

Hydrothermal cooling of the oceanic lithosphere and the square root age law

using parameterized porous convection

H. Schmeling¹ and G. Marquart²,
¹Goethe University Frankfurt, ²RWTH Aachen

see also:

Schmeling, H, and G. Marquart, 2014: A scaling law for approximating porous hydrothermal convection by an equivalent thermal conductivity: theory and application to the cooling oceanic lithosphere. *Geophys. J. Int.*, 197 (2): 645-664 doi:10.1093/gji/ggu022

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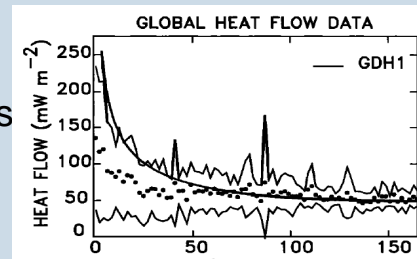
H. Schmeling¹ and G. Marquart²,
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Introduction

- Heatflow of the earth
- The \sqrt{t} - law

Overview:

Cooling of the oceanic lithosphere follows approximately the \sqrt{t} - law, but significant deviations imply the importance of hydrothermal convection. Here we include hydrothermal convection into models of lithospheric plate cooling using a new approach.



New approach

Approximating hydrothermal convection by an equivalent thermal conductivity based on parameterized convection

Result

Cooling plate with hydrothermal convection

Sedimentary sealing of the ocean floor at a given age

Deviation from \sqrt{t} - law:
→ Measurable heat flow
→ Increased total heat loss

Effect on Bathymetry

see also:

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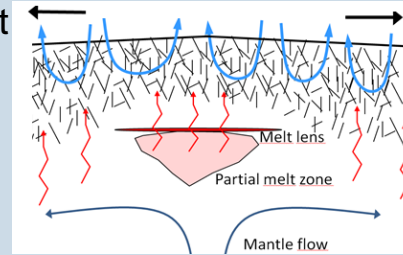
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Overview:

2D porous convection models with depth-dependent permeability are used to derive **parameterized convection** scaling laws. From these laws an equivalent thermal conductivity is employed in conductive cooling models of the lithosphere mimicking hydrothermal cooling.



← 2/3 →

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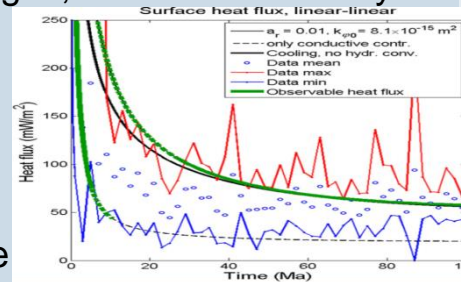
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Clicking on the boxes gives details and figures

Overview:

Hydrothermal convection **increases the total heat flux and loss** with respect to the classical \sqrt{t} - law. At young ages, heat flow data may miss the hydrothermal contribution, at high ages sediment cover most likely changes the hydrothermal cooling mode



3/3

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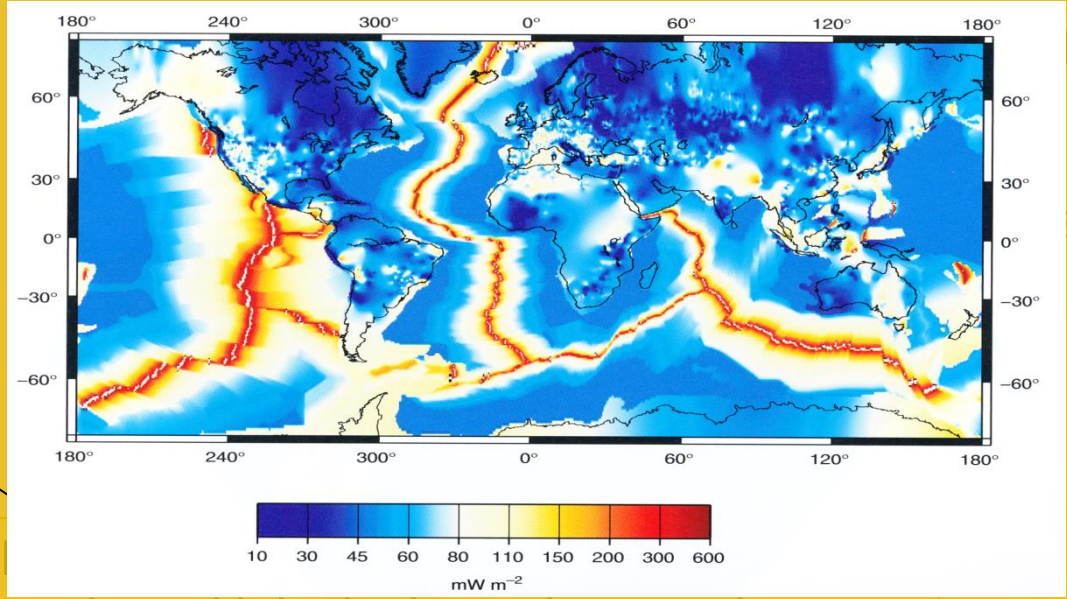
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Result
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Sedimentary sealing of the ocean floor at a given age

This map shows the global heat flow data (Jaupart & Mareshal, 2011)

- Continents: heat flow measurements
- Oceanic lithosphere: plate cooling model assumed, thus hydrothermal cooling is not included

