FORMAL SEMANTICS





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LOGICAL [or FORMAL] SEMANTICS

Meanings \approx (certain) **communicative functions** of expressions, viz.:

- **Content:** *Which information* is expressed ...
- **Reference:** ... and *what* this information is *about*



LOGICAL [or FORMAL] SEMANTICS

The **meaning** of any expressions has (at least) **two components**, viz. its:

- **intension** \approx its contribution to the content of expressions in which it occurs
- **extension**: \approx its contribution to the reference of expressions in which it occurs
- ... and maybe more (but not in this course)

In the simplest cases:

- Intension is content.
- Extension is reference.

We will start with the latter ...



Some examples:

- (3) — Tübingen, Prof. Arnim v. Stechow (**proper names**)
- the president of the US (**definite descriptions**)
- table, horse, book (**nouns**)
- bald, red, stupid, alleged (**adjectives**)
- nobody, nothing, no dog (**negative quantifiers**)

- What do these expressions refer to?
- What is their contribution to reference?



[What do these expressions refer to?]

Referential expressions like

- proper names (like *Stuttgart*, *Edward Snowden*, ...)
- definite descriptions (like *the capital of Baden-Württemberg*, *the whistle blower...*)
- (some uses of) personal pronouns (like *she*)
- ...

(are used to) refer to persons, places, or other **individuals**.

The referent of a referential expression also forms its **extension**.



[What do these expressions refer to?]

- **common (count) nouns** like *table, car, ...*

as well as some ('intersective')

- **adjectives** like *blond, rectangular, ...*

do not refer to single individuals but show **multiple** reference.

The **set** of all its referents forms the **extension** of such a multiply extensional expression.



- A **set** is an abstract collection of (possibly, but not necessarily concrete) objects, their elements.
- Elementhood is a **relational** concept: an object x is or is not an **element of** a given set y .

Notation: $x \in y$ vs. $x \notin y$

- A set A is a **subset** of a (not necessarily distinct) set B iff [= if and only if] every element of A is an element of B and *vice versa*.

Notation: $A \subseteq B$

- The identity criterion for sets A and B is sharing the same elements ('extensionality'):

$A = B$ iff $A \subseteq B$ and $B \subseteq A$

- Sets are defined by **set abstraction**:
 $\{x : \dots x \dots\}$ is that set whose elements are precisely those objects x such that the condition $\dots x \dots$ holds.

Notation: \emptyset is $\{x: x \neq x\}$



[What do these expressions refer to?]

- **common (count) nouns** like *table, car, ...*

as well as some ('intersective')

- **adjectives** like *blond, rectangular, ...*

do not refer to single individuals but show **multiple** reference.

The **set** of all its referents forms the **extension** of such a multiply extensional expression.



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NB1: The extension of

- *the current German chancellor*

is Angela Merkel

but this will change . . .

In four years from now the extension of *the current German chancellor* is going to be another person and it used to be 20 years ago . . .



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SO:

- The extension of *the current German chancellor* is changing over time ... and so are extensions in general.



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NB2: The extension of

- *current German chancellor*

is the set of all current German chancellors – i.e., a set with one member.



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However, the extension of

- ***the current German chancellor***

is the current German chancellor, i.e., a person.



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SO:

- *current German chancellor* (whose extension is $\{A.M.\}$),

and:

- *the current German chancellor*

do not have the same extension¹!

¹on standard set-theoretic assumptions



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NB3: The (current) extension of

■ *current French king*

is the set of all current French kings – i.e., the empty set.



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However, the extension of

■ ***the current king of France***

would have to be the current French king
... but there is no such (existing) person!



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SO: unlike

- *current king of France* (whose extension is \emptyset),
- *the current king of France*

appears to have no extension. We will henceforth ignore such **void** descriptions. (Read chapter 9 for more on this ...)



Not alle nouns are count nouns — some are:

- **mass nouns:** *milk, information,...*
Hallmark: no plural (without meaning shift)
- **relational nouns:** *brother, copy,...*
Hallmark: possessives receive “special” meaning
- **functional nouns:** *father, surface,...*
Hallmark: relational plus inherent uniqueness

Mass nouns will be ignored in the following.



The extensions of relational and functional nouns can be identified with sets of **(ordered pairs)** of individuals.

Relational examples:

(4)

brother:

{⟨Ethan, Joel⟩, ⟨Joel, Ethan⟩, ⟨Deborah, Joel⟩, ⟨Deborah, Ethan⟩, ... }

arm:

{⟨Ludwig, Ludwig's right arm⟩, ⟨Ludwig, Ludwig's left arm⟩, ⟨Paul, Paul's left arm⟩, ... }

idea:

{⟨Albert, $E = mc^2$ ⟩, ⟨René, *COGITO*⟩, ⟨Bertie, $R \in R \Leftrightarrow R \notin R$ ⟩, ... }



Functional examples:

(5)

birthplace:

{⟨Adam, Paradise⟩, ⟨Eve, Paradise⟩, ⟨John, Liverpool⟩, ⟨Yoko, Tokyo⟩, ... }

mother:

{⟨Cain, Eve⟩, ⟨Abel, Eve⟩, ⟨Stella, Linda⟩, ⟨Sean, Yoko⟩, ... }

surface:

{⟨Mars, Mars's surface⟩, ⟨Earth, Earth's surface⟩, ... }

In addition to being relational, the extensions f of functional nouns in (5) are **functions**, i.e., they satisfy a **uniqueness** condition:

(6) If both $\langle a, v_1 \rangle \in f$ and $\langle a, v_2 \rangle \in f$, then $v_1 = v_2$.

The **extension** of a functional noun is a **function mapping** individuals to individuals.



Taking stock:

The extension of a **referential expression** — a name, a (non-void) definite description, a referential pronoun, etc. — is an **individual**.

The extension of a **count noun** (or **intersective adjective**) is a **set** of individuals.

The extension of a **relational noun** is a **binary relation** among [= set of ordered pairs of] individuals.

The extension of a **functional noun** is a **function** mapping individuals to individuals.

