# Composing intensions

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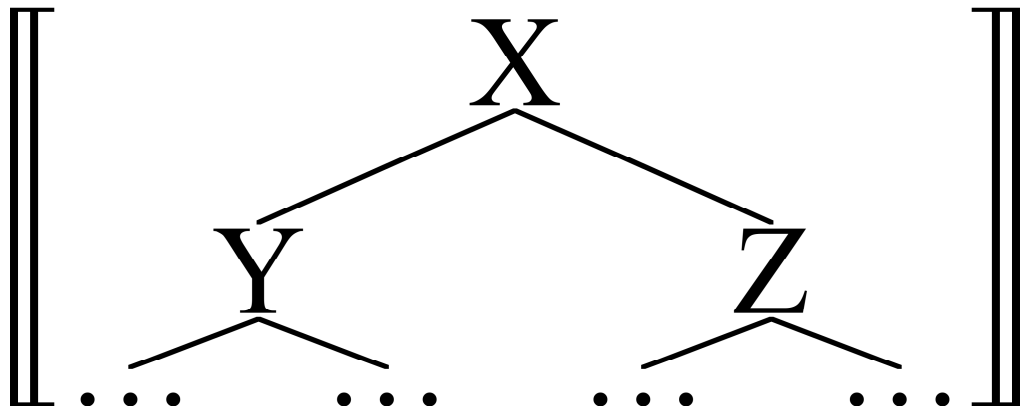
March 2012

# PLAN

0. Compositionality
1. Composing Extensions
2. Intensions
3. Intensional Contexts
4. Afterthoughts

# 0. Compositionality

*Semantic values of complex expressions are determined by combining semantic values of their immediate parts:*

