

Since Hans Kamp's comments contain a detailed criticism of Chierchia's treatment of donkey sentences, I will confine myself to a few remarks concerning the general framework.

1. *Dynamic Logic: basic ideas*

The version of dynamic logic underlying Chierchia's analysis can be understood from comparing the ordinary predicate logic analyses of (1) and (2) with a formalization of the text (1.2) consisting of (1) followed by (2):

- (1) ***A young lady entered the office.***
- (2) ***She wore a fashionable hat.***
- (1') $(\exists x) [YL(x) \ \& \ EO(x)]$
- (2') $(\exists y) [FH(y) \ \& \ W(x,y)]$

Two problems arise when we try to use (1') and (2') to account for the meaning (1.2') of (1.2):

- (1.2') $(\exists x) [YL(x) \ \& \ EO(x) \ \& \ (\exists y) [FH(y) \ \& \ W(x,y)]]$

The first problem (a) is one of freedom and bondage: (2') contains free x reflecting the deictic nature of the 3rd person pronoun *she* in (2); but in (1.2') the same x is bound by an existential quantifier corresponding to the indefinite subject ***a young lady*** to which *she* stands in an anaphoric relation. In particular, the meaning of (1.2') should no longer depend on the contextual value of x . The other problem (b) has to do with compositionality: (1') is no proper part of (1.2') and it is not hard to see that there is no way to obtain the meaning of (1.2') from the meanings of (1) and (2).

Both (a) and (b) can be solved in an *ad hoc* way: we may first bind the x in (2') by a λ -operator thus obtaining the predicate:

- (2'') $\lambda x (\exists y) [FH(y) \ \& \ W(x,y)]$

This already solves (a) if we agree to construe contextual dependence by *applying* (2'') to the contextually relevant individual (given by free x). With this new interpretation of (2), we may now reanalyze (1) by including in the scope of the existential quantifier a predicate variable Q for which (2'') may be inserted as a possible value:

- (1'') $\lambda Q (\exists x) [YL(x) \ \& \ EO(x) \ \& \ Q(x)]$

We can now combine (1'') and (2'') by functional application and obtain (1.2'), as desired.

The basic idea of dynamic logic can be seen as a generalization of this *ad hoc* way of deriving (1.2') from (1'') and (2''). The first aspect of this generalization concerns the number n of anaphoric relations to be found in texts like (1.2):

- (3) ***A young lady with a gun entered the office.***
- (4) ***She pointed it at me.***

Adopting the above analysis to (3.4) would result in an interpretation of (4) as a binary relation (4') for which the interpretation (3') of (3) must contain a suitable *slot*:

- (3') $\lambda R (\exists x) (\exists y) [YL(x) \ \& \ G(y) \ \& \ W(x,y) \ \& \ EO(x) \ \& \ R(x,y)]$
- (4') $\lambda y \lambda x P(x,y,ego)$

Since there seems to be no natural upper limit to n , the predicate corresponding to the anaphorically linked sentence would in general have

arbitrary many places; consequently, the variable slot would have to allow for arbitrarily long predicates. To unify the treatment of (2) and (4), we can thus conceive of them as special cases of predicates with indefinitely many places, obtained by *infinitary abstraction* from all (or all anaphorically relevant) individual variables, i.e. unselective binding:

$$(5) \quad \lambda x_1 \lambda x_2 \lambda x_3 \dots \lambda x_n \lambda x_{n+1} \dots \varphi$$

Since we only need this one special case of infinitary abstraction (from the *standard sequence* of variables denoting the *current case*), we need not define this operation in full generality (although we certainly could). So we may introduce some abbreviation for the abstraction in (5):

$$(5') \quad \hat{\varphi}$$

Note that, in a type-theoretic framework, infinitary abstraction also forces us to introduce a new basic type *s* corresponding to infinite sequences of individuals or *cases*. The generalization of (1'') and (3') then is:

$$(6) \quad (\exists x_1) \dots (\exists x_n) [\varphi \ \& \ \mathbf{R}(x_1, \dots, x_n, x_{n+1}, \dots)]$$

where \mathbf{R} is a variable of type *st* and ' $x_1, \dots, x_n, x_{n+1}, \dots$ ' is the standard sequence. Again we may introduce some notation for applying a functor to the standard sequence. (6) then becomes:

