Endogenizing the Scope of the Stigma of Failure

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Abstract

We analyze the conditions under which credit markets are efficient in providing loans for both new and restarting entrepreneurs. An entrepreneur needs credit for a project which can be run with either high or low risk. Its probability of success depends both on chosen risk and on her entrepreneurial skills which are unknown to everybody. If the probability of high skills is sufficiently large, two types of equilibria may coexist: a \textit{conservative} equilibrium with one-off project financing, and an \textit{experimental} equilibrium with fresh project financing even after a (limited and endogenously determined) number of failures. If previous risk choices are observable, only the welfare-maximizing experimental equilibrium is a sequential equilibrium. However, if they are unobservable, inefficient sequential equilibria arise in which the entrepreneur chooses the low-risk project too early. These results have novel implications for policy making and capital market design.

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1 Introduction

Imperfect traceability of the reasons for business failures attaches a “stigma of failure” to bankrupt entrepreneurs. When trying a fresh start, they are often left discriminated by business partners, employees, and in particular investors. Despite extensive research, it still remains unclear why the extent of this discrimination varies across countries, sectors and over time. European and Japanese financiers, for instance, are perceived to be more reluctant to finance a failed entrepreneur’s restart than their American counterparts. It therefore became commonplace to praise the US’ lower “stigma of failure” as the source of its higher entrepreneurship rates\(^1\) and consequently of its competitive edge in terms of the ability to innovate, commercialize and grow.\(^2\)

In this paper, we study to what extent different scopes of the “stigma of failure” (captured by the maximal number of times a failed entrepreneur is able to get fresh start financing) can simultaneously be equilibrium outcomes. Our main result is that as soon as the riskiness of failed projects cannot be evaluated by investors, two types of equilibria may coexist: a conservative equilibrium, where a once-failed entrepreneur is excluded from further finance, or experimental equilibria, where she can start projects even after a (limited and endogenously determined) number of failures.

In our model, a wealthless entrepreneur seeks funding from a competitive banking sector\(^3\) in order to launch a project. This project can be run with high or low risk of failure. Its probability of success does not only depend on this risk, but also on the entrepreneur’s inherent skills\(^4\), which can be high or low. Unlike in Stiglitz and Weiss (1981), neither the entrepreneur nor banks know her skills. Only the distribution of skills is publicly known. If the project is successful, the entrepreneur continues her business, payoffs are realized and the game is over. If the project fails, the bank that financed the project loses its investment and the entrepreneur asks for further entrepreneurial finance in order to start a new project in the next period. The structure of the game is the same in each period. However, after each failure, banks update their belief about the probability whether the entrepreneur has high skills or not. This belief is not only dependent on the initial distribution of skills, but also on the level of risk, which has been chosen in the preceding periods: if this risk has been high (low), the belief about the probability that the entrepreneur has high skills is also relatively high (low).

\(^1\)GEM (2008) reports that in 2007, 10.8% of adults were engaged in early-stage entrepreneurship in the US as compared to only 5.4% in the EU or 5.4% in Japan.
\(^3\)The results can also be applied to alternative forms of entrepreneurial finance.
\(^4\)For example, entrepreneurial skills can represent whether the ideas of an entrepreneur have a high or low probability of success.
Different scopes of the “stigma of failure” can occur in equilibrium only if the entrepreneur can trade off the expected return of a project against its maximal return: therefore we assume that a low-risk project has a higher expected return, while the return from the high-risk project in case of success exceeds the return from the low-risk project. We show that if the risk of failure of the high-risk project is not too high and the probability of having high skills is sufficiently close to unity, then the first-best outcome is as follows: the entrepreneur realizes high-risk projects in the first periods and then (if all these projects were unsuccessful) switches to the low-risk project. Finally, if she also fails with the low-risk project, she stops realizing projects as it becomes relatively certain that she has low skills.

We will analyze three informational settings: (I.) Under perfect information, banks can observe both the entrepreneur’s past and present risk choices, i.e. there is no moral hazard. We show that any sequential equilibrium is efficient in this setting. (II.) Under private information of banks, these can only assess the riskiness of projects financed by themselves. Conservative and experimental equilibria can then simultaneously exist and be sequential equilibria. This is due to the fact that not all banks can observe the entrepreneur’s decisions. A bank may then become a monopolistic supplier of finance to the entrepreneur if all of its competitors believe that the entrepreneur’s skills are low. The credit market outcome might be inefficient, as the entrepreneur chooses the low-risk project too early. (III.) Finally, the same result obtains under moral hazard, where banks can neither observe the riskiness of past, nor present projects: if banks believe that the entrepreneur chooses high (low) risk, they charge a high (low) loan rate, which makes the entrepreneur choose the high (low) risk. There is, however, one exception: there may also arise a situation, in which a conservative equilibrium is more efficient than any experimental equilibrium.

We provide a novel explanation on why economies with identical cultural and institutional constraints can suffer from different scopes of the “stigma of failure”. Our results lead to a number of policy implications. A banks’ ability to observe both past and present risk choices of entrepreneurs proves crucial in preventing credit market inefficiencies. This supports the view that an efficient system of entrepreneurial finance may be based on small banks or venture capital firms who know their clients’ business well. We argue that most of the EU’s envisaged policies to reduce the “stigma of failure” might not be effective, since the expectations and actions of many market participants must be changed simultaneously. Likewise, potential gains from an increase in entrepreneurial skills in the population might not fully be realized unless the risk of both past and present projects can be evaluated by investors.
Related Literature Varying levels of the “stigma of failure” have typically been attributed to either persistent cultural or institutional differences between countries. There is nevertheless still widespread dispute about which and how cultural traits might shape attitudes towards entrepreneurial failure. Burchell and Hughes (2006) obtain that GDP growth is not related to failure tolerance, but positively to society’s positivity towards second chancing. Yet, as respondents in the US show higher levels of failure tolerance but less willingness to grant a second chance to failed entrepreneurs than Europeans, more entrepreneurial activity in the US cannot be attributed to a more favorable cultural perception of second chancing. Institutional constraints show limited impact on agents’ decision to start new firms. This suggests the experience of the EU-15, where entrepreneurial activity remained quite stable - even after firm setup costs had declined by a third between 2002 and 2007 (see EurActiv 2007).

We focus instead on capital market constraints, which we endogenize. The closest paper to ours is Landier (2006). In his model, high-skill entrepreneurs liquidate mediocre projects in the experimental equilibrium despite their positive net present values. In the conservative equilibrium, entrepreneurs maintain mediocre projects and therefore only low-skill entrepreneurs start a second-time business, which then increases the loan rate. Landier thus rather scrutinizes the liquidation decision and not, like our paper, second chancing after bankruptcy. Groom and Scharfstein (2002) study an organizational choice model with labor market rigidities as barriers to entrepreneurship. When managerial incentives depend on the career prospects, agents might prefer dependent to self-employment. Due to asymmetric information, financing capital is then shifted to lower quality and younger firms. Our key driver, in contrast, is the interplay between the entrepreneur’s skills and risk choices. Finally, the setting with private information of banks builds on Petersen and Rajan (1995), Sharpe (1990) and von Thadden (2004). They show that long-term bank-firm relationships enable banks to gather valuable costly information on their customers. That overrides bank competition for older customers, so that banks can capture some of their rents.

The paper is organized as follows. Section 2 introduces the model and derives the first-best outcome. In Sections 3 to 5 we analyze credit market equilibria for different informational settings: perfect information, private information of banks and imperfect information. After having studied welfare and policy implications in Section 5, Section 6 concludes. All proofs are in the Appendix.