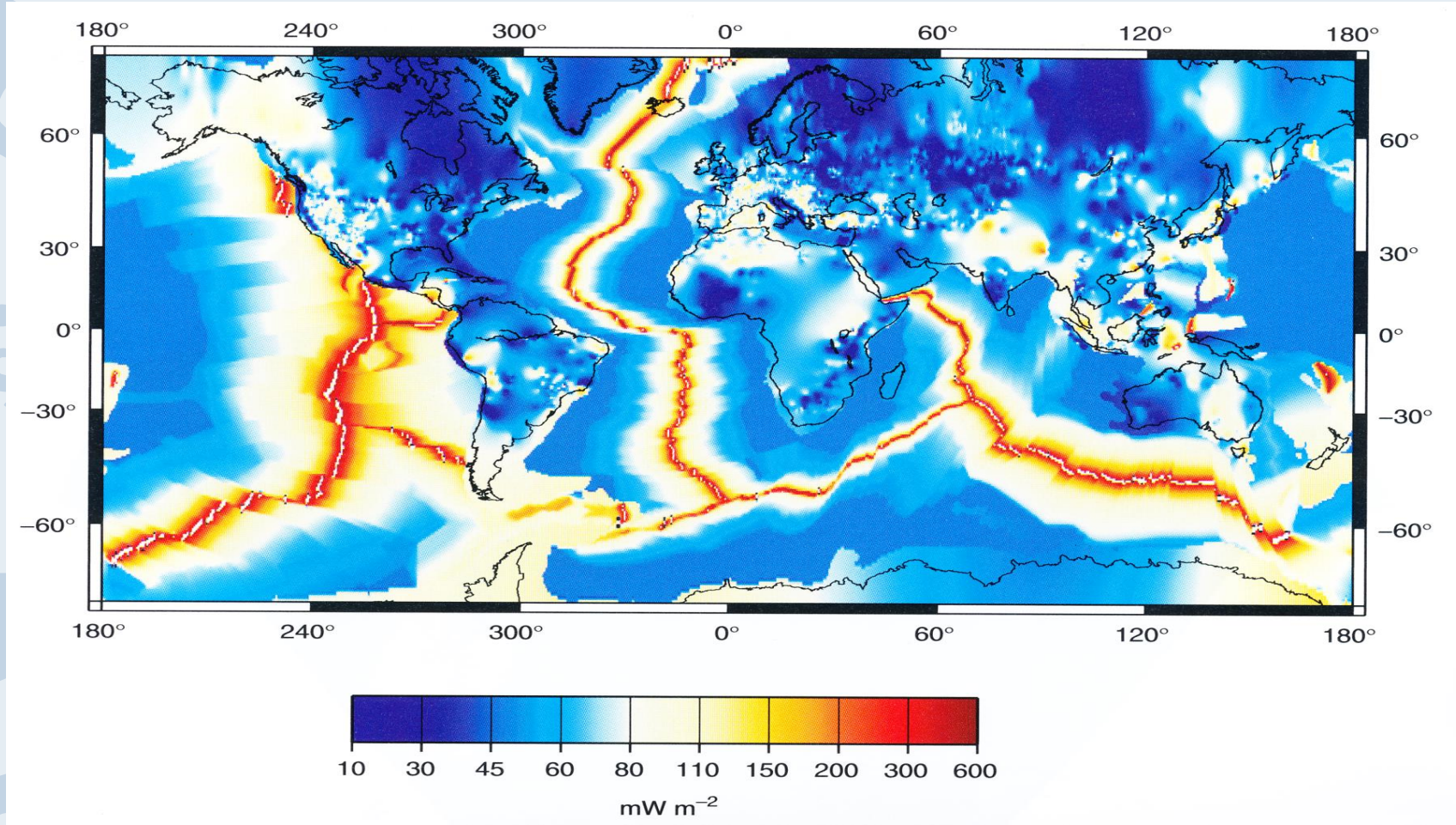
Deviation from
→ Measurable
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Global heat flow data



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Introduction

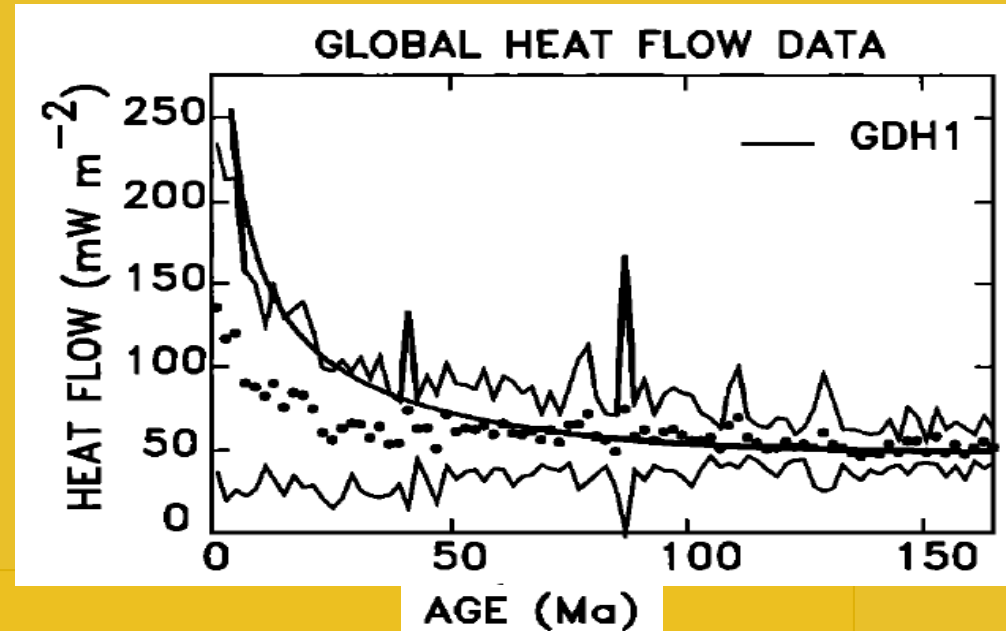
- Heatflow of the earth
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Result