$$(6') \quad (\exists x_1) \dots (\exists x_n) [\varphi \ \& \ \mathbf{R}]$$

One more step is needed for a full generalization of the ideas underlying (1'') and (2''): since virtually any sentence can be used to either start or carry on anaphoric chains, the analysis of any sentence should contain both an \mathbf{R} -slot and an abstraction from all individual variables. Thus the above examples finally end up as having the following *dynamic logical forms*:

$$(1d) \quad \lambda \mathbf{R} \hat{(\exists x) [YL(x) \ \& \ EO(x) \ \& \ \mathbf{R}]}$$

$$(2d) \quad \lambda \mathbf{R} \hat{(\exists y) [FH(y) \ \& \ W(x, y) \ \& \ \mathbf{R}]}$$

$$(3d) \quad \lambda \mathbf{R} \hat{(\exists x) (\exists y) [YL(x) \ \& \ G(y) \ \& \ W(x, y) \ \& \ EO(x) \ \& \ \mathbf{R}]}$$

$$(4d) \quad \lambda \mathbf{R} \hat{[P(x, y, ego) \ \& \ \mathbf{R}]}$$

In particular, then, the dynamic sentence type will be $(st)(st)$. And the combination of adjacent sentences is interpreted by the operation \oplus of *dynamic conjunction*, which turns out to be functional composition:

$$[[S_1]] \oplus [[S_2]] = \lambda \mathbf{R} [[S_1]] ([[S_2]](\mathbf{R}))$$

2. Quantification over cases

Chierchia's analyzes adverbs of quantification as quantifiers over cases. The motivation for his analysis seems to derive from the following paraphrases (as becomes apparent from an earlier version of Chierchia's paper):

(7) *When John is in the bathtub he always sings.*

(7') *All properties of occasions on which John is in the bathtub are instantiated by occasions on which he (John) sings.*

(8) *If a man is in the bathtub he always sings.*

(8') *All relations holding between a man and occasions on which that man is in the bathtub hold between a man and occasions on which that man is in the bathtub and sings.*

(7') and (8') are readily expressed in the dynamic framework sketched

above; and, more importantly, this can be done by combining the expected formalizations of their respective parts. To see this, we may consider second-order translations (7'') and (8'') and then check that they are equivalent to the dynamic formulae (7d) and (8d):

$$(7'') \quad (\forall P) [(\exists o) [B(j,o) \ \& \ P(o)] \rightarrow (\exists o) [B(j,o) \ \& \ S(j,o) \ \& \ P(o)]]$$

$$(7d) \quad (\forall \mathbf{R}) [(\exists o) [B(j,o) \ \& \ \mathbf{R}] \rightarrow (\exists o) [B(j,o) \ \& \ S(j,o) \ \& \ \mathbf{R}]]$$

$$(8'') \quad (\forall \mathbf{R}) [(\exists o) (\exists x) [M(x) \ \& \ B(x,o) \ \& \ \mathbf{R}] \rightarrow (\exists o) (\exists x) [M(x) \ \& \ B(x,o) \ \& \ S(x,o) \ \& \ \mathbf{R}]]$$

$$(8d) \quad (\forall \mathbf{R}) [(\exists o) (\exists x) [M(x) \ \& \ B(x,o) \ \& \ \mathbf{R}] \rightarrow (\exists o) (\exists x) [M(x) \ \& \ B(x,o) \ \& \ S(x,o) \ \& \ \mathbf{R}]]$$

This would suggest the following treatment of sentences of the form *always*(S_1, S_2):

$$(A) \quad \lambda \mathbf{R} \wedge (\forall \mathbf{R}) [\sim [S_1](\mathbf{R}) \rightarrow \sim ([S_1] \oplus [S_2])(\mathbf{R})]$$

Similarly, *sometimes* (S_1, S_2) would become:

$$(S) \quad \lambda \mathbf{R} \wedge (\exists \mathbf{R}) [[S_1](\mathbf{R}) \ \& \ \sim ([S_1] \oplus [S_2])(\mathbf{R})],$$

which does give us the correct truth-conditions. It thus seems that we may think of these quantificational adverbs as the type-lifted determiners *every* and *a* (that also happen to change their dynamic properties through this type-shift): *always* quantifies over infinitary relations just as *every* quantifies over individuals (viz. universally), etc. We would therefore expect the following rule for *usually*, reducing it directly to the determiner *most* (= M):

$$(U) \quad \lambda \mathbf{R} \wedge (M\mathbf{R}) (\sim [S_1](\mathbf{R}), \sim ([S_1] \oplus [S_2])(\mathbf{R}))$$