See e.g. Licht and Siegel (2006), Hayton et al. (2002) or Giannetti and Simonov (2004).
2 The model

We consider an economy populated by an entrepreneur $E$ and $N > 1$ banks $B_k$, $k \in \{1, \ldots, N\}$. Time is discrete and denoted by $t \in \{1, 2, \ldots\}$. All players are risk-neutral.

2.1 The Entrepreneur

The entrepreneur is endowed with entrepreneurial skills $\theta_i$ which are either high ($i = H$) or low ($i = L$), but with no wealth on her own. The level of skills $i$ is time-invariant and unobservable to her and banks. However, both $E$ and banks know that the ex-ante probability of high skills is equal to $\alpha_1 \in (0, 1)$. In period 1, $E$ has access to a project of size 1, which she can realize or not. If $E$ does not realize the project or she does not get a loan, the game is over and the payoff is 0 for all players. Otherwise, $E$ chooses a risk of failure $p_j$ of either high ($j = H$) or low ($j = L$) value. The project’s return structure $y_{ij}$ is determined by $E$’s level of skills $i$ and choice of risk $j$:

$$ y_{ij} = \begin{cases} y_j, & \text{with probability } (1 - p_j)\theta_i \\ 0, & \text{with probability } 1 - (1 - p_j)\theta_i \end{cases} $$

Thereby, $y_j$ is the risk-dependent project return in case of success and $(1 - p_j)\theta_i$ the probability of success when $E$’s skill is $i \in \{L, H\}$ and her risk choice is $j \in \{L, H\}$. In order to simplify matters, we set $p_L = 0$ and $\theta_H = 1$. If $E$’s project with risk of failure $j$ is successful, she exits the game and her payoff is equal to $y_j$ minus the loan rate for this project. As $E$ has no own wealth, this payoff cannot be negative: if the loan rate is higher than the project return, her payoff is equal to 0 and the bank gets the project return. If $E$’s project is not successful, she does not pay anything to the bank that granted the loan and moves on to the next period. The structure of the game is the same in each period. Thus, $E$ asks for finance in period $t$ only if she realized $t - 1$ times a project that failed. As tie-breaking rule we assume that $E$ chooses $j = L$ whenever she is indifferent between the high- and the low-risk project.

In period 1, $E$ has a belief $\hat{\alpha}_1^E = \alpha_1$ about the probability that she has high skills. She updates this belief according to Bayes’ rule. If she chooses the risk of failure $j$ in period $t$ and the project fails, then her belief is given by

$$ \hat{\alpha}_{t+1}^E(\hat{\alpha}_t^E, j) = \frac{\hat{\alpha}_t^E p_j}{1 - (1 - p_j)(\hat{\alpha}_t^E + \theta_L - \hat{\alpha}_t^E \theta_L)}, $$

6The results carry over easily to a continuum of entrepreneurs.

7This assumption embodies that $E$ continues her successful business (with $y_j$, $j \in \{L, H\}$, being the net present value of certain future payoff streams), and therefore does not need to ask for entrepreneurial finance another time.
Her expected level of skills in a period $t$ is given by
\[ \tilde{\theta}_t^E = \alpha_t^E + (1 - \tilde{\alpha}_t^E)\theta_L. \] \hspace{1cm} (2)

2.2 Banks

Banks compete in a Bertrand manner by offering loan contracts to $E$. A contract only specifies the loan rate $E$ has to pay in case of success. They also may decide not to offer any loans, however, we will assume that banks offer contracts as long as they can make zero-profits in expectation. We will consider three informational settings:

(I.) Perfect information ($PI$): banks can observe the riskiness of both $E$’s past and present projects.

(II.) Private information of banks ($PRB$): each bank can only observe the riskiness of past and present projects it financed itself.

(III.) Imperfect information ($IM$): banks cannot observe any project’s riskiness.

Each bank $B_k$ has a belief $\tilde{\alpha}_t^{B_k}$ about the probability that $E$ has high skills if she asks for project financing in period $t$. The way this belief is formed depends on the informational setting: under ($PI$), banks observe all of the $E$’s past decisions, therefore they can update their belief using Bayes’ rule like $E$ does in (1). Under ($PRB$), a bank $k$ can update its belief from $\tilde{\alpha}_t^{B_k}$ to $\tilde{\alpha}_{t+1}^{B_k}$ according to Bayes’ rule only if it financed the project in period $t$. Otherwise, $\tilde{\alpha}_{t+1}^{B_k}$ is given exogenously. Under ($IM$), the belief of each bank in each period is given exogenously. Denote the expected level of skills in a period $t$ for $B_k$ by
\[ \tilde{\theta}_t^{B_k} = \tilde{\alpha}_t^{B_k} + (1 - \tilde{\alpha}_t^{B_k})\theta_L. \]

Under ($PI$) and ($PRB$), $B_k$ can condition its loan rate $r_t^k$ both on its belief $\tilde{\alpha}_t^{B_k}$ and on the risk of failure of the present project, i.e. there is no moral hazard. Thus, it offers two contracts with loan rates $r_t^k(\tilde{\alpha}_t^{B_k}, H)$ and $r_t^k(\tilde{\alpha}_t^{B_k}, L)$. Under ($IM$), a bank cannot condition on the risk of failure of the project, therefore it offers only one loan rate $r_t^k(\tilde{\alpha}_t^{B_k})$. We assume that banks cannot commit to certain loan rates in future periods.

2.3 Timing and Equilibrium

Altogether, if $E$ is in the game at the beginning of period $t$, the sequence of events is as follows:

1. Each bank decides whether to offer loan contracts or not. If yes, it chooses the loan rate(s). If no bank offers loan contracts, the game is over and payoffs are 0 for all players.
2. $E$ decides whether to undertake a project or not. If yes, she chooses the risk of failure $j \in \{L, H\}$ and the contract with the lowest loan rate for this risk (if more than one bank offers the lowest loan rate, $E$ chooses each of those offers with equal probability). If not, the game is over and payoffs are 0 for all players.

3. The project is successful or not. In case of success, $E$ receives the payoff from the project, pays the loan rate to the bank and the game is over. Otherwise, she defaults and enters the next period. The bank that financed the project incurs a loss of 1.

Our main focus lies on the sequential equilibria of the game under the different informational settings. In our model, any sequential equilibrium exhibits the following features: Firstly, beliefs are derived from Bayes’ rule whenever $E$’s actions are observable. Secondly, banks correctly anticipate $E$’s actions whenever these are unobservable. We therefore have

$$\tilde{\alpha}_k^B = \tilde{\alpha}_E$$

for $k \in \{1, ..., N\}$ in each period $t$ of an equilibrium. Finally, $E$’s action in period $t$ maximizes her expected payoff for given belief and the banks’ decisions in subsequent periods. A bank’s decisions in period $t$ maximize its expected payoff for given belief and other banks’ decisions in period $t$. To illustrate important results, we will also refer sometimes to Nash equilibria of the game (in which beliefs do not play a role).