Cooling plate with hydrothermal

Sedimentary sealing of the ocean floor at a given age

Deviat
→ Mea
→ Incr



From Stein and Stein, 1994
Global heat flow measurements have been compiled and plotted as a function of age (dots) with their upper and lower bounds. The conductive plate cooling model GDH1 is shown for comparison. At ages less than 40 Ma the observations are below the cooling model, indicating that the hydrothermal contribution may have been missed





Discrepancy: hydrothermal cooling

Fraction of hydrothermally removed heat: **20 - 40%** of total heat flow of the earth (Sclater et al., 1980; Stein and Stein, 1992; Lowell et al., 2008; Spinelli and Harris, 2011)

→ the conductive \sqrt{t} - law needs to be modified

→ no cooling plate model exists which consistently includes hydrothermal convection

Effect on \sqrt{t} - law?

Right: Black or white smokers at the bottom of mid oceanic ridged impressively demonstrate how much heat may be lost by hydrothermal convection



Introduction

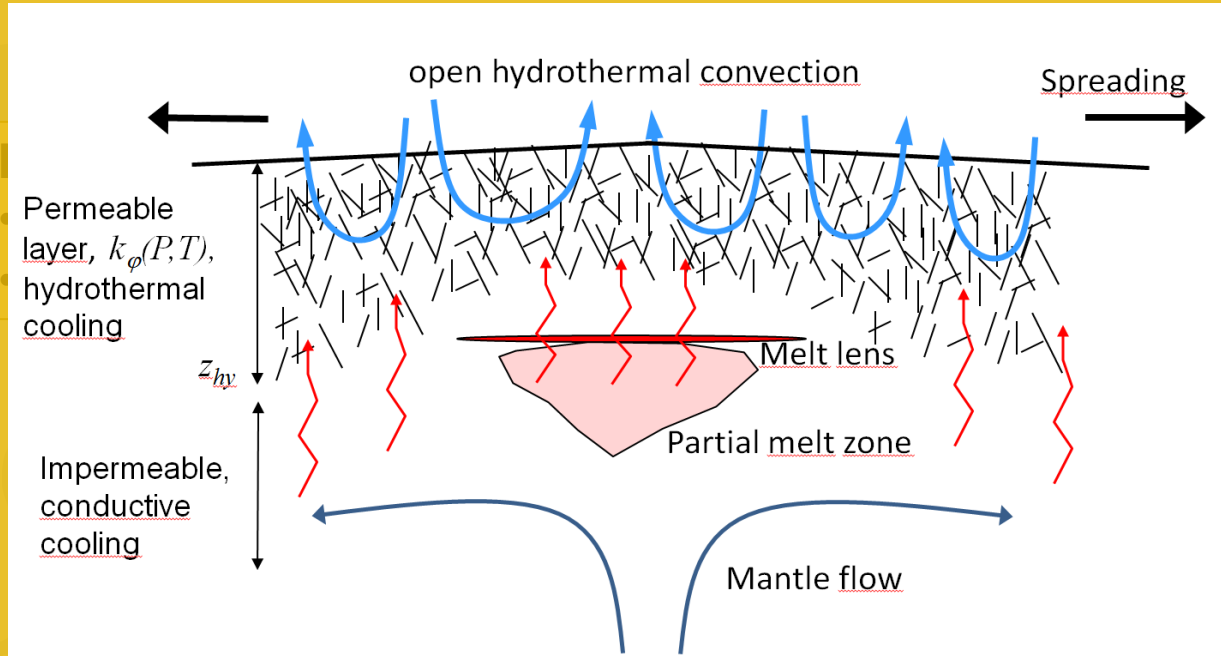
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Result Cooling

