Comparing expressive power in two-dimensional semantics

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Bonn Workshop Context-Sensitivity and Logical Consequence, June 2019

0. The bigger picture

Variables explained away

(0a) \((\exists x) [P(x) \land \lnot Q(x)]\) vs. \(P \land Q \neq \emptyset\)

(b) \((\exists w) [w_0 R w \land P(w)]\) vs. \(\therefore p\)

Explaining index variables away

Köpping & Zimmermann (forthcoming)

Whether two-dimensional logic is expressively equivalent to intensional logic is open to interpretation (and ideology).

Propositionalism

Quine (1953); D. Kaplan (1975); Larson (2002)

Intensionality is (reducible to) clausal embedding.

Law of the instrument

A. Kaplan (1964: 28)

Give a small boy a hammer, and he will find that everything he encounters needs pounding.

1. Comparative Expressivity of Formal Languages

Schematic definitions

- A language \(L^*\) is at least as expressive as a language \(L\) iff for any (relevant) expressions \(\alpha^*\) in \(L^*\) there is a (relevant) expression \(\alpha\) in \(L\) such that \(\alpha^* \sim \alpha\).

where \(\sim\) denotes model-theoretic equivalence, i.e.:

\[
\alpha^* \sim \alpha \text{ iff } \left[ \alpha^* \right]^\theta = \left[ \alpha \right]^\theta
\]

... for all \(L\)-determinants \(\theta\) and matching \(L^*-\)determinants \(\theta^*\).

Examples

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<th>#</th>
<th>(L)</th>
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<th>determinants reversible?</th>
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\(\alpha \in L\)

(1) \((\exists x) [P(x) \land Q(x)]\)

(2) \((\exists \emptyset) [P(x) \land \lnot P(x)]\)

(3) \(\therefore [p \land q]\)

(4) \(\therefore [p \rightarrow \therefore p]\)

(5) \(\lambda p^{*} (i) \cdot (\exists x) \cdot \left[ B(x) \land P(x) \right]\)

(6) \(\lambda f^{*} (i) \cdot B(f(i)(x))\)

... where \(\Sigma\) abbreviates: \(\lambdax^{*} \cdot \therefore \left[ \lambda x^{*} Q(s,t,i) \cdot [p = \left[ [\lambda q. \forall q. = Q] \right]\left[ \lambda q. \forall q. \right] = Q] \right]\)

... and \(\Phi\) abbreviates: \(\lambda R. (\emptyset R) \cdot \left[ \Sigma (p) \land \left[ p \land B(x) \right] \right]\)

(7) \(\lambda p^{*} [\lambda x^{*} \cdot (\forall j^{*} [i_0 Epi_j \rightarrow p(j)]]\)

\(\alpha^* \in L^*\)

cf.

\(\emptyset \in \emptyset\)

\(\emptyset \in \emptyset\)

Quine (1960)

D. Kaplan (1974)

Gallin (1975)

Gallin (1975)

Zimmermann (1989)
3. Two-dimensional Languages

Determinants of denotation
\[ \llbracket \alpha \rrbracket^{M,c,i} \], where
- \( M \) is an interpretation (of non-logical constants)
- \( c \) is a context
- \( i \) is an index
- ‘…’ could be empty or contain more determinants (e.g. a variable assignment) and will be suppressed