2.4 Projects and the First-Best Outcome

The high-risk project has a higher return than the low-risk project, i.e. $y_H > y_L > 1$. Moreover, it holds that high skills and low risk increase the probability of success, i.e. $\theta_H > \theta_L$ and $p_H > p_L$. The projects taken by the high- (low-) skill entrepreneur always have a positive (negative) net present value (NPV):

$$(1 - p_j)y_j > 1 \text{ and } (1 - p_j)\theta_Ly_j < 1 \text{ for } j \in \{L, H\}. \quad (3)$$

$E$ can trade off the expected return of a project against the maximal return, i.e. the expected return from the low-risk project is higher than from the high-risk project:

**Assumption (A1):** We have $y_L > (1 - p_H)y_H$. 

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8To keep matters simple we suppress some notation here, which would be needed to define the sequential equilibrium formally. The first two points follow from the concept of “consistency” of strategies and beliefs, the last point follows from “sequential rationality”. To proof “consistency” one usually has to construct a sequence of mixed strategies and beliefs (derived for a given strategy according to Bayes’ rule) which converges against the equilibrium strategy profile and equilibrium beliefs. For details we refer to Fudenberg and Tirole (1991), pages 337 - 338. As equilibria in our model have a very simple structure, we will do without this construction.

9Our results would be similar in a model in which $E$ can trade-off those two variables continuously.
Consider an entrepreneur with “deep pockets” who knows that she has high skills and who can finance projects by herself. Given that this entrepreneur has only one chance to realize a project, she would choose the low-risk project if (A1) holds. If (A1) does not hold, she would go for the high-risk project. Assume now that this entrepreneur can start a new project in infinitely many periods like in our model, i.e. if she succeeds, payoffs are realized and the game is over, otherwise she can start another project. Her expected payoff from always choosing the high-risk project, $V^H_1$, is then given by

$$V^H_1 = (1 - p_H) (y_H - 1) + p_H (-1 + V^H_1).$$

Solving for $V^H_1$ yields us

$$V^H_1 = y_H - \frac{1}{1 - p_H}.$$ 

Her expected payoff from choosing the low-risk project, $V^L_1$, is given by

$$V^L_1 = y_L - 1.$$ 

We will assume that $V^H_1 > V^L_1$:

**Assumption (A2):** We have $y_H - \frac{1}{1 - p_H} > y_L - 1$.

If (A2) holds, then the entrepreneur with deep pockets and high skills would choose $j = H$ in each period. If (A2) does not hold, she would choose the low-risk project. The assumptions in (A1) and (A2) can be fulfilled at the same time if and only if $y_L > 1$. This is ensured by the fact that both projects have a positive NPV as long as the high-skill entrepreneur runs them.

We now derive the first-best outcome if (A1) and (A2) hold and the entrepreneur is uncertain about her skills. Again assume that $E$ has deep pockets and can finance all projects by herself. Note that in period 1, her expected payoff from realizing the low-risk project is positive if and only if

$$\alpha_1 > \frac{1}{1 - \theta_L} \left( \frac{1}{y_L} - \theta_L \right).$$

If (4) does not hold, then the entrepreneur does not start any projects. As $\alpha_1 < 1$, she will not finance high-risk projects in infinitely many periods, as her belief $\hat{\alpha}^E_t \to 0$ for $t \to \infty$, according to (1). If she anticipates in period $\bar{t}$ that her belief $\hat{\alpha}^E_{\bar{t}+1}$ will be below the right-hand side of (4) in case of failure, then she chooses $j = L$ in this period (and stops realizing projects if this project fails). Define

$$I(\theta_L, y_L) \equiv \left( \frac{1}{1 - \theta_L} \left( \frac{1}{y_L} - \theta_L \right), 1 \right).$$

Note that this interval is always non-empty. We then get the first-best outcome:
Proposition 1 (First-Best Outcome) Assume that (A1) and (A2) hold. Then for each $\alpha_1 \in I(\theta_L, y_L)$ there is a number $\tilde{\alpha}_1 \in \mathbb{N}$, such that the entrepreneur with deep pockets chooses $j = H$ in the periods $t \in \{1, ..., \tilde{\alpha}_1 - 1\}$ and $j = L$ in period $t = \tilde{\alpha}_1$. For $\tilde{\alpha} \in \mathbb{N}$ there is a $\tilde{\alpha}_1 < 1$, such that $\tilde{\alpha}_1 > \tilde{\alpha}$ whenever $\alpha > \tilde{\alpha}_1$.

3 Equilibria under perfect information (PI)

In our first informational setting, banks can evaluate the riskiness of past and present projects. As the NPV of projects run by a low-skill entrepreneur is negative, projects will only be financed in finitely many periods. Facing Bertrand competition, banks only offer loan rates, which generate zero profits in equilibrium. Hence, the expected repayment equals the investment sum. If $B_k$ then sells a loan contract to $E$ to finance a project of risk $j \in \{L, H\}$ in period $t$, the loan rate must be

$$ r^k_t(\tilde{\alpha}^B_k, j) = \frac{1}{(1 - p_j)\tilde{\theta}^E_t}, $$

where $\tilde{\alpha}^B_k = \tilde{\alpha}^E_t$. As $\theta_L < 1$, the loan rate decreases in $\tilde{\alpha}^B_k$.

Let banks’ loan rates and $E$’s decisions be given for all $t$. Denote by $V_t$ be the expected payoff of $E$ in the beginning of period $t$. If banks do not offer loans anymore (or if $E$ does not realize a project) in period $t$, $V_t = 0$. If $E$ gets a loan from $B_k$ in period $t$, her expected payoff from realizing a project with risk $j$ is

$$ V_t = (1 - p_j)\tilde{\theta}^E_t (y_j - r^k_t(\tilde{\alpha}^B_k, j)) + (1 - (1 - p_j)\tilde{\theta}^E_t) V_{t+1}. $$

This allows us to calculate recursively $E$’s expected payoff $V_1$.

3.1 The conservative equilibrium

We first show that there is a Nash equilibrium in which $E$ gets finance only if she never went bankrupt, i.e. in period 1 and never thereafter. If banks do not offer loans in periods $t \in \{2, 3, ...\}$, we have $V_2 = 0$. Equations (5) and (6) imply that $E$ picks $j = L$ in period 1 if

$$ \tilde{\theta}^E_1 y_L \geq (1 - p_H)\tilde{\theta}^E_1 y_H. $$

This expression is equivalent to (A1). A bank $B_k$ provides funding for a low-risk project as long as

$$ \tilde{\theta}^B_k y_L - 1 \geq 0, $$

which is satisfied if $\alpha_1 \in I(\theta_L, y_L)$. It remains to show that banks do not provide loans in $t \in \{2, 3, ...\}$. Note that failure of a low-risk project reveals low skills. As all banks can observe $E$’s decisions, it is rational for them not to finance any more projects.
Lemma 1 If and only if (A1) holds and $\alpha_1 \in I(\theta_L, y_L)$, then under PI there is a Nash equilibrium, in which $E$ chooses $j = L$ in period 1. Banks finance the project in this period, but do not provide loans in periods $t \in \{2, 3, \ldots\}$.

This equilibrium may, however, not be a sequential equilibrium. The threat that no offers are made in period 2, even if $E$ chooses $j = H$ in period 1, may not be credible, as all banks observe $E$’s decisions and can update their belief via Bayes’ rule. If $E$ deviates and chooses the high-risk project, it can be profitable for a bank to finance her after failure given that $\alpha_1$ is sufficiently high. This is what we are going to show now.