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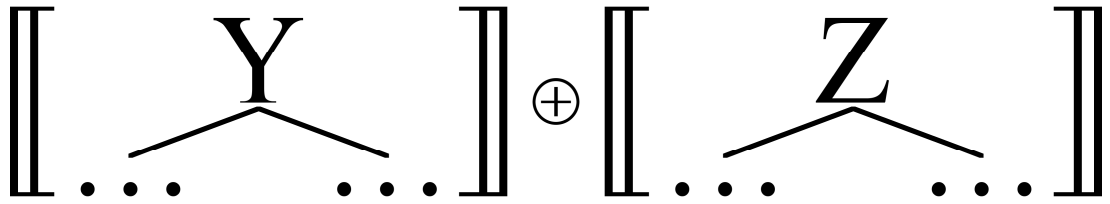
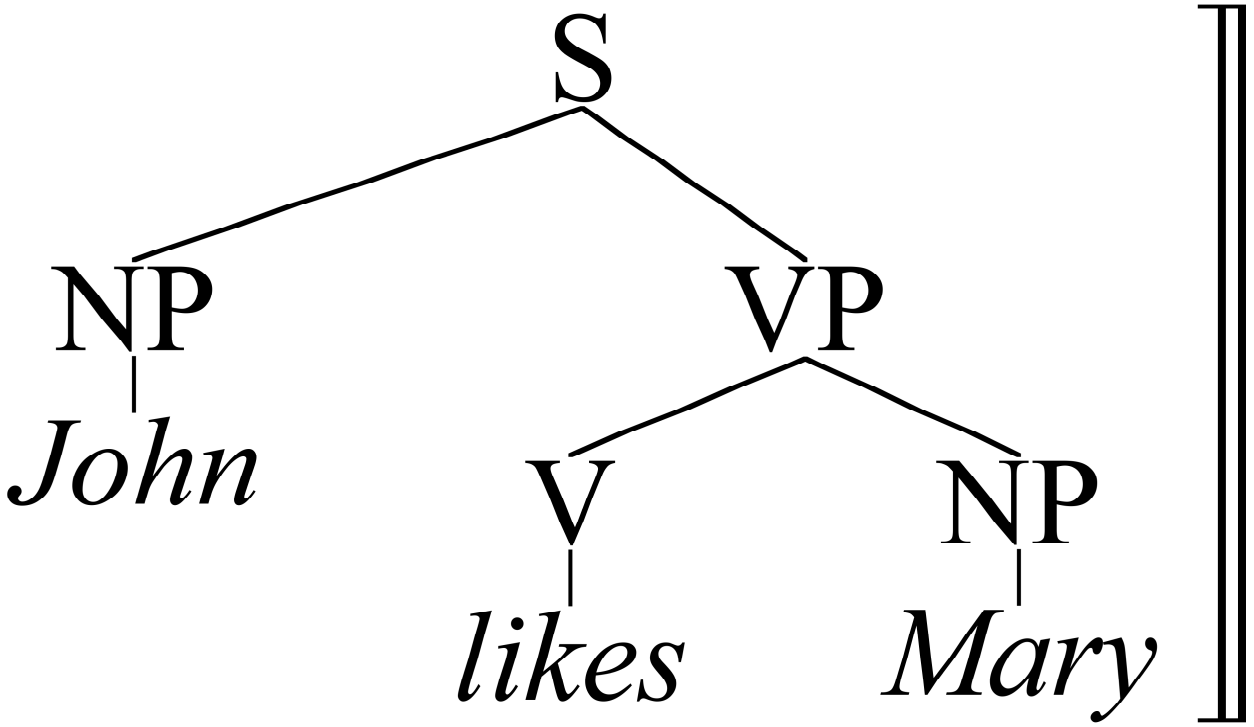
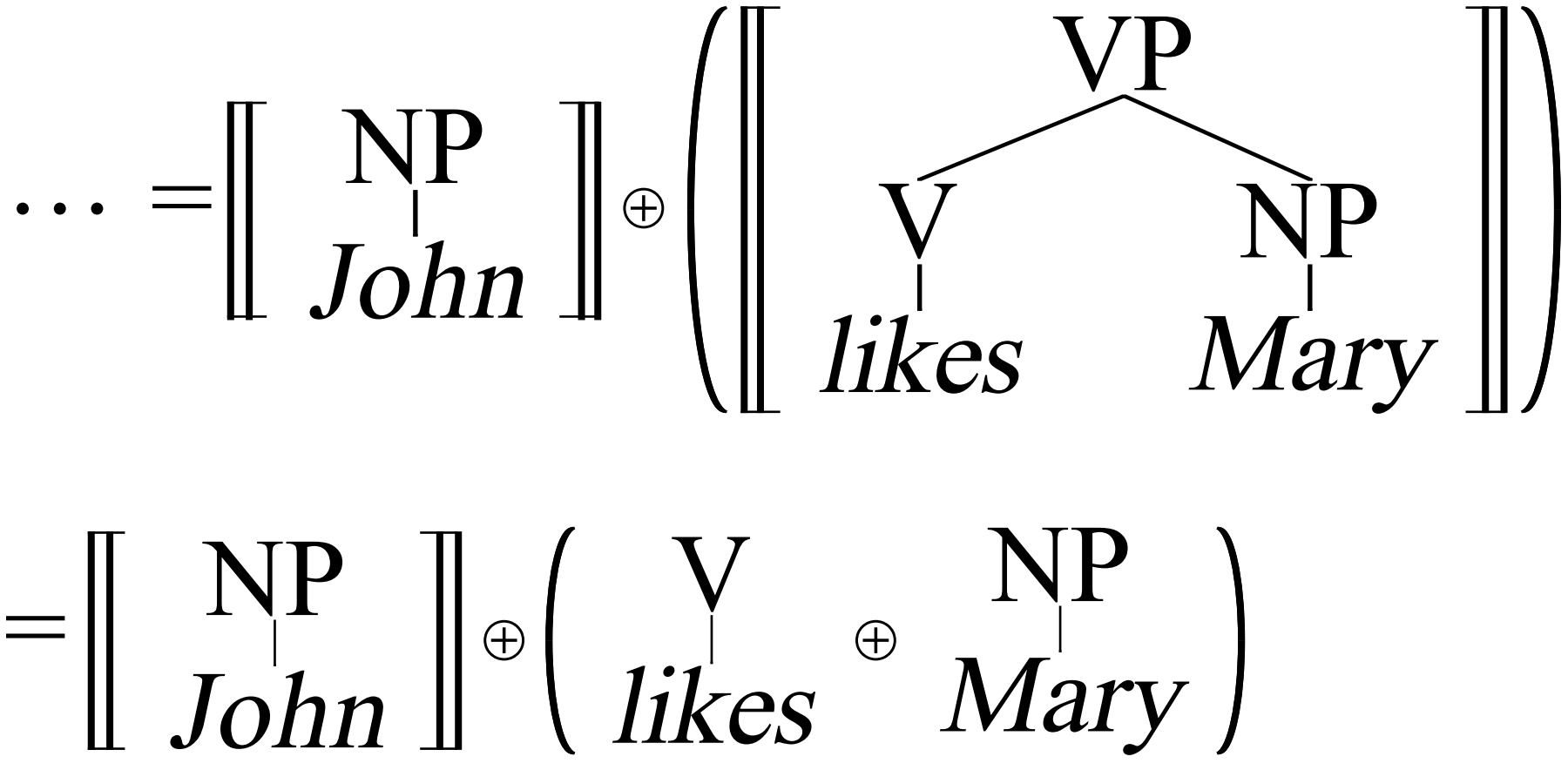