However, (U) does *not* give the correct truth-conditions, as can be verified by a simple counter-example: if only two out of five dogs bark, then the number of sets containing a barking dog (= 24) exceeds the number of non-empty dog-sets divided by 2 (= 15.5). The relation between a determiner and the corresponding quantifier over infinitary relations must therefore be more complicated. Now, although it is in principle possible to construe this relation as the result of a messy type-shift, there is a more straightforward way suggested by a careful analysis of equivalences like (7) and (7'): the latter essentially depends on the fact that, among the set quantified over in the paraphrase, there are all singletons satisfying the *when*-clause; in particular, the equivalence still holds if we restrict the quantifier to singletons. And the same restriction turns (U) into an acceptable analysis of *usually* while not changing the content of (S), as desired. None of this is very surprising: quantification over singletons of cases amounts to quantification over cases, so that this interpretation of quantificational adverbs turns out to be a variant the classical treatment of Lewis (1975). But it is a remarkable fact that the dynamic framework is flexible enough to integrate this analysis: the existential character of indefinites does not render them immune to the force of unselective binding. However, it is clear (and can be directly read off (A), (S), and (U)) that, once a quantificational adverb has been applied, the resulting formula is completely closed.

The incorrectness of (U) raises an interesting question about shifting determiners and quantifiers in general: what is it that makes the second-order quantification (a) and (S) collapse into their first-order counterparts? More precisely: for which (global) determiners D do we get

the equivalence of (D₁) and (D₂):

$$(D_1) \quad (D_{Ax}) [\varphi x, (\varphi x \ \& \ \psi x)]$$

$$(D_2) \quad (D_{\varphi A Q}) [(\exists x) (\varphi x \ \& \ Qx), (\exists x) (\varphi x \ \& \ \psi x \ \& \ Qx)]$$

We have seen that \forall and \exists happen to satisfy the equivalence, whereas M fails. But does this difference reflect some more basic (or even: known) distinction between quantifiers?

Do we really have to quantify over singletons rather than quantifying directly over cases? Chierchia suggests that the latter option is out, due to some intrinsic expressive weakness of dynamic logic. However, this is not so: quantification only necessitates abstraction; and abstraction (\wedge) we have got. Thus, e.g., (binary) universal quantification over cases can be expressed by:

$$(\square) \quad \lambda R \lambda S [\wedge (\sim R \rightarrow \sim S) = \wedge T],$$

where **T** is a tautology. However, in order to apply a determiner like (\square) to a pair of sentences, we would need free variables corresponding to their indefinite noun phrases. But if the latter are already existential quantifiers, we would have to add new variables and identify them with the variables bound by the existentials. Thus, the arguments of a quantifier like (\square) would have to have the form (9') rather than (9) – for otherwise quantification would be vacuous:

$$(9) \quad \lambda x_1 \lambda x_2 \dots \lambda x_n \dots (\exists x_1) (\exists x_2) \dots (\exists x_n) \varphi$$

$$(9') \quad \lambda y_1 \lambda y_2 \dots \lambda y_n \dots (\exists x_1) (\exists x_2) \dots (\exists x_n) [\varphi \ \& \ x_1=y_1 \ \& \ x_2=y_2 \ \& \ \dots \ x_n=y_n],$$

where we take the y_i 's to be new variables (i.e. distinct from the x_i 's). Now we have the problem of expressing (9') in a dynamic formula: since the only type-*s*-expression is the current case sequence, (9') can only be expressed in a roundabout way. Applying the method of Zimmermann (1989) gives us something like (9d) as the dynamic version of (9'):

$$(9d) \quad [\lambda F \wedge F(\lambda R \sim R)] (\lambda G (\exists x_1) (\exists x_2) \dots (\exists x_n) [\varphi \ \& \ (\lambda R \sim R) = G])$$

This way of achieving direct quantification over cases is certainly not very elegant; nor is it ontologically parsimonious: the order of quantification is even higher than in Chierchia's *st*-quantifiers.

References

- Lewis, D. : 'Adverbs of Quantification'. In: E. L. Keenan (ed.): **Formal Semantics of Natural Language**. Cambridge 1975, 3 - 15.
- Zimmermann, T. E.: 'Intensional logic and two-sorted type theory'. *Journal of Symbolic Logic* 54 (1989), 65 - 77.