<u>The Method of Extension and Intension</u>

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1. *<u>Extensions</u>*

1.1 *<u>Anti-Psychologism</u>*

Psychologism

Q What are meanings?

Naive A *The meaning of an expression is the speaker's mental association connected with this expression*

Objections

Mental associations are …

… *subjective*: Different speakers make different associations with the same expression without thereby changing the meaning of that expression.

… *restricted*: One might imagine associated mental images as meanings of concrete nouns like **table** or **horse**, but which associations does one have with **and**, **frequently**, **only**,…?

… *irrelevant*: Due to their personal experience, speakers may have all kinds of associations when a word is mentioned without that influencing its meaning.

… *private*: A person's associations are princially inaccessible to other speakers, so how can they then be used in communicating among speakers?

Starting point of logical semantics

Meaning must be determined in terms of communicative function. Two aspects of communciation play a key role:

- the aspect of *reference*: language is used to *talk about* things, persons, events, etc. –> *EXTENSION*
- the aspect of *informativity*: language is used to exchange *information* –> *INTENSION*

Strategy

Given any expression, try to find an object (in a very wide sense) that the expression refers to. This will be the *extension* of the expression.

1.2 Extensions: Simple Cases

- *Proper names*

Springfield refers to a city
(Disambiguation of **Springfield** into 'underlying forms' **Springfield**$_{Mass.}$, **Springfield**$_{NJ}$, etc.; similarly for names like **John Smith**)

- *Definite descriptions*

the president of the United States refers to a person (Bill Clinton)
the largest city of Switzerland refers to a place (Zurich)

- *Nouns*

city refers to several places, i. e. its extension is the *set* of all cities.

A set is identified by its *members*. In order to designate a set, one may write a list of its elements and surround it by curly brackets: e.g., {Madrid, Lisbon, Rome} is a set with three elements each of which is a city. Neither the order nor the frequency of the list members are of any relevance – nor is the way they are called. Hence the three element set just mentioned is the same set as: {Rome, Madrid, Rome, the capital of Portugal}. The elements of a set may be sets themselbes. For instance, {Madrid, {Madrid, Lisbon, Rome}} is a set with two elements on e of which is the capital of Spain whereas is other is a set of cities. There is one particular set that does not have any elements: the empty sete Ø (aka { }); we will soon meet it. In order to express that an object x is an element of a set M, one writes: '$x \in M$'; '$x \notin M$' means the opposite. We thus have: Rome∈{Madrid, Lisbon, Rome}, but Rom∉{Madrid, {Madrid, Lisbon, Rome}}!

- *Intransitive verbs*

sleep refers to all individuals that are asleep.

- *Transitive verbs*

kiss refers to all kissing individuals, i.e. all x and y where x kisses y, i.e. all *pairs* (x,y), where y is kissed by x

In the case of pairs, the order of the *components* does play a role and so does the frequency. Given pairs, it is not hard to assume triples, quadruples, quintuples, etc., i.e. 'lists' cosisting of three, four, five etc. component. In general, one speaks of *n-tuples*. Tus a pair is a 2-tuple, a triple a 3-tuple, etc.

expression (category)	*type of extension*	*example*	*extension of example*
proper name	individual (bearer)	**Fritz**	*Fritz Hamm*
definite description	Individual (*describe object*)	**the fouth largest city in France**	*Nice*
count noun	set (of individuals)	**table**	*set of tables*
intransitive verb	set (of individuals)	**sleep**	*set of sleepers*
transitive verb	set of pairs (of individuals)	**eat**	set of pairs *(eater,food)*
ditransitive verb	set of triples (set of individuals)	**give**	*set of triples (giver,recipient, present)*

1.3 Truth values

Parallelism [of Syntactic and Semantic Saturation]
The extension of a n-valent verb is always a set of n-tuples.

Frege's Observation
The extension of a sentence is a set of 0-tuples.
Which set? Assume $(a) = a$ and observe:

verb	*valency*	*extension*
shows	3	all triples (a,b,c) such that: a shows b to c
shows the US president	2	all pairs (a,b) such that: a shows b to Clinton
shows the US president the Vatican	1	all 1-tuples (a) such that: a shows the Vatican to Clinton

Then generalize downwards (by analogy):

Te pope shows the US president the Vatican	0	all 0-tuples ( ) such that: John Paul II shows the Vatican to Clinton

Q: What is a 0-tuple?