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Extensions of verbs and verb phrases

(7)

sleep: the set of sleepers

kiss: a relation between kissers and kissees, i.e., the set of pairs $\langle x, y \rangle$ such that x kisses y

donate: a **three-place relation**, a set of triples



(8)

type of expression	type of extension	example	extension
intransitive verb	set of individuals	<i>sleep</i>	the set of sleepers
transitive verb	set of pairs of individuals	<i>eat</i>	the set of pairs (eater, eaten)
ditransitive verb ditransitive verb	set of triples of individuals	<i>donate</i>	the set of triples (donator, recipient, donation)

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- (9) *Parallelism between valency and type of extension:*
The extension of an n -place verb is always a set of n -tuples.



(10) *The Pope shows the President the Vatican Palace*

(11)	verb or verb phrase <i>shows</i>	valency 3	extension the triples $\langle a, b, c \rangle$ where <i>a</i> shows <i>b</i> to <i>c</i>
	<i>shows</i> <i>the President</i>	2	the pairs $\langle a, b \rangle$ where <i>a</i> shows <i>b</i> to the President
	<i>shows</i> <i>the President</i> <i>the Vatican Palace</i>	1	the 1-tuples $\langle a \rangle$ where <i>a</i> shows the Vatican Palace to the President
(12)	sentence <i>The Pope shows the</i> <i>President the</i> <i>Vatican Palace</i>	valency 0	extension the 0-tuples $\langle \rangle$ where the Pope shows the Vatican Palace to the president



(13)	sentence	valency	extension
	<i>The Pope shows the President the Vatican Palace</i>	0	the 0-tuples $\langle \rangle$ where the Pope shows the Vatican Palace to the president

Standard Assumption 1

There is precisely one zero-tuple, viz., the empty set \emptyset .

Two cases:

- IF the Pope does NOT show the Vatican Palace to the president, then NO zero-tuple satisfies the condition that the Pope shows the Vatican Palace to the president and so the extension in (13) is empty, i.e.: \emptyset .
- IF the Pope DOES show the Vatican Palace to the president, then ANY zero-tuple satisfies the condition that the Pope shows the Vatican Palace to the president and so the extension in (13) is the set of all 0-tuples, i.e.: $\{\emptyset\}$.



Two cases:

- If the Pope does not show the Vatican Palace to the president, then the extension in (13) is: \emptyset .
- If the Pope does show the Vatican Palace to the president, then the extension in (13) is: $\{\emptyset\}$.

(Wildly) generalizing:

- If a (declarative) sentence is false, its extension is: \emptyset .
- If a (declarative) sentence is true, its extension is: $\{\emptyset\}$.



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(Wildly) generalizing:

- If a (declarative) sentence is false, its extension is: \emptyset .
- If a (declarative) sentence is true, its extension is: $\{\emptyset\}$.

Standard Assumption 2

$$\emptyset = 0, \{\emptyset\} = 1.$$

(14) *Frege's Generalization*

The extension of a sentence S is its truth value, i.e., 1 if S is true and 0 if S is false.

ESSLI Summerschool 2014: Intro to Compositional Semantics

Thomas Ede Zimmermann, Universität Frankfurt
Wolfgang Sternefeld, Universität Tübingen

Third Lecture: Composing Extensions

The set theoretic object that constitutes the reference of an expression **in a particular situation s** is called its **extension in s** .

If α is such an expression, its extension is denoted by $[\alpha]_s$.

Given two expressions α and β forming a constituent $\boxed{\alpha \beta}$, what is

$$\boxed{\boxed{\alpha \beta}}_s ?$$

Given Frege's Principle, this must be a function f such that

$$(1) \quad \boxed{\boxed{\alpha \beta}}_s = f([\alpha]_s, [\beta]_s)$$

But which function?

This depends on the nature of α and β , but also on the mode of syntactic combination.

We assume roughly 4 different modes of combination:

- functional application
- “plugging” or arity-reduction
- “predicate modification”
- set abstraction

Example 1: Functional nouns

Assume Berta is John's mother. Then:

$$(2) \quad \llbracket \text{John's mother} \rrbracket_s = \llbracket \text{mother} \rrbracket_s (\llbracket \text{John} \rrbracket_s) = \text{Berta}$$

General rule:

$$(3) \quad \llbracket \text{term's functional noun} \rrbracket_s = \llbracket \text{functional noun} \rrbracket_s (\llbracket \text{term} \rrbracket_s)$$

Convention: in mixed expressions that contain both meta-language and object language, the object language part is colored blue.

Terminology: by a **term** we mean any referential expression (proper name, definite description, pronoun, ...)

Example 2: Truth tables

(4) Harry is reading or Mary is writing

(5)

$\llbracket \text{Harry is reading} \rrbracket_s$	$\llbracket \text{Mary is writing} \rrbracket_s$	$\llbracket (4) \rrbracket_s$
1	1	1
1	0	1
0	1	1
0	0	0

Accordingly:

$$\llbracket \text{or} \rrbracket = \{ \langle \langle 1, 1 \rangle, 1 \rangle, \langle \langle 1, 0 \rangle, 1 \rangle, \langle \langle 0, 1 \rangle, 1 \rangle, \langle \langle 0, 0 \rangle, 0 \rangle \}$$

Compositional semantic rule:

$$(6) \quad \boxed{\boxed{S_1 \text{ or } S_2}}_s = [\text{or}] (\langle [S_1]_s, [S_2]_s \rangle)$$

Example 3: Definite descriptions

Assume that in a certain situation s , the teacher in s is Harry. Then

$$(7) \quad \llbracket \boxed{\text{the teacher}} \rrbracket_s = \llbracket \text{the} \rrbracket (\llbracket \text{teacher} \rrbracket_s) = \text{Harry}$$

For this to work we assume that *the* denotes a function. Which one? The function that assigns to a singleton set its only element (undefined for non-singletons):

$$(8) \quad \llbracket \text{the} \rrbracket = \{ \langle X, y \rangle : X = \{ y \} \}$$

Compositional semantic rule:

$$(9) \quad \llbracket \boxed{\text{the noun phrase}} \rrbracket_s = \llbracket \text{the} \rrbracket (\llbracket \text{noun phrase} \rrbracket_s)$$

Syntactic terminology: nouns are special noun phrases.