*Example*

$[\![$ 

$]\!]$

= …


… = ⟦ NP | John ⟧ ⊕ ( ⟦ VP / V | likes , NP | Mary ⟧ )
= ⟦ NP | John ⟧ ⊕ ( V | likes ⊕ NP | Mary )

$$\cdots = [\![\ \mathrm{NP}\ |\ \textit{John}\ ]\!] \oplus \left( [\![\ \begin{array}{c} \mathrm{VP} \\ \mathrm{V} \quad\quad \mathrm{NP} \\ |\quad\quad\ | \\ \textit{likes} \quad\quad \textit{Mary} \end{array}\ ]\!] \right)$$

$$= [\![\ \mathrm{NP}\ |\ \textit{John}\ ]\!] \oplus \left( \begin{array}{c} \mathrm{V} \\ | \\ \textit{likes} \end{array} \otimes \begin{array}{c} \mathrm{NP} \\ | \\ \textit{Mary} \end{array} \right)$$

(strictly speaking)

$$\cdots = [\![ \begin{array}{c} \mathrm{NP} \\ | \\ John \end{array} ]\!] \oplus \left( [\![ \begin{array}{ccc} & \mathrm{VP} & \\ \mathrm{V} & & \mathrm{NP} \\ | & & | \\ likes & & Mary \end{array} ]\!] \right)$$

$$= [\![ \begin{array}{c} \mathrm{NP} \\ | \\ John \end{array} ]\!] \oplus \left( \begin{array}{c} \mathrm{V} \\ | \\ likes \end{array} \oplus \begin{array}{c} \mathrm{NP} \\ | \\ Mary \end{array} \right)$$

$$= [\![ John ]\!] \oplus \left( [\![ likes ]\!] \oplus [\![ Mary ]\!] \right)$$

# 1. Composing Extensions

- *Extensions: rough characterization*

The extension of an expression is its contribution to extra-linguistic reference (relative to a given possible situation).

- *Extensions: examples* ... in (possible) situation *s**

$$[\![\textit{John}]\!]^{s^*} = \text{j};\ [\![\textit{Mary}]\!]^{s^*} = \text{m};\ldots$$

$$[\![\textit{sleeps}]\!]^{s^*} = \{(x) \mid \text{in } s^*,\ x \text{ sleeps}\} = \{(\text{a}),(\text{c}),\ldots\}$$

$$[\![\textit{likes}]\!]^{s^*} = \{(x,y) \mid \text{in } s^*,\ x \text{ likes } y\}$$

$$= \{(\text{a},\text{a}),\ (\text{b},\text{b}),\ldots,(\text{a},\text{m}),\ldots\}$$

$$[\![\textit{recommends}]\!]^{s^*}$$

$$= \{(x,y,z) \mid \text{in } s^*,\ x \text{ recommends } y \text{ to } z\}$$

$$= \{(\text{a},\text{b},\text{c}),\ldots\}$$

- *Extensional composition: direct objects*

$[\![ \textit{likes Mary} ]\!]^{s^*} = \{(x) \mid \text{in } s^*,\ x \text{ likes Mary}\}$