A : A 3-tuple (or triple) is a list (a,b,c) of length 3,
a 2-tuple (pair) is a list (a,b) of length 2,
a 1-tuple (individual) is a list (a) of length 1.
So a 0-Tupel would have to be a list ( ) of length 0, i.e. a list without an entry.

Convention: () = Ø.

The extension of the above sentence thus is a set of 0-tuples, i.e. a set all of whose elements are 0-tuples. Which set?

- If the pope *does* show the Vatican to Clinton, it will then be true of all 0-tuples *that* the pope shows the Vatican to the US president. So the extension of the sentence turns out to be the set whose sole element is the 0-tuple, i.e.: {Ø}.
- If the pope does *not* show the Vatican to Clinton, it is not true of any 0-tuple that the pope shows the Vatican to the US president. The extension will thus be empty, i.e.: Ø.

CONCLUSION: The extension of the sentence only depends on whether it is true. If so, the extension is {0}; if not, it is Ø. These two sets are known as the truth values. And instead of {Ø} and Ø they are also called: T and F.

1.4 Connectives

Conjunction

A	B	A **and** B
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

A	*B*	*A* **oder** *B*
T	*T*	*T*
T	*F*	*T*
F	*T*	*T*
F	*F*	*F*

Negation

A	**not** *A*
T	*F*
F	*T*

Non-sentential conjunction, disjunction and negation …

(12) **A penguin <u>and</u> two polar bears live in this zoo.**

(13) **She is laughing <u>or</u> crying.**

(14) **One of the girls does <u>not</u> sleep.**

… paraphrased away:

(12') **There is a penguin in this zoo and there are two polar bears in this zoo.**

(13') **She is laughing or she is crying.**

(14') **For one of the girls it holds that she is not crying.**

Not so easily paraphrased:

(15) **In this zoo, a penguin and two polar bears are sharing a cage.**

(16) **She does not know whether she should laugh or cry.**

(17) **One of the girls does not sleep here.**

Pragmatic effects on conective meanings:

(18) **She married and [she] became pregnant.**

(19) **Sie became pregnant und [she] married.**

1.5 <u>Extensional Compositionality</u>

Extensional Principle of Compositionality

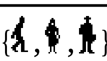
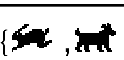
The extension of a complex expression is determined by the extensions of its immediate parts and the way they are combined.

(20) **Fritz doesn't work and Eike is asleep.**

<u>General rules</u> for determing extensions:
The extension of a sentence of the form 'Sentence$_1$ + Connective + Sentence$_2$' is the truth value assigned by the extension of the Connective to the extensions (truth values) of Sentence$_1$ and Sentence $_2$.
The extension of a sentence of the form 'proper name + verb' is the truth value T if the extension of the prper name is an element of the extension of the verb; otherwise the extension of that sentence is F.

1.6 Quantifiers

(21) **Nobody is asleep.**

predicate extension	*sentence extension*
{}	*F*
{}	*F*
{}	*F*
{}	*T*
Ø	*T*
…	…

The left row of the table shows the extension of **is asleep** under varying circumstances, the right one gives the truth value of (21) under the same circumstances.

(22) **Nobody is eating nuts.** Same table as above!

<u>Observation o</u>n sentences with **nobody** in subject position:
Whenever the extension of the predicate contains a person, the sentence will be false; otherwise its extension is *T*.
Why is this so? This must be due to the word **nobody**, and more precisely: to its meaning, which creates a correlation between the reference of the predicate (whose subject it is) and the truth value of the sentence so built.

<u>Simplest hypothesis:</u>
The extension of **nobody** *is a function whose arguemts are sets of individuals and whose values are truth values. It assigns the value W to any set that has no persons as elements; any other set is assigned the value F.*

Similarly:

The extension of **every animal** *is a function whose arguemts are sets of individuals and whose values are truth values. It assigns the value W to any set that has all animals as elements; any other set is assigned the value F.*

2. *Intensions*

2.1 Intensional Contexts

(23) **New York is larger than New Brunswick.**

(24) **Hamburg is larger than Frankfurt.**

(25) **Bill knows that New York is larger than New Brunswick.** attitude report

(26) **Fritz knows that Hamburg is larger than Frankfurt.** attitude report

Bracketing (to determine immediate parts):

(25') [**Fritz** [**knows** [**that** [**Hamburg is larger than Cologne**]]]]

(26') [**Fritz** [**knows** [**that** [**Pfäffingen is larger than Breitenholz**]]]]