Plugging = arity-reduction:

- (10) If R is an n -place relation (i.e. set of n -tuples $\in D_1 \times D_2 \times \dots \times D_n$) and $y \in D_n$, $n \geq 1$, then

Right Edge Plugging (y is a plug for the last argument position):

$$R^{\rightarrow} y := \{ \langle x_1, \dots, x_{n-1} \rangle : \langle x_1, \dots, x_{n-1}, y \rangle \in R \}$$

and Left Edge Plugging (y is a plug for the first argument position):

$$R^{\leftarrow} y := \{ \langle x_2, \dots, x_n \rangle : \langle y, x_2, \dots, x_n \rangle \in R \}$$

We say that the last (first) argument position is **plugged** by y . The result is arity reduction, i.e. an $n - 1$ -place relation.

Recall that since $\langle x \rangle = x$, a 1-place relation is simply a set.

Notational conventions:

- (11) In case R is a one-place relation, $R \overleftarrow{*} y$ and $R \overrightarrow{*} y$ coincide, both saying that $y \in R$; we then simply write $R * y$.
- (12) Sometimes, the syntax of NL places a right edge plug on the left side of a predicate or relation; we then deliberately switch notation to $y \overrightarrow{*} R$ with the same meaning as $R \overrightarrow{*} y$. See below.



(13)

the pope shows the V.P. to the president

(14)

der Papst dem Präsidenten den Vatikanpalast zeigt

Compositional semantic rule:

(15) $\llbracket \text{referential argument expression} + \text{relational expression} \rrbracket_s$

or $\llbracket \text{relational expression} + \text{referential argument expression} \rrbracket_s$

= $\llbracket \text{relational expression} \rrbracket_s \vec{*} \llbracket \text{referential argument expression} \rrbracket_s$



Example:

- (16) Referential argument expressions (= terms used as subjects or objects):

$$\llbracket \text{der Papst} \rrbracket_s = \llbracket \text{the Pope} \rrbracket_s = p$$

$$\llbracket \text{dem Präsidenten} \rrbracket_s = \llbracket \text{the president} \rrbracket_s = o$$

$$\llbracket \text{den Vatikanpalast} \rrbracket_s = \llbracket \text{the V.P.} \rrbracket_s = v$$

Relational expression:

$$\llbracket \text{zeigt} \rrbracket_s = \llbracket \text{shows} \rrbracket_s = \{ \langle p, o, v \rangle, \langle a, b, v \rangle, \langle a, b, c \rangle \}$$

- (17) Syntactic combinations:

$$\llbracket \text{shows the V.P.} \rrbracket_s = \{ \langle p, o, v \rangle, \langle a, o, v \rangle, \langle a, b, c \rangle \} \vec{*} v = \{ \langle p, o \rangle, \langle a, o \rangle \}$$

$$\llbracket \boxed{\text{shows the V.P.}} \text{ (to) the president} \rrbracket_s =$$

$$\{ \langle p, o \rangle, \langle a, o \rangle \} \vec{*} o = \{ \langle p \rangle, \langle a \rangle \} = \{ p, a \} = R_1$$



$$(17) \quad \left[\left[\text{the Pope} \right]_s \left[\left[\text{shows the V.P. (to) the president} \right]_s \right] \right]_s = p * \{ \langle p \rangle, \langle a \rangle \} = \{ \langle p \rangle, \langle a \rangle \} * p = \{ \langle p \rangle, \langle a \rangle \} \vec{*} p = \{ \langle \rangle : \langle p \rangle \in \{ \langle p \rangle, \langle a \rangle \} \} = \{ \langle \rangle \} = 1$$

$$(18) \quad \left[\left[\text{John} \right]_s \left[\left[\text{shows the V.P. (to) the president} \right]_s \right] \right]_s = j * \{ \langle p \rangle, \langle a \rangle \} = \{ \langle \rangle : \langle j \rangle \in \{ \langle p \rangle, \langle a \rangle \} \}, \text{ hence } (18) = \{ \} = \emptyset = 0$$

Summary:

$$\begin{aligned} (17) &= \left[\left[\text{the Pope} \right]_s * \left[\left[\left[\text{shows} \right]_s \vec{*} \left[\left[\text{the V.P.} \right]_s \right] \vec{*} \left[\left[\text{the president} \right]_s \right] \right] \right]_s \right]_s \\ &= \left[\left[\text{the} \right] \left(\left[\text{Pope} \right]_s \right) * \right. \\ &\quad \left. \left[\left[\left[\text{shows} \right]_s \vec{*} \left[\left[\text{the} \right] \left(\left[\text{V.P.} \right]_s \right) \right] \vec{*} \left(\text{to} \right) \left[\left[\text{the} \right] \left(\left[\text{president} \right]_s \right) \right] \right] \right]_s \right]_s \end{aligned}$$



$$(19) \quad \llbracket \text{der Papst dem Präsidenten den Vatikanpalast zeigt} \rrbracket_s = \\ \llbracket \text{der Papst} \rrbracket_s * \llbracket \llbracket \text{dem Präsidenten} \rrbracket_s \vec{*} \llbracket \llbracket \text{den V.P.} \rrbracket_s \vec{*} \llbracket \text{zeigt} \rrbracket_s \rrbracket$$

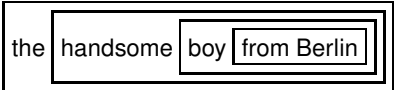
Note:

Since the subject and the objects in (19) are terms (ie. referential expressions denoting individuals), and given that the relation *zeigt* holds of/between individuals, we can use the notation $y \vec{*} R$ instead of $R \vec{*} y$ as defined in (12) above.


The notation thus reveals that semantic composition is the same in English and German.