Sedimentary sealing of the ocean floor at a given age

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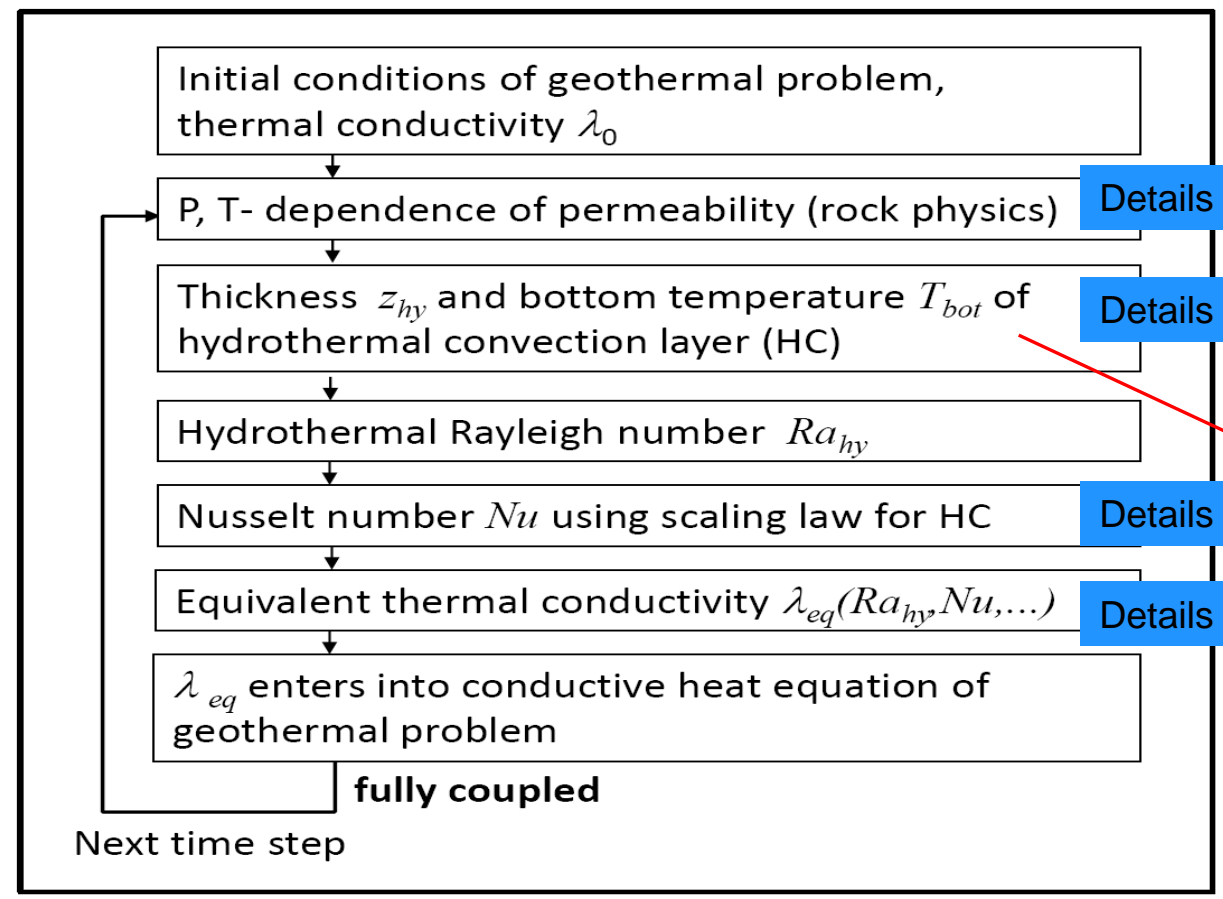
New approach

Approximating hydrothermal convection by an equivalent thermal conductivity based on parameterized convection

Cooling plate with hydrothermal convection

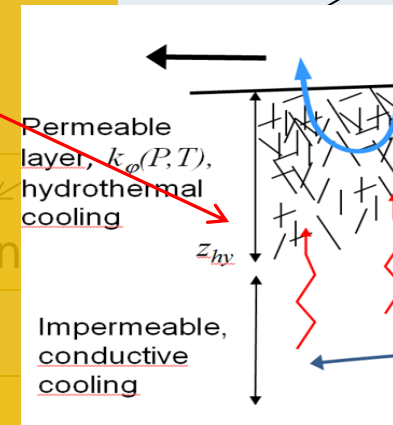
- Thermal contraction due to cooling from above produces cracks → permeable layer
 - Penetration of hydrothermal fluids amplify cooling
 - Permeability is a function of temperature (thermal contraction) and pressure (compaction)
 - Total penetration depth, z_{hy} , is a function of cooling history
- Consistent cooling model should include feedback between cooling – P-T-dependent permeability and porous convection

Flow chart of approach



New approach

Approximating hydrothermal convection by an equivalent thermal conductivity based on parameterized convection



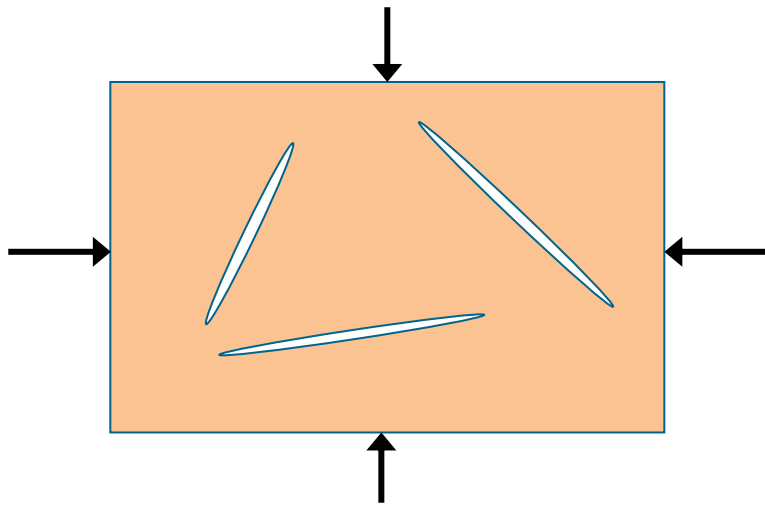
$$Ra_{hy} = \frac{\alpha_f g \rho_f^2 c_{pf} k_{\phi 0} \Delta T_{hy} z_{hy}}{\eta_f \lambda_m}$$