*E.g.*: $[\![ \textit{likes} ]\!]^{s^*} =$

{(a,a), (b,b), (c,b),(m,b),(h,c),…,(a,m),(h,m),
(m,m),(a,n),(d,n),…}

$\Rightarrow [\![ \textit{likes Mary} ]\!]^{s^*} =$

{(a),(h),(m)} =

~~{(a,a), (b,b), (c,b),(m,b),(h,c),…,~~(a,~~m~~),(h,~~m~~),
(m,~~m~~),~~(a,n),(d,n),…~~}

$$[\![ \textit{likes Mary} ]\!]^{s^*} = \{(x) \,|\text{in } s^*,\ x \text{ likes Mary}\}$$

$$= \{(x) \mid (x,[\![ \textit{Mary} ]\!]^{s^*}) \in [\![ \textit{likes} ]\!]^{s^*}\}$$

$$= [\![ \textit{likes} ]\!]^{s^*} \oplus [\![ \textit{Mary} ]\!]^{s^*}$$

$$\Rightarrow \boxed{\mathrm{L} \oplus \mathrm{u} = \{(x) \mid (x,\mathrm{u}) \in \mathrm{L}\}}$$

i.e.: $(x) \in \mathrm{L} \oplus \mathrm{u} \Leftrightarrow (x,\mathrm{u}) \in \mathrm{L}$

(where L is the extension of a transitive verb and u is an individual)

- *Extensional composition: indirect objects*

$$[\![ \textit{recommends Mary} ]\!]^{s^*}$$

$$= \{(x,y) \mid \text{in } s^*,\ x \text{ recommends } y \text{ to Mary}\}$$

$$\textit{E.g.}: [\![ \textit{recommends} ]\!]^{s^*} =$$

{(a,b,c),…,(a,c,m),(h,a,m),(f,b,m),(a,b,n),…}

$$\Rightarrow [\![ \textit{recommends Mary} ]\!]^{s^*} =$$

{(a,c),(h,a),(f,b)} =

{~~(a,b,c),…,~~(a,c,~~m~~),(h,a,~~m~~),(f,b,~~m~~),(a,b,n),…}

$$[\![\, \textit{recommends Mary} \,]\!]^{s^*} =$$

$$\{(x,y) \mid \text{in } s^*,\ x \text{ recommends } y \text{ to Mary}\}$$

$$= \{(x,y) \mid (x,y,[\![\, \textit{Mary} \,]\!]^{s^*}) \in [\![\, \textit{recommends} \,]\!]^{s^*}\}$$

$$= [\![\, \textit{recommends} \,]\!]^{s^*} \oplus [\![\, \textit{Mary} \,]\!]^{s^*}$$

$$\Rightarrow \boxed{\mathrm{R} \oplus \mathrm{u} = \{(x,y) \mid (x,y,\mathrm{u}) \in \mathrm{R}\}}$$

i.e.: $(x,y) \in \mathrm{R} \oplus \mathrm{u} \Leftrightarrow (x,y,\mathrm{u}) \in \mathrm{R}$

(where R is the extension of a ditransitive verb and u is an individual)

- *Parallelism between saturation and -arity*

- from 3 places to 2:

$$\mathrm{R} \oplus \mathrm{u} = \{(x,y) \mid (x,y,\mathrm{u}) \in \mathrm{R}\}$$

- from 2 places to 1:

$$\mathrm{L} \oplus \mathrm{u} = \{(x) \mid (x,\mathrm{u}) \in \mathrm{L}\}$$

- from 1 places to 0?

$$\mathrm{P} \oplus \mathrm{u} = \{(\,) \mid (\mathrm{u}) \in \mathrm{L}\}$$

- *Extensional composition: subjects*

If $[\![ \textit{likes Mary} ]\!]^{s^*} = \{(a),(j),(m)\}$:

then: $[\![ \textit{John likes Mary} ]\!]^{s^*} =$

$\{\cancel{(a)}, (j), \cancel{(m)}\} = \{()\}$ !