(20) $\llbracket \text{the handsome boy from Berlin} \rrbracket_S = ?$

(21) $\llbracket \text{the handsome boy from Berlin} \rrbracket_S$



(22) $\llbracket \text{the handsome boy from Berlin} \rrbracket_S$



(23) $\llbracket \text{from Berlin} \rrbracket_S = \llbracket \text{from} \rrbracket_S \vec{*} \llbracket \text{Berlin} \rrbracket_S$

(24) General rule (*Predicate Modification*):

$$\begin{aligned} &\llbracket \text{noun phrase + modifying expression} \rrbracket_S = \\ &\llbracket \text{modifying expression + noun phrase} \rrbracket_S = \\ &\llbracket \text{noun phrase} \rrbracket_S \cap \llbracket \text{modifying expression} \rrbracket_S \end{aligned}$$

- (25) $\llbracket \text{the handsome boy from Berlin} \rrbracket_S$
- a. $\llbracket \text{the} \rrbracket (\llbracket \text{handsome} \rrbracket_S \cap \llbracket \text{boy} \rrbracket_S \cap \llbracket \text{from} \rrbracket_S \overset{*}{\rightarrow} \llbracket \text{Berlin} \rrbracket_S)$
- b. $\llbracket \text{the} \rrbracket (\llbracket \text{handsome} \rrbracket_S \cap \llbracket \text{boy} \rrbracket_S \rrbracket \cap \llbracket \text{from} \rrbracket_S \overset{*}{\rightarrow} \llbracket \text{Berlin} \rrbracket_S)$

$$A \cap (B \cap C) = ((A \cap B) \cap C)$$

Cautionary notes:

- Some adjectives cannot be handled by Predicate Modification, ie. treated as intersective
- Some adjectives require in addition a standard of comparison

Assume

$$(26) \quad \llbracket \text{John is a murderer} \rrbracket_S = 1 \text{ iff} \\ \text{John} \in \llbracket \text{murderer} \rrbracket_S \text{ iff} \\ \text{John} * \llbracket \text{murderer} \rrbracket_S$$

In general:

$$(27) \quad \llbracket \text{term is a noun phrase} \rrbracket_S = \llbracket \text{term} \rrbracket_S * \llbracket \text{noun phrase} \rrbracket_S$$

(28) $\llbracket \text{John is an alleged murderer} \rrbracket_S = 1$

incorrectly implies that John is a murderer (and that *John is alleged).

Rather, *alleged* should be analysed as a function from sets to sets, taking as argument the set of murderers and yielding the set of alleged murderers as value. As not all alleged murderers need to be murderers, on the contrary, this function is **not intersective**, it does not hold that $\llbracket \text{alleged} \rrbracket_S (M) \subseteq M$.

We then get an ambiguity caused by the scope of *alleged*:

(29) a. alleged murderer from Berlin

b. alleged murderer from Berlin

(30) a. $\llbracket \text{alleged} \rrbracket_S (\llbracket \text{murderer} \rrbracket_S \cap \llbracket \text{from Berlin} \rrbracket_S)$

b. $\llbracket \text{alleged} \rrbracket_S (\llbracket \text{murderer} \rrbracket_S) \cap \llbracket \text{from Berlin} \rrbracket_S$

Second problem:

- (31) a. Jumbo is a small elephant
- b. Jumbo is a big animal
- c. Jumbo is big and small

Sounds like a contradiction. . .

Solution: Adjectives have an additional, syntactically not expressed argument:

- (32) a. Jumbo is small (for an elephant)
- b. Jumbo is big (for an animal)

The additional argument is a property X (*elephant*, *animal*, . . .) that has to be supplied pragmatically by the context of utterance. This property supplies the adjective with a **standard of comparison**.

$$(33) \quad \llbracket \text{small}_X \rrbracket_S = \{ y : \\ y \text{ is small compared to the standard size of objects in } X \}$$

Our fourth mode of operation, namely set formation (or comprehension in set theory) will become important at the level of LF. This will be discussed in the next chapter.



ESSLI Summerschool 2014: Intro to Compositional Semantics

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Fourth Lecture: Determiners and Quantifiers

Determiners
and
Quantifiers

Type
Shifting and
Flexible
Types

QDPs in
Object
Position

Quantifier
Raising

On
Variables

- (1) a. Every student snored
- b. A woman snored
- c. No fly snored

every, a, no (and sometimes also *the*) are called **quantifying determiners**.
The subject phrases are QDPs (quantifying determiner phrases).

- (2) What are the truth conditions for (1)?

- (1) a. Every student snored
- b. A woman snored
- c. No fly snored

every, a, no (and sometimes also *the*) are called **quantifying determiners**.
The subject phrases are QDPs (quantifying determiner phrases).

- (2) What are the truth conditions for (1)?
 - a. $\llbracket \text{every} + \text{NP} + \text{Predicate} \rrbracket_s = 1$ iff $\llbracket \text{NP} \rrbracket_s \subseteq \llbracket \text{Predicate} \rrbracket_s$
 - b. $\llbracket \text{a} + \text{NP} + \text{Predicate} \rrbracket_s = 1$ iff $\llbracket \text{NP} \rrbracket_s \cap \llbracket \text{Predicate} \rrbracket_s \neq \emptyset$
 - c. $\llbracket \text{no} + \text{NP} + \text{Predicate} \rrbracket_s = 1$ iff $\llbracket \text{NP} \rrbracket_s \cap \llbracket \text{Predicate} \rrbracket_s = \emptyset$

- (1) a. Every student snored
- b. A woman snored
- c. No fly snored

every, a, no (and sometimes also *the*) are called **quantifying determiners**.
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- (2) What are the truth conditions for (1)?
 - a. $[\text{every} + \text{NP} + \text{Predicate}]_S = 1$ iff $[\text{NP}]_S \subseteq [\text{Predicate}]_S$
 - b. $[\text{a} + \text{NP} + \text{Predicate}]_S = 1$ iff $[\text{NP}]_S \cap [\text{Predicate}]_S \neq \emptyset$
 - c. $[\text{no} + \text{NP} + \text{Predicate}]_S = 1$ iff $[\text{NP}]_S \cap [\text{Predicate}]_S = \emptyset$
- (3) From (2) we may construe quantifiers as 2-place relations:
 - a. $[\text{every}]_S := \{ \langle X, Y \rangle : X \subseteq Y \}$
 - b. $[\text{a}]_S := \{ \langle X, Y \rangle : X \cap Y \neq \emptyset \}$
 - c. $[\text{no}]_S := \{ \langle X, Y \rangle : X \cap Y = \emptyset \}$

X and Y stand for sets of individuals. X is called the **restriction** of the quantifier, Y is called its **scope**. By convention, the restriction in (3) precedes the scope!

