Zu: "Permutation Invariance and Definability". o.D.

Tübingen, May 15th, 1987

Dear Johan,

Thank you very much for sending me your paper on polyadic quantifiers as well as your notes on invariance. As to the former, I feel very proud indeed to be quoted in an article of yours. Concerning the latter, I would like to make a few comments that might be of interest to you.

I believe that I knew your result before though I think that I had proved it in a slightly different and, may I say, more direct manner: starting with a finite domain $D_e = \{d_1,\dots,d_n\}$, you can define an expression U_X of category τt (for any $X \in D_\tau$) that denotes $\{X\}$ at any assignment starting with $d_1,\dots,d_n$: if $\tau = e$, $U_{d_i} = \lambda x_{i+1}\,(x_{i+1} = x_i)$, and for functional F one lets U_F be $\lambda f\, \forall z \bigwedge_{y \in D_\sigma} [U_y(z) \rightarrow U_{F(y)}(f(z))]$. For invariant X, we then have:

$\| U_X \|^{u_1,\dots,u_n} = \| U_X \|^{\pi(u_1,\dots,u_n)} = \{X\}$, (for any π) and hence:

$\| \lambda y\, \forall x_1,\dots x_n \,(\bigwedge_{1 \le i < j \le n} x_i \neq x_j \Rightarrow U_X(y) \|$, at any assignment. That would have been my definition (or maybe it was). I feel that this is a more direct approach to what you call definability but I might be mistaken here. (Talking of directness: there is, by the way, a slight redundancy in your proof: on top of p. 3 you make a 2nd use of '$\equiv$'. I think this is unnecessary because, according to your construction, a_i and b_i are of the same X type and hence inclusion implies identity anyway.) I must admit that I do not like this kind of approach to definability. For one thing, as you do mention, it doesn't generalize to the infinite case, and I would claim that it does not work properly in the finite case either. At least not if you widen your local perspective. The definitions that I get by the above procedure only produce unit sets if you stick to the ontology you started out with. Otherwise you might end up with larger (or empty) sets and hence with no definition (in your sense). It thus seems to be interesting whether one can get global definitions (= formulae that are definitions on <u>any</u> ontology) for arbitrary invariant objects. I think (I'm not sure, though) that your definitions don't behave any better than mine so that the problem still seems to be open.

X functional

Another issue that arises when leaving the local perspective is the problem of defining counterparts of (logical and other) objects through different ontologies: maybe some possible definitions of (locally logical and invariant) objects should be filtered out because they do not respect some natural (and yet to be defined) counterpart relation. I am very interested in this question and would like you to send me any of your work relating to it. Also, I would like to know whether you've got any results on term definability (in the sense that α defines X iff $\|\alpha\|$ = X, for any assignment). From your letter to Ed I had got the impression that your theorem concerned this latter notion.

I hope that not everything I have said about your definability paper is wrong or obscure. If you have meanwhile extended it, I would be extremely grateful if you sent me a copy of a more recent version.

Thanks again, and hope to see you some time:

Ede

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