3.2 Experimental Equilibria

Assume that banks provide finance up to period $\bar{t} > 1$. As the failure of a low-risk project reveals low skills, this only happens if $E$ picks $j = H$ in periods $t \in \{1, \ldots, \bar{t} - 1\}$. Provided that (A1) holds, $E$ faces the same trade-off in period $\bar{t}$ as in the previous subsection. In view of a zero-payoff in case of failure and a higher expected payoff from the low-risk project, $E$ chooses $j = L$. She might be willing to realize high-risk projects in periods $t \in \{1, \ldots, \bar{t} - 1\}$ if condition (A2) holds. With banks’ beliefs being derived from Bayes’ rule, (1) entails that for any values of $p_H, \theta_L$ and $k \in \{1, \ldots, N\}$:

$$\lim_{\tilde{\alpha}_{t-1}^{B_k} \to 1} \tilde{\alpha}_t^{B_k}(\tilde{\alpha}_{t-1}^{B_k}, H) = 1.$$  

For any $t$, we get that $\tilde{\alpha}_t^{B_k} \to 1$ as $\alpha_1^{B_k} \to 1$. We obtain

$$\begin{align*}
\tilde{\theta}_t^{B_k} y_L - 1 & \geq 0, \\
(1 - p_H)\tilde{\theta}_t^{B_k} y_H - 1 & \geq 0,
\end{align*}$$

for $t \in \{1, \ldots, \bar{t} - 1\}$ if $\alpha_1$ is sufficiently high. In these periods, projects’ NPV is positive, so that banks provide loans to $E$. This allows us to establish the existence of experimental equilibria:

Lemma 2 Let $\bar{t} \in \mathbb{N}$ be given. If (A1), (A2) hold and $\alpha_1$ is sufficiently high, then under PI there is a Nash equilibrium in which $E$ chooses $j = H$ in periods $t \in \{1, \ldots, \bar{t} - 1\}$ and $j = L$ in period $\bar{t}$. Banks finance all projects in periods $t \in \{1, \ldots, \bar{t}\}$, but not in periods $t \in \{\bar{t} + 1, \bar{t} + 2, \ldots\}$.

Again, not every experimental equilibrium is a sequential equilibrium: If in period $\bar{t}$, belief $\tilde{\alpha}_t^{B_k}$ is sufficiently large, then banks can profitably finance projects in period $\bar{t} + 1$, given that $E$ chooses $j = H$ in period $\bar{t}$. The following result owes to the perfect observability of past and present risk choices:
Proposition 2 (Unique sequential equilibrium outcome under $PI$) If $(A1), (A2)$ hold and $\alpha_1 \in I(\theta_L, y_L)$, then under $PI$ in any sequential equilibrium, $E$ chooses $j = H$ in the periods $t \in \{1, \ldots, \bar{t}_{\alpha_1} - 1\}$ and $j = L$ in period $\bar{t}_{\alpha_1}$. Projects are financed in periods $t \in \{0, \ldots, \bar{t}_{\alpha_1} \}$, but not in periods $t \in \{\bar{t}_{\alpha_1} + 1, \bar{t}_{\alpha_1} + 2, \ldots\}$.

Therefore, if entrepreneurial risk choices are perfectly observable, countries with similar entrepreneurial skills and similar institutional constraints should expose the same scope of the “stigma of failure”.

4 Equilibria under private information of banks (PRB)

In this section, we relax the assumption that banks can perfectly observe the riskiness of all past projects. Instead, a bank only knows the risk of projects which it financed itself. As in Sharpe (1990) and von Thadden (2004), this enables banks to acquire private information about $E$. The risk of projects financed by other banks remains unknown. We thereby implicitly assume that banks cannot (or do not) infer the risk of past projects from past loan rates.\footnote{This assumption is not innocuous if there are detailed credit registers. If banks infer previous risk choices from the loan rates of past projects, then the results of the setting with perfect information apply.}

4.1 The conservative equilibrium

As in the last chapter, we can show that there is a conservative equilibrium if $(A1)$ holds and $\alpha_1$ is sufficiently large. Now this is a sequential equilibrium. To see why, assume that $E$ deviates and chooses $j = H$ in period 1 instead of $j = L$. Further assume that she gets financed by bank $B_k$. If her project fails, $B_k$ updates its belief about her type to $\tilde{\alpha}_1^{B_k}$ according to Bayes’ rule as in (1). All other banks assume that $E$ has chosen the low-risk project in period 1. Their belief about $E$ is $\tilde{\alpha}_t^{B_k} = 0$, $l \in \{1, \ldots, k - 1, k + 1, \ldots, N\}$. Thus, they will refuse to finance $E$’s project in a period $t > 1$. This makes $B_k$ a monopolistic supplier of finance to $E$. It can charge the maximal loan rates, $r_t^k(\tilde{\alpha}_t^{B_k}, j) = y_j$, in all subsequent periods $t > 1$. $E$’s expected payoff then equals zero. Therefore, it pays off for $E$ to pick the project with the highest expected return in period 1. We conclude:

Lemma 3 If and only if $(A1)$ holds and $\alpha_1 \in I(\theta_L, y_L)$, then under PRB there is a sequential equilibrium, in which $E$ chooses $j = L$ in period 1. Banks finance projects in this period, but do not provide loans in periods $t \in \{2, 3, \ldots\}$.
4.2 Experimental Equilibria

Experimental equilibria have the same form as in the last section: $E$ chooses $j = H$ in the first $\tilde{t} - 1$ periods and $j = L$ in $\tilde{t}$. Given that $E$ and banks (regardless of whether they financed the projects of $E$ or not) have the same beliefs on the equilibrium path, banks charge a loan rate according to (5). For the same reasons as for a conservative equilibrium, these experimental equilibria must be also sequential equilibria. We therefore obtain:

**Lemma 4** Let $\tilde{t} \in \mathbb{N}$ be given. If (A1), (A2) hold and $\alpha_1$ is sufficiently high, then under PRB there is a sequential equilibrium in which $E$ chooses $j = H$ in periods $t \in \{1, ..., \tilde{t} - 1\}$ and $j = L$ in period $\tilde{t}$. Banks finance all projects in periods $t \in \{1, ..., \tilde{t}\}$, but not in periods $t \in \{\tilde{t} + 1, \tilde{t} + 2, ...\}$.

Several experimental equilibria with different numbers of periods with project financing exist simultaneously if $\alpha_1$ is sufficiently close to unity. Note that for a given $\alpha_1 \in I(\theta_L, y_L)$, an equilibrium with $\tilde{t}_{\alpha_1}$ periods of project financing may not exist: $E$ could probably gain by choosing $j = L$ in all periods $t \in \{1, ..., \tilde{t}\}$ and switch to another bank after each failure (as all banks which financed previous projects know for sure that $E$ has low skills). However, she will refrain from doing so as long as she feels reasonably comfortable that she has high skills (i.e. as long as $\alpha_1$ and therefore $\tilde{t}_{\alpha_1}$, $t \in \{1, ..., \tilde{t}\}$, is sufficiently high). Combining Lemmata 3 and 4 gives rise to our next result:

**Proposition 3 (Multiple equilibria under PRB)** Let $\tilde{t} \in \mathbb{N}$ be given. If (A1), (A2) hold and $\alpha_1$ is sufficiently high, then under PRB both a conservative equilibrium (in which banks only finance projects in period 1) and an experimental equilibrium (in which banks finance all projects in periods $t \in \{1, ..., \tilde{t}\}$, but not in periods $t \in \{\tilde{t} + 1, \tilde{t} + 2, ...\}$) exist and are sequential equilibria.

The multiplicity of sequential equilibria implies that the “stigma of failure” may differ among countries with the same institutional environment and the same average level of entrepreneurial skills. The outcome in the credit market depends on banks’ expectations and $E$’s risk choices. If both cannot be altered simultaneously, changes in the institutional environment may not have an impact on the “stigma of failure”. Before we discuss this result’s welfare- and policy implications, we show that the same also obtains if banks cannot control $E$’s current risk choice.