- (4) a. $\langle [\text{student}]_S, [\text{snore}]_S \rangle \in [\text{every}]_S$
b. $\langle [\text{woman}]_S, [\text{snore}]_S \rangle \in [\text{a}]_S$
c. $\langle [\text{fly}]_S, [\text{snore}]_S \rangle \in [\text{no}]_S$

The problem of compositionality:

- (5) every student snores

- (6) a. $[\text{every} + \text{NP}]_S = \{ X : \langle [\text{NP}]_S, X \rangle \in [\text{every}]_S \}$
b. $[\text{a} + \text{NP}]_S = \{ X : \langle [\text{NP}]_S, X \rangle \in [\text{a}]_S \}$
c. $[\text{no} + \text{NP}]_S = \{ X : \langle [\text{NP}]_S, X \rangle \in [\text{no}]_S \}$

We thus have to plug in the NP at the first position of the quantifier.

(7) General scheme:

$$\begin{aligned}
 \llbracket \text{QDet} + \text{NP} \rrbracket_s &= \llbracket \text{QDet} \rrbracket_s^* \llbracket \text{NP} \rrbracket_s \\
 &= \{ X : \langle \llbracket \text{NP} \rrbracket_s, X \rangle \in \llbracket \text{QDet} \rrbracket \}
 \end{aligned}$$

(8) a. $\llbracket \text{every} + \text{NP} \rrbracket_s = \{ X : \llbracket \text{NP} \rrbracket_s \subseteq X \}$

b. $\llbracket \text{a} + \text{NP} \rrbracket_s = \{ X : \llbracket \text{NP} \rrbracket_s \cap X \neq \emptyset \}$

c. $\llbracket \text{no} + \text{NP} \rrbracket_s = \{ X : \llbracket \text{NP} \rrbracket_s \cap X = \emptyset \}$

(9) $\llbracket \text{QDP} + \text{Predicate} \rrbracket_s = 1$ iff

$\llbracket \text{Predicate} \rrbracket_s \in \llbracket \text{QDP} \rrbracket_s$ iff

$\llbracket \text{QDP} \rrbracket_s^* \llbracket \text{Predicate} \rrbracket_s = 1$

(10) $\llbracket \text{no fly snored} \rrbracket_s = 1$ iff

$\llbracket \text{snored} \rrbracket_s \in \llbracket \text{no fly} \rrbracket_s$ iff

$\llbracket \text{snored} \rrbracket_s \in (\llbracket \text{no} \rrbracket_s^* \llbracket \text{fly} \rrbracket_s)$ iff

$\{ x : x \text{ snored in } s \} \in (\{ x : x \text{ is a fly in } s \}^* \{ \langle X, Y \rangle : X \cap Y = \emptyset \})$ iff

$\{ x : x \text{ snored in } s \} \in \{ Y : \{ x : x \text{ is a fly in } s \} \cap Y = \emptyset \}$ iff

$\{ x : x \text{ is a fly in } s \} \cap \{ x : x \text{ snored in } s \} = \emptyset$



Note that for subject + predicate we actually have two cases:

- (11) a. $\llbracket \text{referential argument expression} + \text{predicate} \rrbracket_S =$
 $\llbracket \text{referential argument} \rrbracket_S * \llbracket \text{predicate} \rrbracket_S = 1$ iff
 $\llbracket \text{referential argument} \rrbracket_S \in \llbracket \text{predicate} \rrbracket_S$
- b. $\llbracket \text{QDP} + \text{predicate} \rrbracket_S =$
 $\llbracket \text{QDP} \rrbracket_S * \llbracket \text{predicate} \rrbracket_S = 1$ iff
 $\llbracket \text{QDP} \rrbracket_S \ni \llbracket \text{predicate} \rrbracket_S$

This is because our notation $\alpha * \beta$ actually allows for two interpretations:

- a. $\alpha = y$ (a referential expression), $\beta = R$ (a predicate), so that
 $\alpha * \beta = y * R = 1$ iff $\alpha \in \beta$ (cf. (11-a)), or
- b. $\alpha = R$ (a quantifying expression) and $\beta = y$ a predicate, so that
 $\alpha * \beta = R * y = 1$ iff $\beta \in \alpha$ (cf. (11-b)).

The correct interpretation depends on the “logical types” of α and β . This kind of semantics is also called **type driven interpretation**.

In more classical approaches, however, this flexibility is not allowed. In particular, the logical types of the corresponding components of semantic rules are fixed. In particular, there is no such convention that $R * y = y * R$. We would therefore need two rules:

- (12) a. $\llbracket \text{term} + \text{predicate} \rrbracket_s = 1$ iff $\llbracket \text{term} \rrbracket_s \in \llbracket \text{predicate} \rrbracket_s$
b. $\llbracket \text{QDP} + \text{predicate} \rrbracket_s = 1$ iff $\llbracket \text{predicate} \rrbracket_s \in \llbracket \text{QDP} \rrbracket_s$

However, some more restrictive theories require a one-to-one-correspondance between syntactic and semantic rules, and moreover one between syntactic categories and semantic types. In such a theory, the semantic difference between term and QDP in (12) must be ignorable.

In these approaches, it is assumed that all subjects, even terms, are sets of sets (have the logical type of quantifying DPs):

$$(13) \quad \llbracket \text{subject} + \text{predicate} \rrbracket_S = 1 \text{ iff } \llbracket \text{predicate} \rrbracket_S \in \llbracket \text{subject} \rrbracket_S$$

For referential expressions, a rule called **type shifting** or **Montague Lifting** converts a referential expression into a set of sets:

$$(14) \quad \text{LIFT}(a) = \{ X : a \in X \}$$

Accordingly,

- (15) $\llbracket \text{John snores} \rrbracket_s = 1$ iff
 $\llbracket \text{snores} \rrbracket_s \in \llbracket \text{John} \rrbracket_s$ iff
 $\llbracket \text{snores} \rrbracket_s \in \text{LIFT}(\text{John})$ iff
 $\llbracket \text{snores} \rrbracket_s \in \{ X : \text{John} \in X \}$ iff
 $\text{John} \in \llbracket \text{snores} \rrbracket_s$

Or alternatively,

- (16) $\llbracket \text{John snores} \rrbracket_s = 1$ iff
 $\llbracket \text{snores} \rrbracket_s \in \llbracket \text{John}^{DP} \rrbracket_s$ iff
 $\llbracket \text{snores} \rrbracket_s \in \text{LIFT}(\llbracket \text{John} \rrbracket_s)$ iff
 $\llbracket \text{snores} \rrbracket_s \in \{ X : \llbracket \text{John} \rrbracket_s \in X \}$ iff
 $\llbracket \text{John} \rrbracket_s \in \llbracket \text{snores} \rrbracket_s$

(17) Paul loves every girl

The problem: a simple rule like argument reduction is not applicable!