$$Nu = \frac{q_{top} z_{hy}}{\lambda_0 \Delta T_{hy}}$$

Details of approach: P – T – dependent porosity and permeability



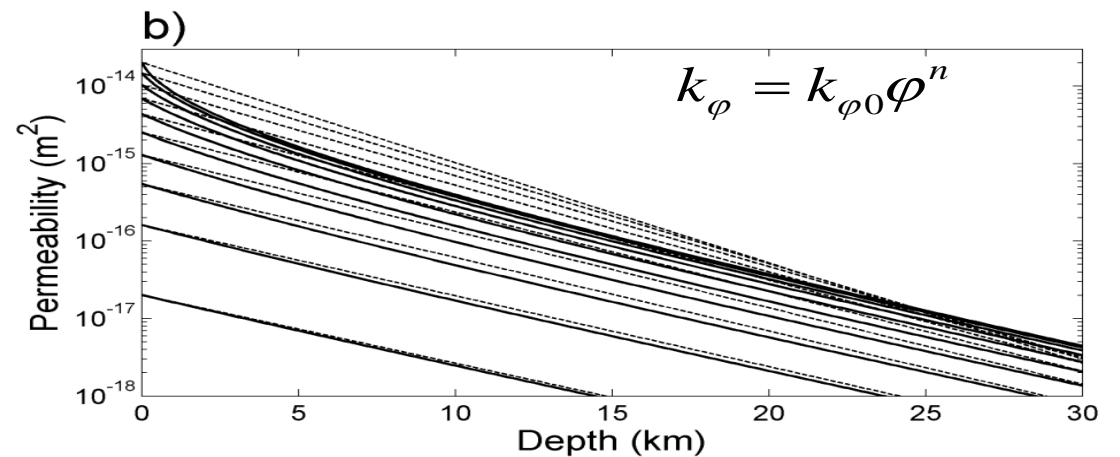
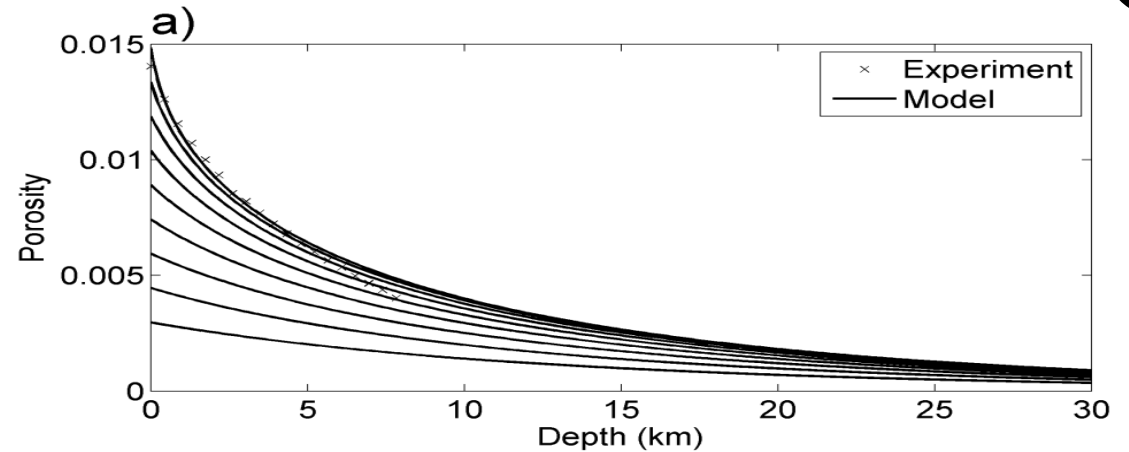
Effect of pressure from elastic crack model (based on Schmeling(1986))



Crack porosity

$$\varphi(z) = \varphi_0 \cdot \exp\left(-\frac{z}{z_{ch}}\right)$$

$$z_{ch} = \frac{\tilde{a}_{r0} K_0}{(\bar{\rho} - \rho_f) g} \cdot \left(1 - c_{fit} \frac{1 - c_v}{\tilde{a}_{r0}} \cdot \varphi_0 \cdot \left(1 - \frac{1}{e}\right)\right)$$



Details of approach: P – T – dependent porosity and permeability

Pore closure by thermal expansion at const volume and relaxed thermal stress

Germanovich et al. (2000)

and ductile creep

The final permeability law

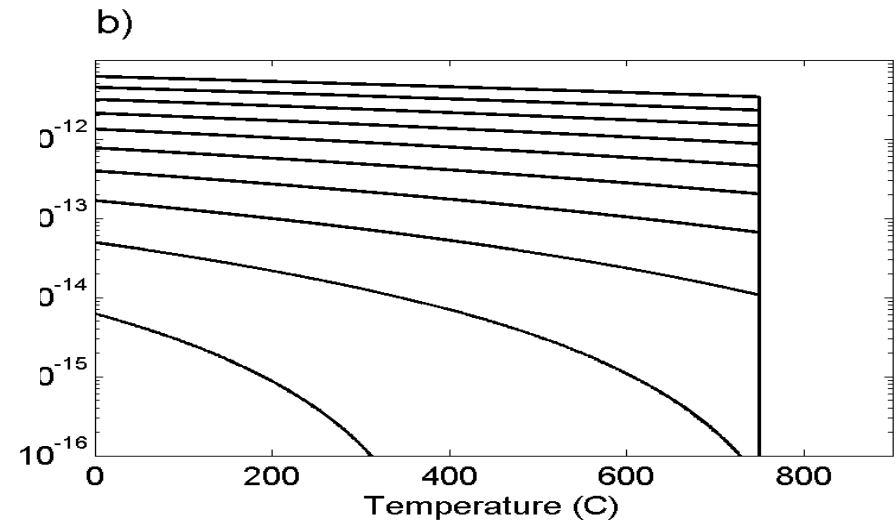
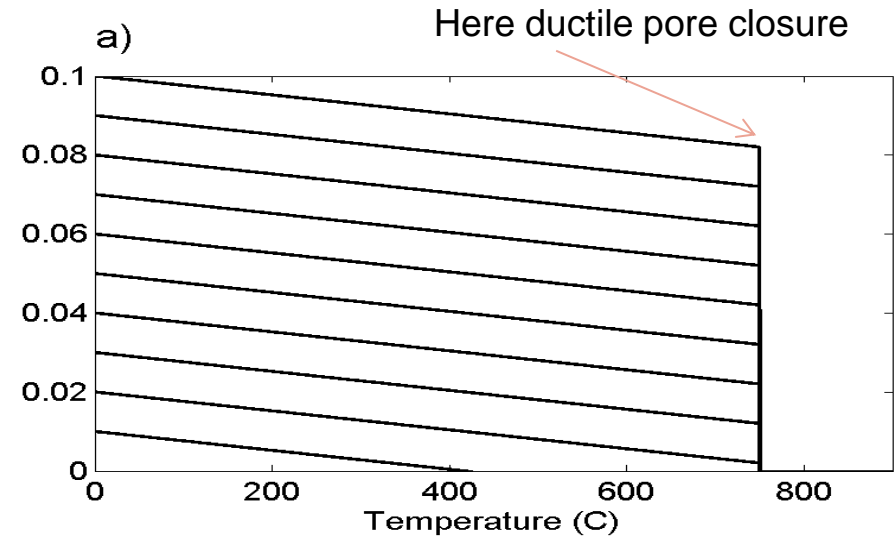
$$k_{\varphi}(T, z) = k_{\varphi 0} \cdot [1 - \gamma \cdot (T - T_0)]^n \cdot H(1 - \gamma \cdot (T - T_0)) \cdot$$