(27) **that New York is larger than New Brunswick**

(28) **that Hamburg is larger than Frankfurt**

The extensions of (27) and (28) according to the extensional principle of compositionality

$|S|$ is the extension of sentence S; '$\oplus$' indicates the way in which the extension of **that** is combined with that of the extension of the embedded clause.

$$|(27)| = |\textbf{that}| \oplus |(23)| = |\textbf{that}| \oplus T$$

$$|(28)| = |\textbf{that}| \oplus |(24)| = |\textbf{that}| \oplus T$$

Hence: $|(27)| = |\textbf{that}| \oplus T = |(28)|$

(29) **knows that New York is larger than New Brunswick**

(30) **knows that Hamburg is larger than Frankfurt**

The extensions of (29) and (30) according to the extensional principle of compositionality

'$\otimes$' indicates the way in which the extension of **know** is combined with that of the **that**-clause.

$$|(29)| = |\textbf{know}| \otimes |(27)| = |\textbf{know}| \otimes |(28)| = |(30)|$$

This cannot be: the two above predicates (29) and (30) clearly have distinct extensions!

Whenever extensions appear to disregard the extensional principle of compositionality, semanticists speak of an *intensional contexts*; in the case at hand this was the gap in **knows that** _____.

2.2 *Propositions*

(31) **Four coins were tossed.**
(32) **At least one of the four coins tossed came up heads.**
(33) **At least one of the four coins tossed came up tails.**
(34) **Exactly two of the four coins tossed came up heads.**
(35) **Exactly two of the four coins tossed came up tails.**

Comparison of informative values:
(31) is the least informative of the five sentences.
(32) is less informative than (34).
(35) is more informative than (33).
(34) and (35) are equally informative.
(32) and (33) contain the same *amount* of information, but they differ in the information they convey: they are *quantitatively* equal and *qualtitatively* different in informativity (or information value).

Observation

A sentence A is *quantitatively* more informative than a sentence B if the *number* of cases of which A is true is smaller than the number of cases in which B is true.
A sentence A is *qualitatively* more informative than a sentence B if B is true in each case in which A is true.

Carnap's Idea

The proposition expressed by a sentence is the set of cases of which it is true.
Notation: $[\![S]\!]$.

Cases

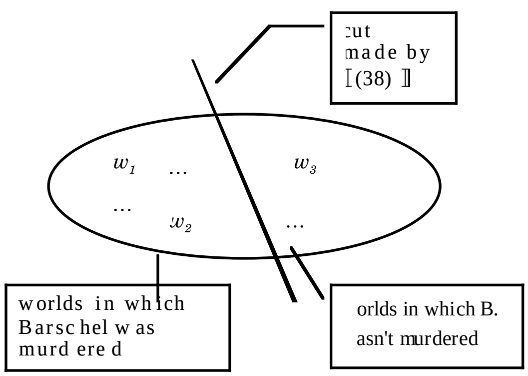
… judging from (31) – (35):
HHHH (4 times heads), *HHHT*,…, *TTTT* etc.
$[\![(31)]\!] = \{HHHH,\ldots,TTTT\}$ (all cases)
$[\![(32)]\!] = \{HHHH, HHHT,\ldots,TTTH\}$ (all cases with Hs)
… taking further examples into account:
(36) **Four coins were tossed, while someone was coughing.**
(36) **Four coins were tossed, while no one was coughing.**
HHHHC, *TTTTN*, ….
$[\![(31)]\!] = \{HHHHC,\ldots,TTTTN\}$ (all cases)
… and so on …

Conclusion
Cases are specified with respect to arbitrary aspects. Semanticists therefore prefer to call them *possible worlds* instead of cases. The set of all possible worlds is called *Logical Space*. It contains all possible cases, even the most bizarre ones, provided they are spelt out in every detail. The points of Logical Space, the possible words, all differ from each other in at least one detail or another: the time of origin of our universe, the number of grains of sand in the Sahara, etc. Only one of these many possibilities actually occurs; it is the actual world. Since we are ill-informed about all the details fo this relaity, we do not know which point in Logical Space it corresponds to. We cannot completely *localize* reality.

2.3 *From Propositions to Intensions*

(38) **Barschel was murdered.**

The proposition $[\![(38)]\!]$ divides Logical Space:



Proposition $[\![(38)]\!]$ *as a set …*

World	*Truth values*
w_1	T
w_2	T
…	…
w_n	F
…	

… and as a function

Alternative characterization of $[\![(38)]\!]$ *:*

The intension of (38) is a function whose arguments are possible worlds and whose values are truth values. It assigns the truth value W to any world in which Barschel was murdered, and it assigns the value F to all other worlds.