First solution: *In situ* interpretation

(18) Let R be an n -place relation and \mathcal{Q} a set of sets.

$$R \xrightarrow{*Q} \mathcal{Q} = \mathcal{Q} \xrightarrow{*Q} R = \{ \langle x_1, \dots, x_{n-1} \rangle : \{ y : \langle x_1, \dots, x_{n-1}, y \rangle \in R \} \in \mathcal{Q} \}$$

(19) $\llbracket \text{loves } \boxed{\text{every girl}} \rrbracket_s = \llbracket \text{loves} \rrbracket_s \xrightarrow{*Q} \llbracket \text{every girl} \rrbracket_s =$
 $\llbracket \text{loves} \rrbracket_s \xrightarrow{*Q} \{ X : \llbracket \text{girl} \rrbracket_s \subseteq X \} =$
 $\{ x_1 : \{ y : \langle x_1, y \rangle \in \llbracket \text{loves} \rrbracket_s \} \in \{ X : \llbracket \text{girl} \rrbracket_s \subseteq X \} \} =$
 $\{ x_1 : \llbracket \text{girl} \rrbracket_s \subseteq \{ y : \langle x_1, y \rangle \in \llbracket \text{loves} \rrbracket_s \} \}$

$$(20) \quad \boxed{\boxed{\text{John loves every girl}}}_s = 1 \text{ iff}$$

$$j \in \{x_1 : \boxed{\text{girl}}_s \subseteq \{y : \langle x_1, y \rangle \in \boxed{\text{loves}}_s\}\}$$

$$\boxed{\text{girl}}_s \subseteq \{y : \langle j, y \rangle \in \boxed{\text{loves}}_s\}$$

Note: The rule that applies $\overrightarrow{*}_Q$ also covers the case of quantified subjects.

More generally, we can dispense with the simple rule for terms in favor of the more complicated one for QDPs.



$$(21) \quad \left[\left[\left[\text{A carpet touches every wall} \right] \right]_s \right] = 1 \text{ iff}$$

$$\left[\left[\text{a carpet} \right]_s *_{Q} \left[\left[\left[\text{touches} \right]_s \overrightarrow{*_{Q}} \left[\left[\text{every wall} \right]_s \right] \right] \right] \right] = 1 \text{ iff}$$

$$\left[\left[\text{a carpet} \right]_s \right] \ni \left[\left[\left[\text{touches} \right]_s \overrightarrow{*_{Q}} \left[\left[\text{every wall} \right]_s \right] \right] \right]$$

This derives the reading with *every wall* in the scope of *a carpet*. To get the reverse reading, we apply QR:

$$(22) \quad \left[\left[\text{every wall} \right]_x \left[\left[\text{a carpet touches } t_x \right] \right] \right]$$

Now we have to interpret (22) as “the set of walls is a subset of the set of x being touched by a carpet.” More generally:

$$(23) \quad \mathcal{Q} \ni \{ x : x \text{ is touched by a carpet} \} \text{ iff}$$

$$\mathcal{Q} * \{ x : x \text{ is touched by a carpet} \} = 1 \text{ iff}$$

$$\mathcal{Q} * \{ x : \text{a carpet touches } x \} = 1 \text{ iff}$$

$$\mathcal{Q} * \{ x : \left[\left[\text{a carpet touches } x \right]_s \right] \} = 1$$

General rule:

$$(24) \quad \left[\left[\boxed{\text{DP}}_x \boxed{\dots t_x \dots} \right] \right]_S = \left[\text{DP} \right]_S * \{ x : \left[\dots t_x \dots \right]_S \}$$

Assumptions:

- $\left[t_x \right]_S = x$;
- t_x is a referential expression, x is a term.
- the second box is a clause (a sentence, a CP, anything the extension of which is a truth value)

Note: if we want to generalize to QDPs, $\left[t_x \right]_S = \{ Y : x \in Y \}$



Recall that QDPs in object position cannot be interpreted by $\vec{*}$. A second way to resolve the problem is the application of QR:

$$(25) \quad \boxed{\text{John}} \boxed{\text{loves}} \boxed{\text{every girl}} \rightsquigarrow$$

$$\boxed{\boxed{\text{every girl}}_x} \boxed{\boxed{\text{John}} \boxed{\text{loves}} \boxed{t_x}}$$

$$(26) \quad \left\| \boxed{\boxed{\text{every girl}}_x} \boxed{\boxed{\text{John}} \boxed{\text{loves}} \boxed{t_x}} \right\|_s =$$

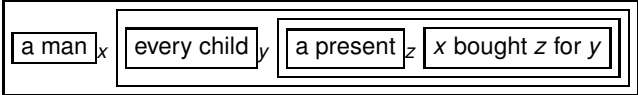
$$\llbracket \text{every girl} \rrbracket_s * \{ x : \llbracket \text{John} \rrbracket_s * [\llbracket \text{loves} \rrbracket_s \vec{*} \llbracket t_x \rrbracket_s]$$

Notes on the use of variables:

Variables are essential for multiple applications of QR. They relate the QDP to the argument position of the verb.

(27) A man bought a present for every child

Assume we want a reading with *every child* having wide scope with respect to a present, and *a man* having wide scope with respect to *every child*.