If $[\![ \textit{likes Mary} ]\!]^{s^*} = \{(a),(r)\}$:

then: $[\![ \textit{John likes Mary} ]\!]^{s^*} =$

$\{\cancel{(a)}, \cancel{(r)}\} = \{\} = \emptyset$ !

- *Truth values*

$$[\![ NP\ VP ]\!]^{s^*} = \begin{cases} \{()\} \text{ if } ([\![ NP ]\!]^{s^*}) \in [\![ VP ]\!]^{s^*} \\ \emptyset \quad \text{if (u)} \notin \text{P} \end{cases}$$

**1** := {()} TRUE

**0** := Ø FALSE

## 2. Intensions

- *Intensions: rough characterization*

The intension of an expression is its contribution to informational content.

- The intension of a sentence may depend on the contex of utterance; this aspect will be suppressed in what follows.

- *Propositions and Logical Space*

| (Declarative) Sentence | Truth value in situation | | | | |
|---|---|---|---|---|---|
| | $s_0$ | $s_1$ | $s_2$ | $s_2$ | … |
| *It is raining.* | 1 | 1 | 1 | 0 | … |
| *It is not raining.* | 0 | 0 | 0 | 1 | … |
| *It is raining heavily.* | 1 | 0 | 0 | 0 | … |

- The truth value profile of a sentence mirrors its information value: the possible situations it rules out, make up the information it conveys.

- Information value measured by truth value profiles is qualitative, not (just) quantitative.

- Information value depends on (epistemic) background: the elimination of a particular situation is only valuable if it has not been ruled out before.

Definition

The *proposition* expressed by a sentence is its maximal truth value profile, i.e. the truth value profile relative to Logical Space, the set of all situations possible.

- The information value of a particular sentence on a particular background can be obtained from the proposition it expresses by relativizing it to the situations compatible with the background.

- *Extension profiles*

| Referential term | Referent in $s_0$, $s_1$, … | | | | |
|---|---|---|---|---|---|
| *the mayor of Paris* | **a** | **b** | **c** | **a** | … |
| *the oldest person alive* | **c** | **d** | **e** | **a** | … |
| *Mary* | **m** | **m** | **m** | **m** | … |

| Predicate | Extension in $s_0$, $s_1$, … | | | | |
|---|---|---|---|---|---|
| *is sleeping* | {**a**,**b**,**c** …} | Ø | {**a**, **c** …} | {**f**,**g**, …} | … |
| *is snoring* | {**a**} | Ø | Ø | {**f**,**g**, …} | … |

Definition

The *intension* of an expression $A$

$$[\![ \hat{A} ]\!]$$

is its maximal extension profile, i.e. the extension profile relative to **Logical Space**, the set of all situations possible.

- The intension of a sentence is the proposition it expresses.
- $[\![ A ]\!]^{s} = [\![ \hat{A} ]\!](s)$

- *Pointwise composition*

For any $s$:

$$[\![\textit{John lik}\hat{\textit{es}}\ \textit{Mary}]\!](s) = [\![\textit{John likes Mary}]\!]^{s} =$$

$$[\![\textit{John}]\!]^{s} \oplus \left([\![\textit{likes}]\!]^{s} \oplus [\![\textit{Mary}]\!]^{s}\right) =$$

$$[\![\textit{Jo}\hat{\textit{hn}}]\!](s) \oplus \left([\![\textit{lik}\hat{\textit{es}}]\!](s) \oplus [\![\textit{M}\hat{\textit{ary}}]\!](s)\right)$$

where $\oplus$ is as above

- The intension of *John likes Mary* is that function that assigns the above value to any $s$ .

*Generalizing*

$$[\![ NP\hat{}VP ]\!] = \lambda s.\ [\![ \hat{NP} ]\!](s) \oplus [\![ \hat{VP} ]\!](s)$$