In general:

The intension is the extension as depending on the possible world.

Examples of intensions that are not propositions

- $[\![\textbf{Fritz}]\!]$

World	*Individual*
w_1	
w_2	
w_3	
…	…

The proper name as a rigid designator (= expression with a constant intension)

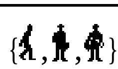
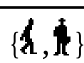
<u>NB</u>: Although Fritz could have been *called* 'Hans', Fritz would still *be* Fritz (and niot Hans).

- $[\![\textbf{the president of the US}]\!]$

World	*Individual*
w_1	
w_2	
w_3	
…	…

The intension of a definite description as an individual concept (= world-dependent individual)

- $[\![\textbf{communist}]\!]$

World	*Set*
w_1	
w_2	
w_3	Ø
…	…

The intension of a definite description as an property (= world-dependent set of individuals)

- $[\![$**no communist**$]\!]$

World	*Set*
w_1	{Ø , {}, …,{, },…}
w_2	{Ø , {},{}, …,{, },{, },{,},…,}
w_3	{Ø , {},{},{ }, {}…,{, },…} (= *all* sets of individuals)
…	…

The intension of a quantifier as a function

Intensional principle of compositionality
The intension of a complex expression is determined by the intensions of its immediate parts and the way they are combined.

Example
(39) **The president of the US is a communist.**

<u>Notation:</u> For any world w and expression X, '$[\![X]\!]^w$' is the extension of X for w, i.e. the value to the right of w in the table representing X's intension.

$[\![(39)]\!]$ can be determined 'pointwise': given $[\![$**the president of the US**$]\!]$ and $[\![$**communist**$]\!]$, for any w, the value will be T if the extension of former (for w) is an element of the extension of the latter (for w), i.e. if:

$$[\![\textbf{the president of the US}]\!]^{\boldsymbol{w}} \in [\![\textbf{communist}]\!]^{\boldsymbol{w}}.$$

2.4 From Intension to Extension – and Back

Frege's Road

The extension of an expression X is completely determined by its intension (and the facts):

$$\| X \| = [\![X]\!]^{r}.$$

Carnap's Road

The intension of an expression X is completely determined by its extensions in Logical Space:

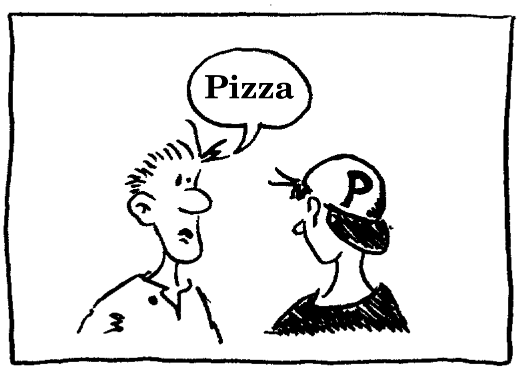
$[\![X]\!]$ =

World	*Extension*
w_1	$[\![X]\!]^{w_1}$
w_2	$[\![X]\!]^{w_2}$
w_3	$[\![X]\!]^{w_3}$
…	…

Intensions are conventionally associated with expressions, extensions must be discovered.

3. <u>*From Meaning to Use*</u>

Flow of Information



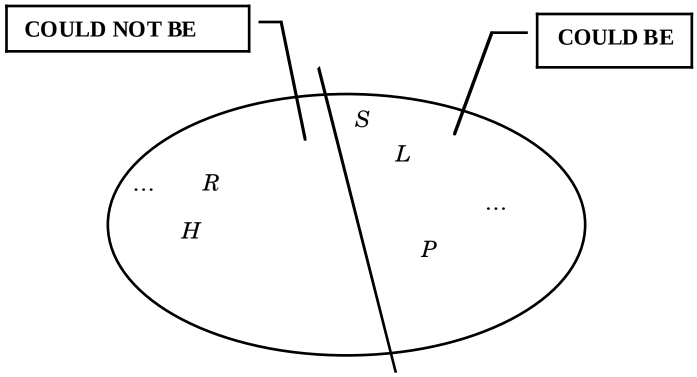
[© Regine Eckardt]