(28) 



A note on so-called bound variable pronouns (BVPs):

- (29) every man loves his mother
 (\neq every man loves every man's mother)

(30) LF: $\boxed{\boxed{\text{every man}}_x \text{ } t_x \text{ loves } \boxed{\text{his}}_x \text{ mother}}$

Assume $\text{his}_x = \text{he}_x$'s and $\llbracket \text{he}_x \rrbracket_S = x$

$$\begin{aligned}
 (31) \quad & \llbracket \text{every man} \rrbracket_S * \{ x : \llbracket x \text{ loves he}_x \text{'s mother} \rrbracket_S = 1 \} = \\
 & \llbracket \text{every man} \rrbracket_S * \{ x : x * \llbracket \text{loves} \rrbracket_S \overset{\rightarrow}{*} \llbracket \text{he}_x \text{'s mother} \rrbracket_S \} = \\
 & \llbracket \text{every man} \rrbracket_S * \{ x : x * \llbracket \text{loves} \rrbracket_S \overset{\rightarrow}{*} \llbracket \text{mother} \rrbracket_S (\llbracket \text{he}_x \rrbracket_S) \} = \\
 & \llbracket \text{every man} \rrbracket_S * \{ x : x * \llbracket \text{loves} \rrbracket_S \overset{\rightarrow}{*} \llbracket \text{mother} \rrbracket_S (x) \}
 \end{aligned}$$

In this framework, BVPs can be interpreted as bound by a QDP **only if** the QDP is QRed. The reason is that only after quantifier raising, the quantifying expression gets attached a variable, parallel to expressions like $(\forall x)$ or $(\exists x)$ in Predicate Logic.



Another cautionary note:

The interpretation of QR uses the operation of set building or comprehension by forming the set $\{x : \llbracket \dots t_x \dots \rrbracket_s\}$. We also assumed that $\llbracket t_x \rrbracket_s = x$. But x is strictly speaking not a denotation or reference, but an element of the language we use to describe denotations. This is a serious flaw which can be overcome by using various methods, the most popular being the use of assignment functions for variables, i.e. functions that assign values to x .

It would then follow, that $\llbracket t_x \rrbracket_s = g(x)$, where g is such a function. But then all interpretations must depend not only on s , but on g . Unfortunately, there is still a problem for compositionality. The reason is that set formation cannot depend on a variable assignment $g(x)$ that determines a denotation but must consider **all** such functions h with potentially different values than g . This is again a problem because then the semantics cannot depend on things, situations and truth values alone, but also on such functions (i.e. such functions are part of the ontology).

Determiners
and
Quantifiers

Type
Shifting and
Flexible
Types

QDPs in
Object
Position

Quantier
Raising

On
Variables

This problem is addressed but not completely solved in Chapter 10 of our book.

In fact, there is no straightforward and fully satisfying solution to the problem of compositionality. . .



ESSLI Summerschool 2014: Intro to Compositional Semantics

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Fifth Lecture: Propositions and Intensions

Intensional
Contexts

Cases and
Proposi-
tions

Logical
Space

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Composing
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Hintikka's
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[from Lecture 2]

LOGICAL [or FORMAL] SEMANTICS

The **meaning** of any expressions has (at least) **two components**, viz. its:

- **intension** \approx its contribution to the content of expressions in which it occurs
- **extension**: \approx its contribution to the reference of expressions in which it occurs
- ... and maybe more (but not in this course)

In the simplest cases:

- Intension is content.
- Extension is reference.



- (1) a. Pfäffingen is larger than Breitenholz
- b. Hamburg is larger than Cologne
- c. John knows that Pfäffingen is larger than Breitenholz
- d. John knows that Hamburg is larger than Cologne

- (2) a. There are no thieves
- b. There are no murderers
- c. John is an alleged thief
- d. John is an alleged murderer
- e. The criminologist is looking for a thief
- f. The criminologist is looking for a murderer



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- (3) Four fair coins are tossed
- (4) At least one of the 4 tossed coins lands heads up
- (5) At least one of the 4 tossed coins lands heads down
- (6) Exactly 2 of the 4 tossed coins land heads up
- (7) Exactly 2 of the 4 tossed coins land heads down



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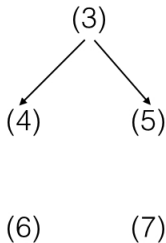
(3)

(4) (5)

(6) (7)

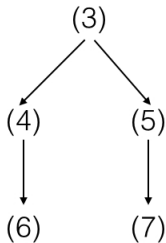


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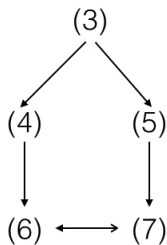


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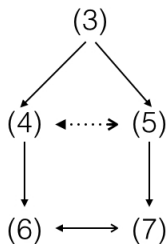


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- (8) John knows that at least one of the 4 tossed coins lands heads up
- (9) John knows that at least one of the 4 tossed coins lands heads down
- (10) *Most Certain Principle*
If a (declarative) sentence S_1 is true and another sentence S_2 is false in the same circumstances, then S_1 and S_2 differ in meaning.
- (11) John knows that exactly two of the 4 tossed coins lands heads up
- (12) John knows that exactly two of the 4 tossed coins lands heads down
- (13) *Definition [to be revised]*
The **proposition** expressed by a sentence is the set of possible cases of which that sentence is true.



(14)

possible cases	C_1	C_2	C_3	C_4
1	1	1	1	1
2	1	1	1	0
3	1	1	0	1
...
14	0	0	1	0
15	0	0	0	1
16	0	0	0	0

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- (15) a. Four coins were tossed when John coughed
b. Four coins were tossed and no one coughed

(16) *[Revised] Definition*

The **proposition** expressed by a sentence is the set of possible worlds of which that sentence is true.

(17) *Definition*

A sentence S is **true of** [or **at**] a possible world w if and only if $\llbracket S \rrbracket_w = 1$.

(18) By $\llbracket S \rrbracket$ we mean the proposition expressed by S :

$$\llbracket S \rrbracket := \{ w : \llbracket S \rrbracket_w = 1 \}$$

(19) A sentence S is true of a possible world w if and only if $w \in \llbracket S \rrbracket$.

(20) $\llbracket S \rrbracket_w = 1$ iff $w \in \llbracket S \rrbracket$.