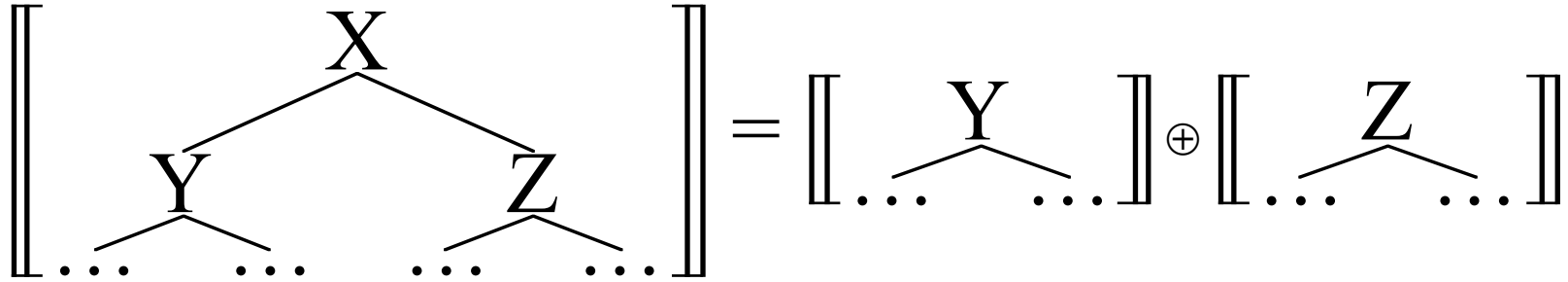
where '$\lambda x.$ …$x$…' means: the function that assigns …$x$… to any $x$

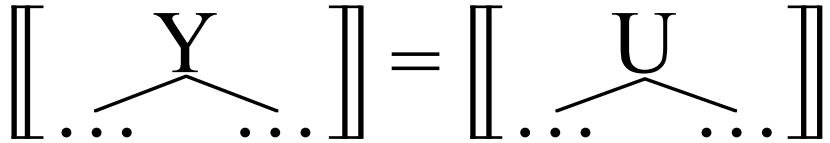
etc.

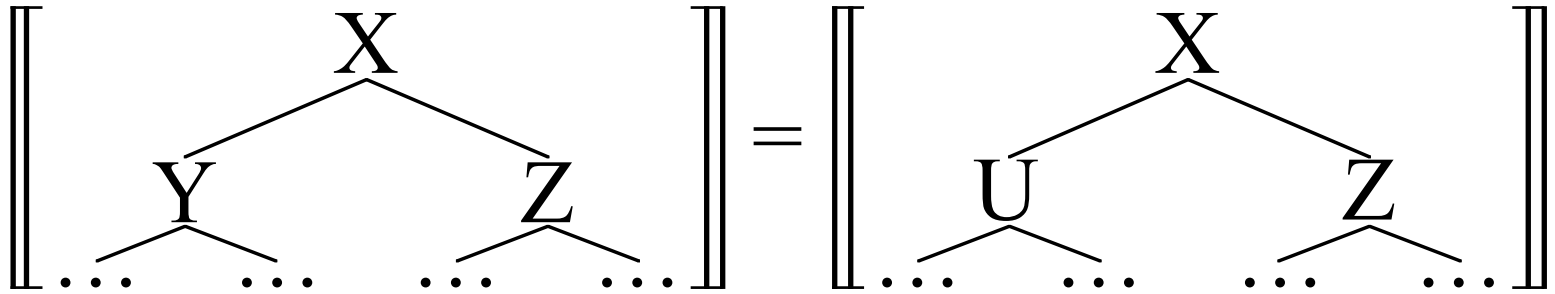
- Pointwise composition of extensions guarantees intensional compositionality.

# 3. Intensional Contexts

- *Compositionality and Substitutivity*

If: 


and: 


then: 

*More generally* (given compositionality)

If *A* derives from *B* by substituting a (not necessarily immediate) constiuent *C* by some expression *D* such that

$$[\![ C ]\!] = [\![ D ]\!]$$

then:

$$[\![ A ]\!] = [\![ B ]\!]$$

*(A) Every person who came to the party brought a bottle*
*(B) Every member of the department brought a bottle*
*(C) member of the department*
*(D) person who came to the party*