Walter answers his son Paul's question *What are we having for dinner tonight?*

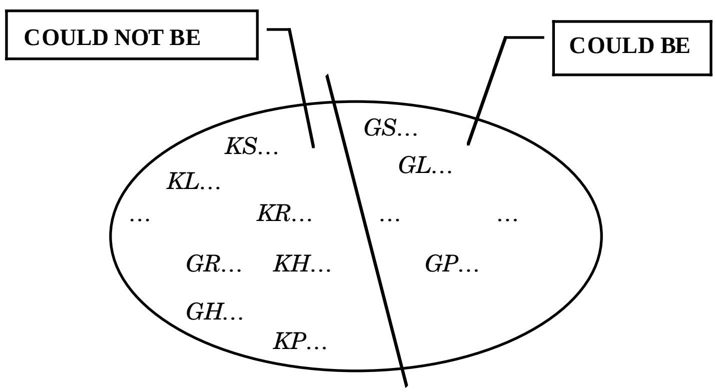
(40) **We are going to have pizza tonight.**

Ideal update scenario

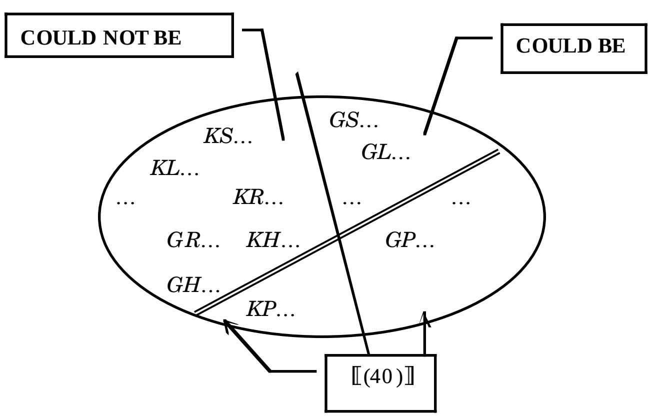
- Paul's background (= his information before Walter's utterance) does not suffice to decide about the truth or falsity of (40).
- However, Walter knows that (40) is true.
- After Walter's utterance Paul too knows that (40) is true.



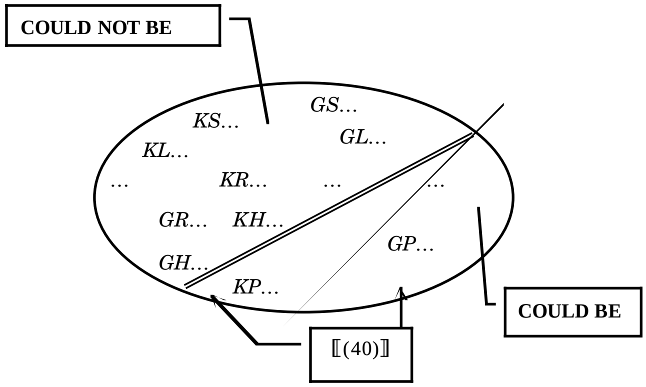
What Paul already knows about tonight's dinner
[H = honey cake; R = rabbit, S = spinach, P = pizza, L = lasagna]



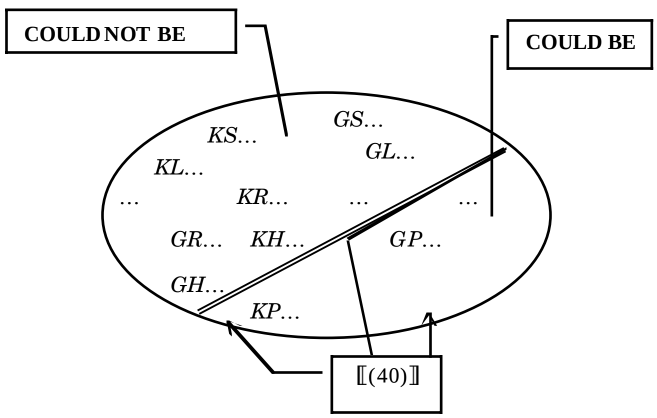
More things Paul knows [G = Paul is in Germany; K = Paul is in Korea]



Paul's background and the proposition expressed by (40)



Walter's background and the proposition expressed by (40)



The communicative effect of Walter's utterance on Paul's background

Attitude reports revisited

(41) **Fritz knows that <u>Hamburg is larger than Cologne</u>.**

(42) **Fritz knows that <u>Pfäffingen is larger than Breitenholz</u>.**

Extensional analysis

The extension of **know** *is a set of pairs* (x,p), *where x's information state excludes all worlds outside of p.*