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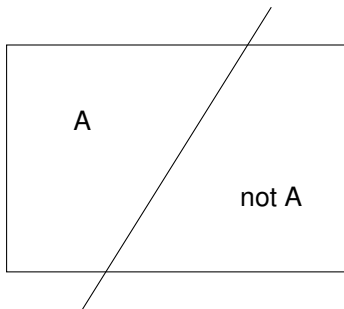
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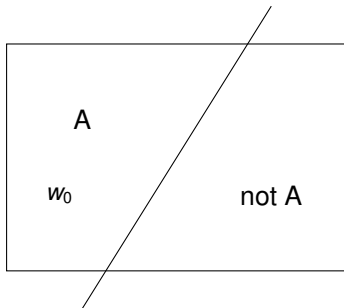
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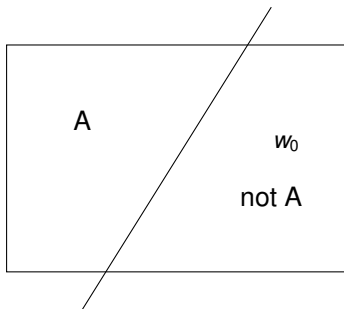
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(21) Barschel was murdered¹



¹Uwe Barschel [1944–1987] was a German politician who had to resign as the prime minister of Schleswig-Holstein under scandalous circumstances (comparable to the Watergate affair) and who was found dead in the bathtub of his hotel room a few days after his resignation. The circumstances of his death could never be fully clarified.



(22)

world	truth value
w_1	1
w_2	0
w_3	1
...	...
w_n	0
...	...

(23) *Definition*

The **intension** of α , written as $\llbracket \alpha \rrbracket$, is that function f such that for every possible world w , $f(w) = \llbracket \alpha \rrbracket_w$.



(24) *Principle of Intensional Compositionality*

The intension of a complex expression is a function of the intensions of its immediate parts and the way they are composed.

EXTENSIONAL CONSTRUCTIONS:

(25) For any world w :

$$\begin{aligned} & \llbracket \text{Paul is sleeping} \rrbracket (w) \\ = & \llbracket \text{Paul is sleeping} \rrbracket_w \\ = & \llbracket \text{Paul} \rrbracket_w * \llbracket \text{is sleeping} \rrbracket_w \\ = & \llbracket \text{Paul} \rrbracket (w) * \llbracket \text{is sleeping} \rrbracket (w) \end{aligned}$$



INTENSIONAL CONSTRUCTIONS

- (26) a. John knows that [Hamburg is larger than Cologne]
b. John knows that [Pfäffingen is larger than Breitenholz]

$$(27) \quad \llbracket \textit{John knows that S} \rrbracket_w = 1 \text{ iff } \langle \llbracket \textit{John} \rrbracket_w, \llbracket \textit{S} \rrbracket \rangle \in \llbracket \textit{know} \rrbracket_w$$

$$(28) \quad \text{For any world } w:$$
$$= \llbracket \textit{attitude verb + that + S} \rrbracket_w$$
$$= \llbracket \textit{attitude verb} \rrbracket_w \overset{*}{\rightarrow} \llbracket \textit{S} \rrbracket$$
$$= \llbracket \textit{attitude verb} \rrbracket (w) \overset{*}{\rightarrow} \llbracket \textit{S} \rrbracket$$



INTENSIONAL CONSTRUCTIONS

(29) John is an alleged thief / murderer

(30) For any world w :

$$\begin{aligned} & \llbracket \text{intensional-adjective + noun} \rrbracket_w \\ = & \llbracket \text{intensional-adjective} \rrbracket_w (\llbracket \text{noun} \rrbracket) \end{aligned}$$

(31) The criminologist is looking for a thief / murderer

(32) For any world w :

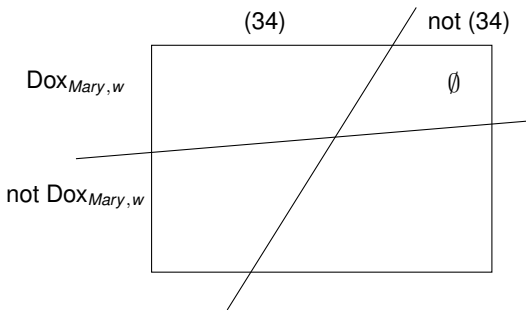
$$\begin{aligned} & \llbracket \text{opaque verb + quantifier phrase} \rrbracket_w \\ = & \llbracket \text{opaque verb} \rrbracket_w \xrightarrow{*} \llbracket \text{quantifier phrase} \rrbracket \end{aligned}$$



(33) Mary thinks that John is in Rome

(34) John is in Rome

(35)





$$(36) \quad \llbracket \text{think} \rrbracket_w = \{ \langle x, p \rangle : \text{Dox}_{x,w} \subseteq p \}$$

$$(37) \quad \llbracket \text{know} \rrbracket_w = \{ \langle x, p \rangle : \text{Epi}_{x,w} \subseteq p \}$$

$$(38) \quad \llbracket \text{want} \rrbracket_w = \{ \langle x, p \rangle : \text{Bou}_{x,w} \subseteq p \}$$

(39) Mary knows that Bill snores
 \models Mary thinks that Bill snores

$$(40) \text{ a. } \text{Epi}_{\text{Mary},w} \subseteq \llbracket \text{Bill snores} \rrbracket$$

$$\text{ b. } \text{Dox}_{\text{Mary},w} \subseteq \llbracket \text{Bill snores} \rrbracket$$

$$(41) \quad \text{Dox}_{x,w} \subseteq p \text{ whenever } \text{Epi}_{x,w} \subseteq p.$$

$$(42) \quad \text{Dox}_{x,w} \subseteq \text{Epi}_{x,w}$$



(43) Mary knows that Bill snores

⊨ Bill snores

(44) #Mary knows that Bill snores, but Bill doesn't snore

[Cf.: Mary believes that Bill snores, but (in fact) Bill doesn't snore]

(45) $w \in \text{Epi}_{x,w}$

(46) Mary doesn't know that Bill snores

⊨ Bill snores

(47) Mary thinks that Bill has two or three children

⊨ Mary thinks that the number of Bill's children is prime