$$H(T_{duc} - T) \cdot \exp\left(-n \cdot \frac{z}{z_{ch}}\right)$$

$\gamma = \alpha / \varphi_0$ thermal expansivity / surface porosity

n = permeability – porosity law power exponent

H – Heaviside function



X

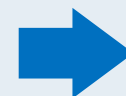
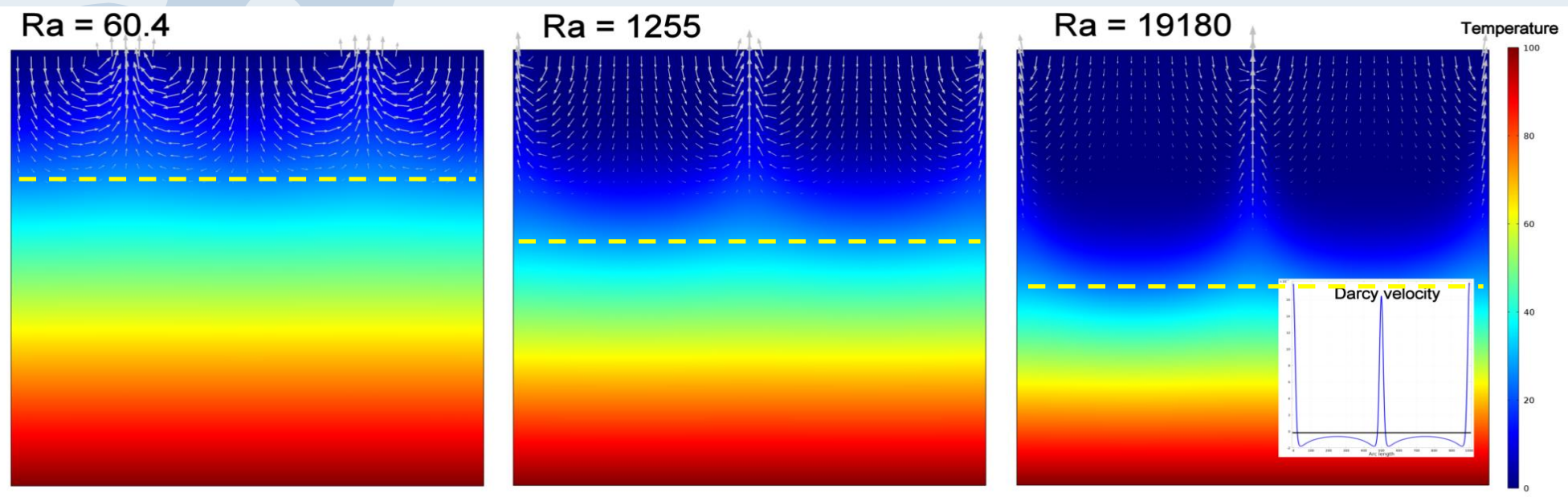
Details approach: Deriving a scaling law $Nu - Ra$ and c_{kthr}



Convection experiments with **permeable** surface, representing regions **near** the ridge axis

- 2-D hydrothermal convection experiments using the FEM code COMSOL
- Depth-dependent permeability
- Dashed lines show penetration depth of porous convection \rightarrow determine z_{hy} , T_{bot} , $k_{\phi bot}$
- Heat flow at surface $\rightarrow Nu(Ra)$