Additional structural assumptions
- Diagonal:
  Each context \( c \) determines its index \( i_c \) due to parameterization:
  \( c = (c_1, \ldots, c_n, \ldots, c_k) \), and: \( i^c = (i^c_1, \ldots, i^c_n) \).
- No monsters:
  \( \alpha \) is an interpretation of \( \alpha \)
  \( \beta \) is an interpretation of \( \beta \)
  then: \( \llbracket \alpha \beta \rrbracket^{M,c,i} = \llbracket \alpha \rrbracket^{M,c,i} \llbracket \beta \rrbracket^{M,c,i} \),
  where \( \llbracket \gamma \rrbracket^{M,c} \) is the intension of \( \gamma \):
\[ \llbracket \gamma \rrbracket^{M,c}(i) = \llbracket \gamma \rrbracket^{M,c'} \], for any index \( i \).
- … or, equivalently:
  All syntactic constructions are (at most) intensional, i.e.: for every context \( c \in C \), there is a corresponding operation \( \Gamma_c \) on (possible) intensions such that for any expression \( \alpha \) built up by \( \Sigma \) from expressions \( \beta \) and \( \gamma \), the following equation holds:
\[ \llbracket \alpha \rrbracket^{M,c} = \Gamma_c(\llbracket \beta \rrbracket^{M,c}, \llbracket \gamma \rrbracket^{M,c}) \).

Relevant determinants
- **characters** assigning denotations \( \llbracket \alpha \rrbracket^{M,c,i} \) relative to models \( M \) and (arbitrary) points of reference \((c,i)\).
  **Motivation**: linguistic meaning, cognitive significance \( \text{Montague (1970), Kaplan (1989)} \)

- **epistemic contents** assigning denotations \( \llbracket \alpha \rrbracket^{M,c,i} = \llbracket \alpha \rrbracket^{M,c,i} \) relative to models \( M \) and contexts \( c \).
  **Motivation**: logical validity; cognitive significance \( \text{Montague (1970); Lewis (1979)} \)

- **intensions** assigning denotations \( \llbracket \alpha \rrbracket^{M,c} \) relative to models \( M \) and contexts \( c \).
  **Motivation**: indirect denotation, expressed content \( \text{Montague (1970); Kaplan (1989)} \)

Notions of Truth
\[ \varphi \text{ is true at (or in) a context } c \text{ [relative to a model } M \text{] iff } \llbracket \varphi \rrbracket^{M,c} = 1. \]
\[ \varphi \text{ is true of an index } i \text{ [relative to a context } c \text{ in a model } M \text{] iff } \llbracket \varphi \rrbracket^{M,c}(i) = 1. \]
[Hence being true in a context is being true of its index]
\[ \varphi \text{ is true of an index-component } i_m \text{ as the } m \text{-component [relative to …] iff } \llbracket \varphi \rrbracket^{M,c}(i_1, \ldots, i_m, \ldots, c_n) = 1. \]

4. Properties as Objects of Intentional Attitudes

*Propositionalism*

**Propositionalism**

Any intentional attitude is [definable in terms of] a propositional attitude.

**Examples**
- To seek a unicorn is to try for it to be the case that one finds a unicorn. \( \text{Quine (1953)} \)
- To want chocolate is to desire for it to be the case that one has chocolate. \( \text{Larson (2002)} \)
Counterexamples
To think of a unicorn is not to think that there is a unicorn.
To like chocolate is not to like for oneself to have chocolate.

Anti-propositionalism
Some intentional attitudes are irreducibly attitudes towards properties. cf. Grzankowski (2013)

Perspectivism
Some intentional attitudes are irreducibly attitudes towards properties.

Question
What distinguishes anti-propositionalism and perspectivism?

Some tentative answers:
The difference between …
… having a property and being exposed to a property
… properties as attributes vs. properties as objects
… truth at a location and truth of an object

References
van Benthem, Johan: ‘Correspondence Theory’. In: D. M. Gabbay & F. Guenthner (eds.), Handbook of
Dosen, Kosta: ‘Second-order logic without variables’. In: W. Buszkowski et al. (eds.), Categorial
Köpping, Jan; Zimmermann, Thomas Ede: ‘Looking Backwards in Type Logic’. Inquiry. Forthcoming. [Early
access version available online]
Larson, Richard: ‘The Grammar of Intensionality’. In: G. Preyer & G. Peter (eds.), Logical Form and
New York 1953: 139-159.
104 (1960), 343–347.
Zimmermann, Thomas Ede: ‘On the Proper Treatment of Opacity in Certain Verbs’. Natural Language
Semantics 1 (1993), 149-179.