• *Substitution failure*

*(A) John knows that Mary is a postdoc*

*(B) John knows that Hannover is the capitol of Lower Saxony*

*(C) Hannover is the capitol of Lower Saxony*

*(D) Mary is a postdoc*

$$[\![\, \textit{Mary is a postdoc} \,]\!]^{s^*} =$$

$$[\![\, \textit{H. is the capital of L. S.} \,]\!]^{s^*} = 1$$

$$[\![\, \textit{John knows that H. is the capital of L.S.} \,]\!]^{s^*} = 0 \neq$$

$$[\![\, \textit{John knows that Mary is a postdoc} \,]\!]^{s^*} = 1$$

- *Frege's solution*

$$[\![ \textit{John knows that Mary is a postdoc} ]\!]^{s^*} =$$

$$[\![ \textit{John} ]\!]^{s^*} \oplus [\![ \textit{knows that Mary is a postdoc} ]\!]^{s^*} =$$

$$[\![ \textit{John} ]\!]^{s^*} \oplus \left( [\![ \textit{knows} ]\!]^{s^*} \oplus \boxed{[\![ \textit{Mary is } \hat{a} \textit{ postdoc} ]\!]} \right) =$$

$$[\![ \textit{John} ]\!]^{s^*} \oplus \left( [\![ \textit{knows} ]\!]^{s^*} \oplus \lambda s.\, [\![ \textit{Mary is a postdoc} ]\!]^{s} \right) =$$

$$[\![ \textit{John} ]\!]^{s^*} \oplus \left( [\![ \textit{knows} ]\!]^{s^*} \oplus \lambda s.\, \left( [\![ \textit{Mary} ]\!]^{s} \oplus [\![ \textit{is a postdoc} ]\!]^{s} \right) \right) =$$

$$[\![ \textit{John} ]\!]^{s^*} \oplus \left( [\![ \textit{knows} ]\!]^{s^*} \oplus \lambda s.\, \left( [\![ \textit{Mary} ]\!]^{s} \oplus [\![ \textit{postdoc} ]\!]^{s} \right) \right) =$$

…

- *Fregean Compositionality*

Extensions of compound expressions are determined by the extensions or intensions of their immediate parts and the mode of composition.

More precisely: (thanks to Peter Pagin)

The extension of a compound expressions is determined by the extensions of their immediate parts (and the relevant mode of composition) if the latter satisfy extensional substitution; otherwise its extension is determined by the intension(s) of the immediate constituent(s) defying extensional substitution and the extension(s) of the other constituents (plus the relevant mode of composition).

- *Fregean Compositionality*

Extensions of compound expressions are determined by the extensions or intensions of their immediate parts and the mode of composition.

*… implies…*

- *Intensional Compositionality*

Intensions of compound expressions are determined by the intensions of their immediate parts and the mode of composition.

*E.g.:*

$$[\![\textit{knows that } \widehat{\textit{Mary}} \textit{ is a postdoc}]\!] =$$

$$\lambda s.\ [\![\textit{knows that Mary is a postdoc}]\!]^{s} =$$

$$\lambda s.\ [\![\textit{knows}]\!]^{s} \oplus [\![\textit{Mary is } \widehat{\textit{a}} \textit{ postdoc}]\!] =$$

$$\lambda s.\ [\![\widehat{\textit{knows}}]\!](s) \oplus [\![\textit{Mary is } \widehat{\textit{a}} \textit{ postdoc}]\!]$$

- *Intensional Compositionality*

Intensions of compound expressions are determined by the intensions of their immediate parts and the mode of composition.

*… does <u>not</u> imply…*

- *Fregean Compositionality*

Extensions of compound expressions are determined by the extensions or intensions of their immediate parts and the mode of composition.

*(Artificial) Counter-examples*

If, for any situation *s*,

$$[\![ \tilde{S} ]\!]^{s} = 1 \Leftrightarrow [\![ S ]\!]^{s} = [\![ S ]\!]^{s^*}$$

(where $\tilde{S}$ is the result of some morpho-syntactic process operating on *S*)

or:

$$[\![ X\ Y ]\!]^{s} = 1 \Leftrightarrow [\![ X ]\!]^{s} \oplus [\![ Y ]\!]^{s} = 1\ \&\ s = s^*$$

(where $\oplus$ is some extensional operation resulting in truth values)

then Fregean Compositionality fails,
but Intensional Compositionality may still hold.

# 4. Afterthoughts

- Compositionality, characters and monsters

- Frege on ‘intensional’ compositionality