$$k_{\phi}(z) = k_{\phi 0} \cdot \exp\left(-n \cdot \frac{z}{z_{ch}}\right)$$



Details to determine penetration depth, bottom temperature

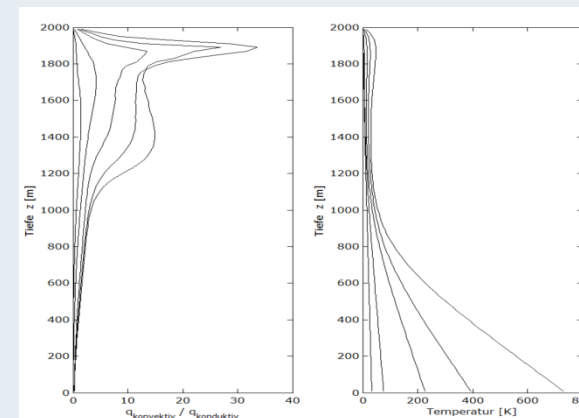
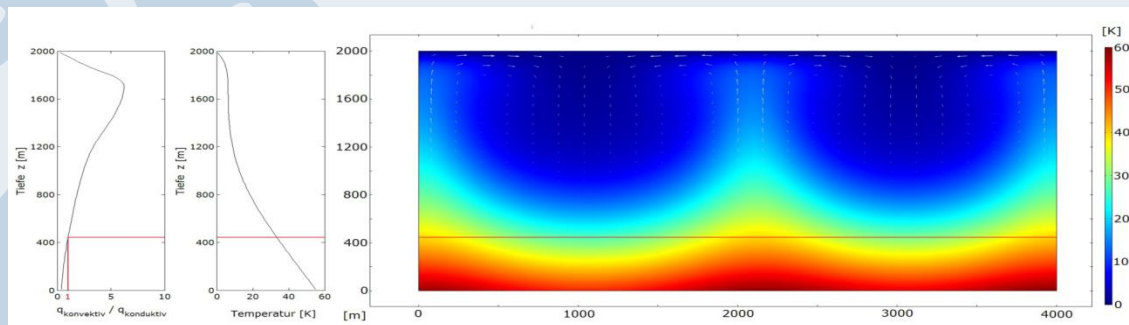


Convection experiments with **impermeable** surface, representing regions **far from** the ridge axis

- 2-D hydrothermal convection experiments using the FEM code COMSOL
- Depth-dependent permeability
- Red lines show penetration depth of porous convection \rightarrow determine z_{hy} , T_{bot} , $k_{\phi bot}$
- Heat flow at surface $\rightarrow Nu(Ra)$

$$k_{\phi}(z) = k_{\phi 0} \cdot \exp\left(-n \cdot \frac{z}{z_{ch}}\right)$$

Click on images to enlarge



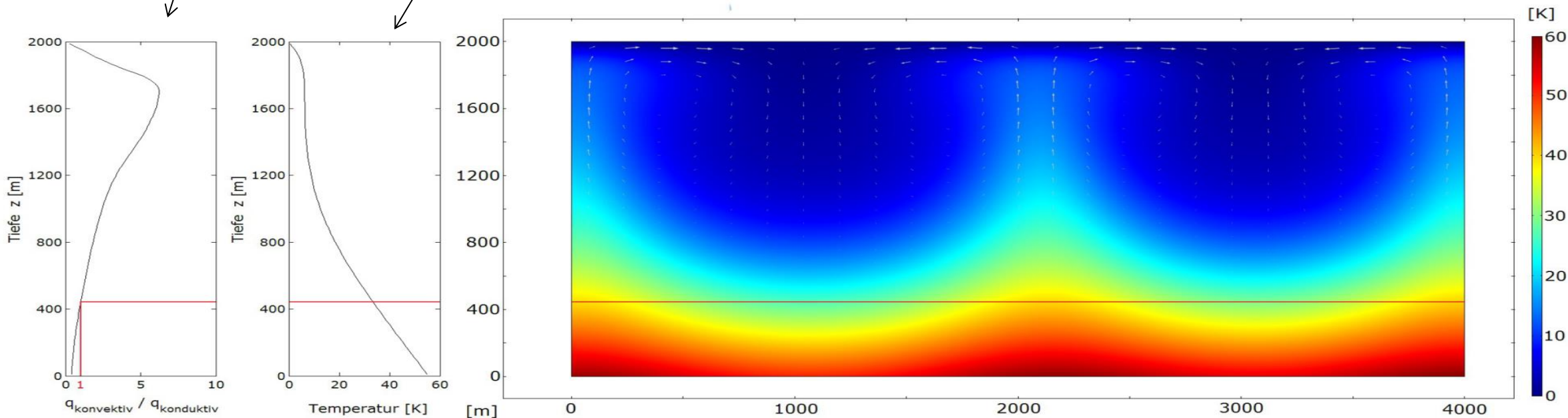
Details to determine penetration depth, bottom temperature



Horizontally averaged advective / conductive heat flow. Convective penetration depth is defined by the depth at which this ratio equals 1

Horizontally averaged temperature used to determine T_{hy}

Example of a 2D model with impermeable top boundary and heat flux bottom boundary condition