5 Equilibria under imperfect information (IM)

Finally, we also relax the assumption about banks’ control of $E$’s currently chosen risk level. Instead, banks only know the period number, i.e. how many times $E$ previously
went bankrupt. They are also aware of the fact that E can choose between a risky and a less risky business strategy. Details, however, remain hidden to banks. This creates moral hazard in the credit market: E may be inclined to choose the high risk if banks charge a loan rate, which only covers low risk. Still, both conservative and experimental equilibria can exist at the same time as sequential equilibria.

5.1 The conservative equilibrium

For a conservative equilibrium, in which E chooses the low-risk project, we must rule out that E can gain from picking a high-risk project. For this, we need to modify assumption (A1):

**Assumption (A1\(^*\))**: We have \( y_L - 1 > (1 - p_H)(y_H - 1) \).

Note that (A1\(^*\)) implies (A1). Assume that E purchases the loan contract from B\(^{k}\) in period 1. For a given loan rate \( r^k_1 \), E prefers the low-risk project in this period if and only if

\[
\theta^E_1 (y_L - r^k_1) \geq (1 - p_H) \theta^E_1 (y_H - r^k_1). 
\]

Rearranging terms yields us the inequality

\[
r^k_1 \leq \frac{y_L - (1 - p_H)y_H}{p_H}. \tag{7}
\]

Provided that E chooses \( j = L \), B\(^{k}\) makes zero-profits if it charges the loan rate

\[
r^k_1 = \frac{1}{\theta^E_1}. \tag{8}
\]

By combining (7) and (8), we can show the existence of a conservative equilibrium:

**Lemma 5** If and only if (A1\(^*\)) holds and \( \alpha_1 \) is sufficiently high, then under IM there is a sequential equilibrium, in which E chooses \( j = L \) in period 1. Banks finance projects in this period, but do not provide loans in periods \( t \in \{2, 3, \ldots\} \).

The threat of not providing further credits in the next periods is credible, as banks cannot observe the risk choice of E. Thus, a conservative equilibrium is robust under imperfect information.

5.2 Experimental Equilibria

In a sequential equilibrium with \( \bar{t} \) periods of project financing, banks correctly anticipate that E chooses \( j = H \) in periods \( t \in \{1, \ldots, \bar{t} - 1\} \) and \( j = L \) in \( t = \bar{t} \). This results in loan
rates of
\[ r^k_t = \frac{1}{(1 - p_H)\hat{\theta}^B_t} \] (9)
in \( t \in \{1, ..., \bar{t} - 1\} \) and
\[ r^k_t = \frac{1}{\hat{\theta}^B_t}, \]
where \( \hat{\alpha}^B_t = \hat{\alpha}^E_t \) in all periods \( t \in \{1, ..., \bar{t} - 1\} \) for \( k \in \{1, ..., N\} \). Again, if \((A1^*)\) holds and \( \hat{\alpha}^B_t \) is sufficiently close to unity, then in period \( t = \bar{t} \), \( E \) cannot gain by choosing \( j = H \) instead of \( j = L \). In order to show that \( E \) cannot profitably deviate in periods \( t \in \{1, ..., \bar{t} - 1\} \), we need to modify assumption \((A2)\):

**Assumption \((A2^*)\):** We have \((1 - p_H)y_H + \frac{1}{1 - p_H} > y_L + 1.\)

\((A2^*)\) requires that a high-risk project’s expected payoff is not too small relative to a low-risk project’s. Assumptions \((A1^*)\) and \((A2^*)\) can hold at the same time if and only if \( p_H > 0 \) (which is implied by the construction of the model). Note that \((A2^*)\) may hold even if \((A2)\) does not and vice versa. We now can show:

**Lemma 6** Let \( \bar{t} \in \mathbb{N} \) be given. If \((A1^*)\), \((A2^*)\) hold and \( \alpha_1 \) is sufficiently high, then under \( IM \) there is a sequential equilibrium, in which \( E \) chooses \( j = H \) in periods \( t \in \{1, ..., \bar{t} - 1\} \) and \( j = L \) in period \( \bar{t} \). Banks finance all projects in periods \( t \in \{1, ..., \bar{t}\} \), but not in periods \( t \in \{\bar{t} + 1, \bar{t} + 2, ...\} \).

Again, it may well be that for given \( \alpha_1 \in I(\theta_L, y_L) \), an equilibrium with \( \bar{t}_{\alpha_1} \) periods of project financing does not exist, as loan rates are inflexible to entrepreneurial decisions: \( E \) does not choose \( j = L \) in periods \( t \in \{1, ..., \bar{t} - 1\} \) if she is relatively convinced of her high skills (and therefore will be successful with the low-risk project in period \( \bar{t} \) with high probability).

Consequently, the existence of multiple equilibria remains unaffected by the introduction of imperfect information. Combining Lemmata 5 and 6 leads us to conclude:

**Proposition 4 (Multiple equilibria under \( IM \))** Let \( \bar{t} \in \mathbb{N} \) be given. If \((A1^*)\), \((A2^*)\) hold and \( \alpha_1 \) is sufficiently high, then under \( IM \) both a conservative equilibrium (in which banks only finance projects in period 1) and an experimental equilibrium (in which banks finance all projects in periods \( t \in \{1, ..., \bar{t}\} \), but not in periods \( t \in \{\bar{t} + 1, \bar{t} + 2, ...\} \) exist and are sequential equilibria.

### 6 Welfare and policy implications

The results under the different informational settings offer a new framework for the analysis of welfare and policy making.
6.1 Welfare

Perfect Information and Private Information of Banks  As banks make zero expected profits in all periods, welfare is given by the \( E \)'s expected payoff \( V_1 \) at the beginning of the first period.\(^{11}\) We have shown that in the setting with perfect information, the first-best outcome with \( \bar{t}_{\alpha_1} \) periods of project financing is realized in any sequential equilibrium. The tie-breaking rule for \( E \) implies that in any other Nash equilibrium with fewer periods of project financing, \( E \)'s expected payoff must be smaller (as the low-risk project is realized too soon). However, in the setting with private information of banks, these equilibria can be sequential equilibria. This implies that the credit market outcome with private information may be inefficient.

Consider now two assessments with \( \bar{t}_1 \) and \( \bar{t}_2 \), \( \bar{t}_1 < \bar{t}_2 \), periods of project financing, where \( E \) picks \( j = H \) in the periods \( t \in \{1, ..., \bar{t}_1 - 1\} \) and \( j = L \) in period \( \bar{t}_l \), \( l \in \{1, 2\} \). By Lemma 4, these assessments can be equilibrium outcomes if \( \alpha_1 \) is sufficiently high. Let \( V_1^{(\bar{t}_1)} \) and \( V_1^{(\bar{t}_2)} \) the expected payoffs of \( E \) in the corresponding equilibria. It must hold that \( V_1^{(\bar{t}_1)} > V_1^{(\bar{t}_2)} \). If \( V_1^{(\bar{t}_1)} < V_1^{(\bar{t}_2)} \), \( E \) could increase her expected payoff in period \( \bar{t}_1 \) of the equilibrium with \( \bar{t}_1 \) periods of project financing by choosing \( j = L \). The loan rate for this project must be the same in both equilibria. If \( V_1^{(\bar{t}_2)} = V_1^{(\bar{t}_1)} \), then the tie-breaking rule implies that \( E \) chooses \( j = L \) in period \( \bar{t}_1 \). We conclude that welfare is higher in an equilibrium with more periods of project financing than in an equilibrium with fewer periods of project financing.

Imperfect Information  Under imperfect information, things are more difficult. We saw that an experimental equilibrium exists even if (A2) does not hold. Then, it is against the \( E \)'s interest to realize high-risk projects. The reason is that after subtracting the bank’s break-even loan rate, this project’s net return in case of success is lower than for the low-risk project. \( E \) would prefer to realize projects with low risk. Yet, if banks assume that \( E \) chooses the high-risk realization of the project, the high loan rate prevents \( E \) from picking the low risk. This effect is the same as in models of asymmetric information in which inefficient high-risk projects crowd out efficient low-risk projects. A conservative then dominates any experimental equilibrium. However, (A1*) and (A2) can be fulfilled at the same time if and only if \( y_L > 2 \). This implies that if \( y_L \leq 2 \) holds, any experimental equilibrium is dominated by a conservative equilibrium under imperfect information.

Provided that \( y_L > 2 \) and that assumptions (A1*), (A2) and (A2*) are fulfilled, an experimental equilibrium may well dominate a conservative equilibrium. We know

\(^{11}\)If we consider a continuum of entrepreneurs of mass 1, banks make zero profits for sure and welfare is the aggregated payoff of entrepreneurs (which is equivalent to the expected payoff in our setting).
from Lemma 6 that if $\alpha_1$ is sufficiently high, there can simultaneously exist equilibria with different numbers of periods of project financing. As before, we can show that an equilibrium with more periods of project financing always dominates an equilibrium with fewer periods of project financing in terms of welfare.

**Example**  Consider a scenario with the following values: $y_L = 2.5$, $y_H = 2.66$, $p_H = 0.1$, $\theta_L = 0.3$, $\alpha_1 = 0.9$. It is straightforward to verify that assumptions ($A1$), ($A1^*$), ($A2$) and ($A2^*$) are satisfied for these values. Let the equilibrium loan rates for $k \in \{1, ..., N\}$ be as follows:

\[
\begin{align*}
   r_1^k(\tilde{\alpha}_1^{B_k}) &= 1.075, \\
   \tilde{r}_1^k(\tilde{\alpha}_1^{B_k}) &= 1.195, \\
   \tilde{r}_2^k(\tilde{\alpha}_2^{B_k}) &= 1.457.
\end{align*}
\]

The expected payoff in the conservative equilibrium (with $\tilde{\alpha}_1^{B_k} = \alpha_1$ and $r_1^k = r_1^k(\tilde{\alpha}_1^{B_k})$, $k \in \{1, ..., N\}$) is therefore $V_1^C = 1,325$. In contrast, an experimental equilibrium with two periods of project financing (with $\tilde{\alpha}_i^{B_k} = \tilde{\alpha}_i^{E}$ and $r_i^k = \tilde{r}_i^k(\tilde{\alpha}_i^{B_k})$, $k \in \{1, ..., N\}$, $t \in \{1, 2\}$), leads to $V_1^E = 1,363$. The loan rates are chosen such that banks make zero-profits in expectation. Both under ($PI$) and ($IM$), the experimental equilibrium dominates the conservative one.

Now stick to the same setting, but with $y_L = 1.5$ and $y_H = 1.55$. Assumption ($A2$) is violated, while the others remain fulfilled. As the underlying risk is the same as before, the loan rates remain unchanged. Under ($IM$), there can be both the conservative and the experimental equilibrium with two periods of project financing. Clearly, this experimental equilibrium is inefficient, because of $V_1^C = 0,395$ and $V_2^E = 0,303$. In contrast, under ($PI$), the experimental equilibrium does not exist.

### 6.2 Policy Implications

**Banking System Design**  Our analysis shows that the observability of entrepreneurs’ past and present risk choices is a crucial feature that prevents inefficiencies in the credit market. We think that a banking system, which is most likely to exhibit this feature, is based on small, specialized and regional banks or on venture capitalists. Such institutions keep close ties to their clients and may well observe the risk involved in past and present business decisions. Some empirical support for this result comes from a comparison of the EU and Japan to the US: by trend, a more (less) pronounced “stigma of failure” seems to go in hand with more bank finance (market finance).
Economies with a financial system in which banks are able to observe past and present risk choices should be left unchanged. As the highest equilibrium level of welfare is attained in any sequential equilibrium, there is no room for policies aimed at changing the nature of the equilibrium. In particular, a conservative equilibrium may be the result of a relatively low level of average entrepreneurial skills. Yet, De Meza (2002) and ABRP (2002) caution that most businesses failures stem from low project quality and management incompetence. Hence, enabling more entrepreneurs might simply result in more costly failures.

On the contrary, large banks may be too distant to their borrowers in order to evaluate the risk of failed projects. These creditors mainly rely on statistical data ("credit scoring"), so that the results of the settings with private information of banks or imperfect information apply. As entrepreneurs choose the low-risk project to early in some equilibria, the outcome in the credit market may be inefficient. Policies aiming at changing the nature of the equilibrium may not be effective, as many entrepreneurs’ actions and banks’ expectations must be changed simultaneously. Consider for example the approach adopted by the European Commission (2000, 2007) through programs initiated in the aftermath of the Lisbon Council in 2000. Among other things, it foresees reducing the stigma of failure by advising entrepreneurs to choose higher risk levels. Entrepreneurs will follow such advice only if banks change their policy at the same time. This remains impossible as long as banks do not understand better the risk involved in their clients’ business.

**Improving Entrepreneurial Skills** Another measure of the EU to increase entrepreneurial activity is education, formation of relevant skills and early support for viable enterprises (see European Commission, 2007). In our model, such policies are reflected by an increase in $\alpha_1$. If banks have perfect information, an increase in the share of skilled entrepreneurs $\alpha_1$ has a direct and an indirect effect on welfare in a sequential equilibrium: the loan rate decreases in all periods, see equation (5), and it (weakly) increases the number $T_{\alpha_1}$ of periods in which projects are financed in equilibrium (see Proposition 1).

However, under private information or imperfect information of banks, inflexible beliefs about entrepreneurial decisions deter policy’s impact on the nature of the equilibrium. Unless banks’ credit offers and agents’ risk-taking behavior becomes simultaneously coordinated to another equilibrium, only the direct effect will materialize.
7 Conclusion

This paper presents a multi-period credit market model where the extent to which failed entrepreneurs are excluded from further start-up financing is determined endogenously. The results’ key driver is the evolution of a banks belief with regard to an entrepreneur’s skills and its interplay with her risk choices. If the probability of high skills is sufficiently large, multiple equilibria may obtain. We observed that under perfect information (i.e. if banks can evaluate both past and present risk choices of an entrepreneur), in any sequential equilibrium the first-best outcome is realized. Second, under private information of banks (i.e. if banks can evaluate only the risk of projects which were financed by themselves), both a conservative and experimental equilibria are sequential equilibria. The multiplicity of equilibria is robust. Finally, the same result obtains if banks cannot evaluate the risk of any projects. We concluded that the outcome in credit markets where banks do not always observe the full history of entrepreneurial risk choices can be inefficient. Policy measures aiming at lowering the “stigma of failure” might not be effective, because banks’ expectations and entrepreneurs’ actions must simultaneously be shifted to a new equilibrium. However, our results also leave room for regulation: a banking system with small banks that know well their clients’ business should be more prone to achieving an efficient allocation than one with arms-length finance.

Altogether, our paper is a starting point that offers ample scope for future research. It allows for the incorporation of numerous additional factors that might influence credit market conditions, such as education, social security, or the tax system. More specifically, the integration of learning would result in a lower decline of financiers’ beliefs about entrepreneurs’ skills over time. A population’s age distribution should also matter, as younger agents have a higher risk appetite and thus readiness to create new firms, see Lévesque and Minniti (2006). Related work suggests taking into account multi-tool contracts (that include risk monitoring or quality screening) or various effects of the creation of innovative firms, such as technological- or demand-spillovers. At last, more convincing empirical evidence is needed to support effective policy making.

8 Appendix

8.1 Proof of Proposition 1

If $E$ chooses $j = L$ and the project fails, $E$ knows about her low skills. She then does not realize any further projects. Therefore, consider the sequence $t^* = 1, 2, ...$ and the set of assessments in which $E$ chooses $j = H$ in periods $t \in \{1, ..., t^* - 1\}$, $j = L$ in period $t^*$, and no more projects thereafter. Denote by $V_t^{(t^*)}$ the expected payoff of $E$ at the beginning.
of period \(t \in \{1, ..., t^*\}\) under the assessment with \(t^*\) periods of project realizations. We have
\[
V^{(t^*)}_{t} = \tilde{\theta}^E_t y_L - 1, \tag{10}
\]
and for \(t \in \{1, ..., t^* - 1\}\)
\[
V^{(t^*)}_t = (1 - p_H)^{\tilde{\theta}^E_t} y_H - 1 + (1 - (1 - p_H)^{\tilde{\theta}^E_t}) V^{(t^*)}_{t+1}. \tag{11}
\]
For given \(\alpha_1 < 1\), there is a finite period \(\tilde{t}\), such that \((1 - p_H)^{\tilde{\theta}^E_t} y_L - 1 < 0\) and \((1 - p_H)^{\tilde{\theta}^E_t} y_H - 1 < 0\) for all \(t \geq \tilde{t}\), regardless of the assessment. That is why \(V^{(t^*)}_1\) is only positive for a finite number of assessments with periods of project realizations \(t^* \in \{1, ..., t^*\}\). Pick two numbers \(g_1, g_2 \in \{1, ..., t^*\}\) with \(g_1 < g_2\). If \(V^{(g_1)}_1 \geq V^{(g_2)}_1\), then we have \(V^{(g_1)}_t \geq V^{(g_2)}_t\) for \(t \in \{1, ..., g_1\}\). Otherwise, we would have
\[
\tilde{\theta}^E_{g_1} y_L - 1 < (1 - p_H)^{\tilde{\theta}^E_{g_1}} y_H - 1 + (1 - (1 - p_H)^{\tilde{\theta}^E_{g_1}}) V^{(g_2)}_{g_1+1},
\]
which contradicts \(V^{(g_1)}_1 \geq V^{(g_2)}_1\). Hence, \(E\) never can gain by switching from one assessment to another after period 0. Because of the tie-breaking rule, we have
\[
\bar{t}_{\alpha_1} = \min \left\{ g \in \{1, ..., t^*\} \mid V^{(g)}_1 \geq V^{(t^*)}_1, t^* \in \{1, ..., t^*\} \right\}.
\]
To prove the second claim, consider two assessments with \(g_1, g_2 \in \mathbb{N}\), \(g_1 < g_2\), periods of project realizations. We can then calculate that
\[
\lim_{\alpha_1 \to 1} V^{(g_1)}_1 = (1 - p_H^{g_1-1}) \left( y_H - \frac{1}{1 - p_H} \right) + p_H^{g_1-1} (y_L - 1)
\]
for \(t \in \{1, 2\}\). If (A2) holds, then
\[
\lim_{\alpha_1 \to 1} V^{(g_2)}_1 > \lim_{\alpha_1 \to 1} V^{(g_1)}_1.
\]
Note that \(V^{(t^*)}_1\) is continuous in \(\alpha_1\) for all \(t^* \in \mathbb{N}\). Thus, there is a \(\hat{\alpha}_1 < 1\), such that \(V^{(g_2)}_1 > V^{(g_1)}_1\) whenever \(\alpha_1 > \hat{\alpha}_1\) and therefore \(\bar{t}_{\alpha_1} > g_1\).

### 8.2 Proof of Lemma 2

Assume that the equilibrium is as stated in the claim. As banks make zero profits in equilibrium, we get
\[
V_t = \tilde{\theta}^E_t \left( y_L - \frac{1}{\tilde{\theta}^E_t} \right), \tag{12}
\]
and for \(t < \bar{t}\),
\[
V_t = (1 - p_H)^{\tilde{\theta}^E_t} \left( y_H - \frac{1}{(1 - p_H)^{\tilde{\theta}^E_t}} \right) + (1 - (1 - p_H)^{\tilde{\theta}^E_t}) V_{t+1}. \tag{13}
\]
First, consider the last period $\bar{t}$: as in Lemma 1, (A1) ensures that $E$ chooses $j = L$ in period $\bar{t}$, given that banks do not finance projects in future periods. Next, focus on a period $t < \bar{t}$. If $E$ chooses $j = L$ in this period, her expected payoff is equal to $\tilde{\theta}_t^E \left( y_L - \frac{1}{\tilde{\theta}_t^E} \right)$, since no loans will be provided to her in future periods. $E$ therefore chooses $j = H$ if

$$V_t \geq \tilde{\theta}_t^E \left( y_L - \frac{1}{\tilde{\theta}_t^E} \right). \quad (14)$$

It follows from (1) and (2) that

$$\lim_{\alpha_1 \to 1} \tilde{\theta}_t^E = 1.$$

With (A2), we can then estimate

$$\lim_{\alpha_1 \to 1} V_t = \left( 1 - p_{H}^{\tilde{\theta}_t^E} \right) \left( y_H - \frac{1}{1 - p_{H}} \right) + p_{H}^{\tilde{\theta}_t^E} (y_L - 1) > y_L - 1.$$

Note that $V_t$ is continuous in $\alpha_1$. Thus, (14) holds if $\alpha_1$ is sufficiently close to unity.

### 8.3 Proof of Proposition 2

In any sequential equilibrium, we have $\tilde{\theta}_t^E = \tilde{\theta}_k^B$ for all $k \in \{1, ..., N\}$ and all periods $t$. From assumption (A1) it follows that in the last period of an equilibrium, in which projects are financed by banks, $E$ chooses $j = L$. It is also clear that in the periods before this last period, $E$ chooses $j = H$. Otherwise, banks would not finance projects any longer. Consider therefore the sequence $t^* = 1, 2, ...$ and the set of assessments in which $E$ chooses $j = H$ in periods $t \in \{1, ..., t^* - 1\}$, $j = L$ in period $t^*$ and banks finance all projects in periods $t \in \{1, ..., t^*\}$, but not in periods $t \in \{t^* + 1, t^* + 2, ...\}$. Denote by $V_t^{(t^*)}$ the expected payoff of $E$ in period $t \in \{1, ..., t^*\}$ under the assessment with $t^*$ periods of project financing. As banks make expected zero profits in each period of a sequential equilibrium, we have

$$V_t^{(t^*)} = \tilde{\theta}_t^E \left( y_L - \frac{1}{\tilde{\theta}_t^E} \right), \quad (15)$$

and for $t \in \{1, ..., t^* - 1\}$

$$V_t^{(t^*)} = (1 - p_{H})\tilde{\theta}_t^E \left( y_H - \frac{1}{(1 - p_{H})\tilde{\theta}_t^E} \right) + (1 - (1 - p_{H})\tilde{\theta}_t^E) V_{t+1}^{(t^*)}. \quad (16)$$

Note that (15) equals (10) and (16) equals (11) from the proof of Proposition 1. Thus, $V_t^{(t^*)}$ is the same as in the proof of Proposition 1 for all $t \in \{1, ..., t^*\}$ and for all $t^*$.

As $\tilde{\theta}_t^E \to 0$ for $t^* \to \infty$, at least one of the following statements must be true for each assessment with $t^*$ periods of project financing: (1.) There exists a $\tau \in \{1, ..., t^* - 1\}$, such that

$$\tilde{\theta}_t^E \left( y_L - \frac{1}{\tilde{\theta}_t^E} \right) \geq (1 - p_{H})\tilde{\theta}_\tau^E \left( y_H - \frac{1}{(1 - p_{H})\tilde{\theta}_\tau^E} \right) + (1 - (1 - p_{H})\tilde{\theta}_\tau^E) V_{t+1}^{(t^*)} .$$
This assessment cannot be a Nash equilibrium as \( E \) would choose \( j = L \) in period \( \tau \). It also cannot be the first-best. (2.) It holds that
\[
y_L - \frac{1}{\theta_{L^*}} < 0.
\]
This assessment cannot be a Nash equilibrium as banks would not any finance projects in period \( t^* \). It also cannot be the first-best. (3.) There exists a \( \tau \in \{1, \ldots, t^*-1\} \), such that
\[
y_H - \frac{1}{(1-p_H)\theta_{\tau}} < 0.
\]
This assessment cannot be a Nash equilibrium as banks would not finance projects with high risk in period \( \tau \). It also cannot be the first-best. (4.) The assessment is a Nash equilibrium. Denote by \( t_{\text{max}}^* \) the maximal number of periods in which projects are financed and the corresponding assessment is an equilibrium. Note that \( t_{\text{max}}^* \) is well-defined as \( \alpha_1 \in I(\theta_L, y_L) \). Then, this assessment must be the only sequential equilibrium outcome.

To see why, consider an alternative Nash equilibrium with \( t^* < t_{\text{max}}^* \) periods of project-financing. \( E \) can gain by choosing \( j = H \) in the periods \( t \in \{t^*, \ldots, t_{\text{max}}^*-1\}, j = L \) in period \( t_{\text{max}}^* \). As beliefs must be given by Bayes’ rule and \( E \)’s decisions are observable, banks cannot credibly threat to stop financing projects (by offering loan contracts with expected zero-profits) in these periods. Otherwise, the assessment with \( t_{\text{max}}^* \) periods of project financing would not be an equilibrium, as statement (1.) and/or (2.) and/or (3.) would be true. It remains to show that \( t_{\text{max}}^* = \tilde{t}_{\alpha_1} \): \( t_{\text{max}}^* \geq \tilde{t}_{\alpha_1} \) follows from the equivalence of expected payoffs (as stated above) and the fact that statements (1.) to (3.) are not true for any period \( t \in \{1, \ldots, \tilde{t}_{\alpha_1}-1\} \) of an assessment with less than \( \tilde{t}_{\alpha_1} \) periods of project financing. \( t_{\text{max}}^* \leq \tilde{t}_{\alpha_1} \) follows from the equivalence of expected payoffs (as stated above) and the fact that any assessment with more than \( \tilde{t}_{\alpha_1} \) periods of project financing violates statement (1.). Thus, we have \( t_{\text{max}}^* = \tilde{t}_{\alpha_1} \).

8.4 Proof of Lemma 4

We must have \( \tilde{\theta}_E = \tilde{\theta}_{B_k} \) for \( k \in \{1, \ldots, N\} \) and all periods \( t \) of a sequential equilibrium. If \( E \) sticks to the proposed strategy, her expected payoffs are as in the proof of Lemma 2, i.e. equations (12) and (13). First, consider the last period \( t = \tilde{t} \). As in Lemma 3, (A1) ensures that \( E \) chooses \( j = L \), as \( V_{\tilde{t}+1} = 0 \). Next, focus on period \( \tilde{t}-1 \). If \( E \) chooses \( j = L \) in period \( \tilde{t}-1 \) and she fails, then she and her bank, say \( B_l \), know that she has low skills, i.e. \( \tilde{\alpha}_{\tilde{t}}^E = \tilde{\alpha}_{B_l}^E = 0 \). Yet, she may get credit from another bank, for example \( B_k \), \( k \neq l \), in period \( \tilde{t} \). Thus, if she chooses \( j = L \) in period \( \tilde{t}-1 \), her expected payoff \( \tilde{V}_{\tilde{t}-1} \) is
\[
\tilde{V}_{\tilde{t}-1} = \tilde{\theta}_{\tilde{t}-1}^E \left( y_L - \frac{1}{\theta_{L^*}} \right) + (1 - \tilde{\theta}_{\tilde{t}-1}^E)\theta_L \left( y_L - \frac{1}{\theta_{B_k}} \right).
\]
where $\hat{\theta}_t^{B_k}$ is the equilibrium value. As in the proof of Lemma 2, we then can calculate that

$$\lim_{\alpha_1 \to 1} V_{t-1} > \lim_{\alpha_1 \to 1} \tilde{V}_{t-1}$$

if (A2) holds. Thus, if $\alpha_1$ is sufficiently large, then $E$ chooses $j = H$ in period $\bar{t} - 1$. By going through the same steps, one can show that $E$ chooses $j = H$ in all periods $t < \bar{t}$ if $\alpha_1$ is sufficiently large.

### 8.5 Proof of Lemma 6

We must have $\hat{\theta}_t^E = \hat{\theta}_t^{B_k}$ for $k \in \{1, ..., N\}$ and all periods $t$ of a sequential equilibrium. Assume that $E$ acts as stated in the claim. Banks charge loan rates $r_t^k$, $k \in \{1, ..., N\}$, $t \in \{1, ..., \bar{t}\}$, such that they make zero-profits in expectation. Denote by $V_t$ the corresponding expected payoff of $E$ at the beginning of period $t \in \{1, ..., \bar{t}\}$. For period $\bar{t}$, the proof proceeds as for Lemma 5. Next, focus on a period $t < \bar{t}$. Note that $E$ has private information about her probability of success whenever she deviates from the equilibrium path in these periods: if she chooses $j = L$, and this project fails, then she knows that she has low skills. However, as banks do not observe $E$’s decisions, the loan rates in the next periods are not affected by $E$’s risk choice. Denote by $\tilde{V}_t$ the expected payoff of $E$ at the beginning of period $t \in \{2, ..., \bar{t}\}$ if she knows for sure that she has low skills but follows the equilibrium path of play. Trivially, it holds that $V_t > \tilde{V}_t$. $E$ chooses $j = H$ in period $t$ if

$$\hat{\theta}_t^E (y_L - r_t^k) + \left(1 - \hat{\theta}_t^E\right) \tilde{V}_{t+1} \leq \left(1 - p_H\right) \hat{\theta}_t^E (y_H - r_t^k) + \left(1 - (1 - p_H) \hat{\theta}_t^E\right) V_{t+1}.$$

If $\alpha_1$ is sufficiently large, then this inequality is implied by

$$(y_L - r_t^k) < (1 - p_H)(y_H - r_t^k).$$

Rearranging terms gives

$$r_t^k > \frac{y_L - (1 - p_H)y_H}{p_H}.$$  

(17)

Recall the loan rate in period $t$ is given by (9). Assumption (A2*) ensures that inequality (17) holds if $\alpha_1$ is sufficiently large. Thus, if (A1*) and (A2*) hold and $\alpha_1$ is sufficiently high, then $E$ chooses $j = H$ in periods $t \in \{1, ..., \bar{t} - 1\}$ and $j = L$ in period $\bar{t}$.

### References


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