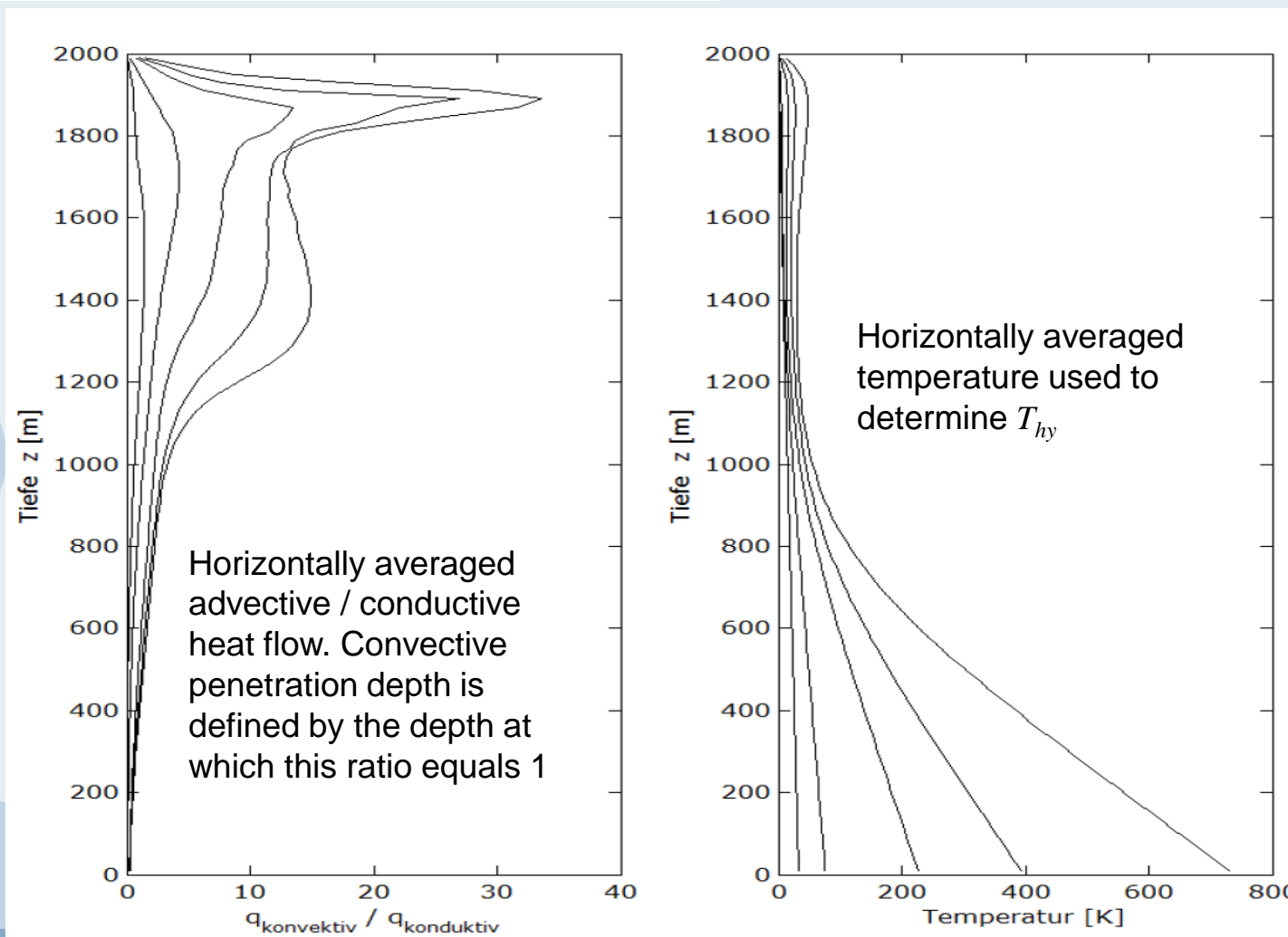


Details to determine penetration depth, bottom temperature

Compilation of profiles for different Rayleigh numbers



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Derived scaling laws

$$k_{\varphi}(z) = k_{\varphi 0} \cdot \exp\left(-n \cdot \frac{z}{z_{ch}}\right)$$

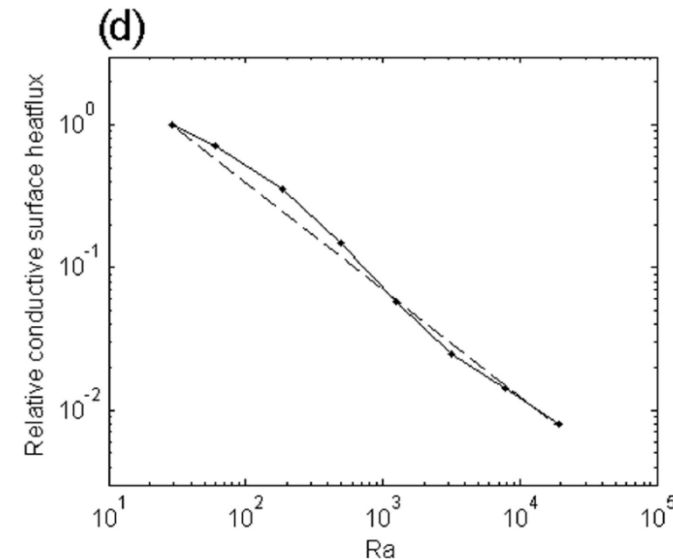
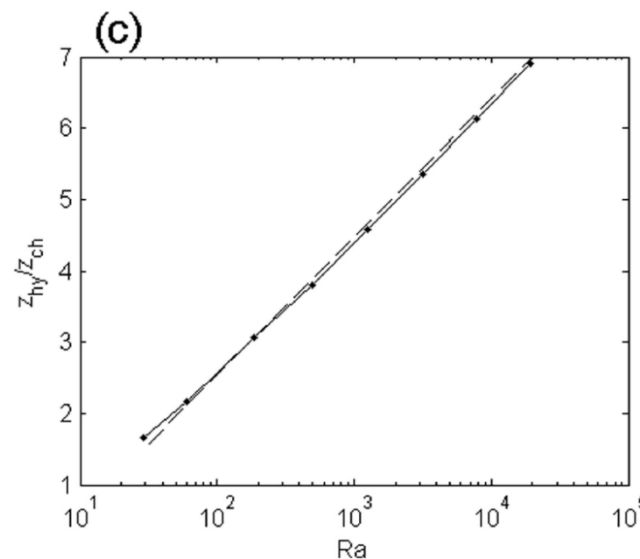
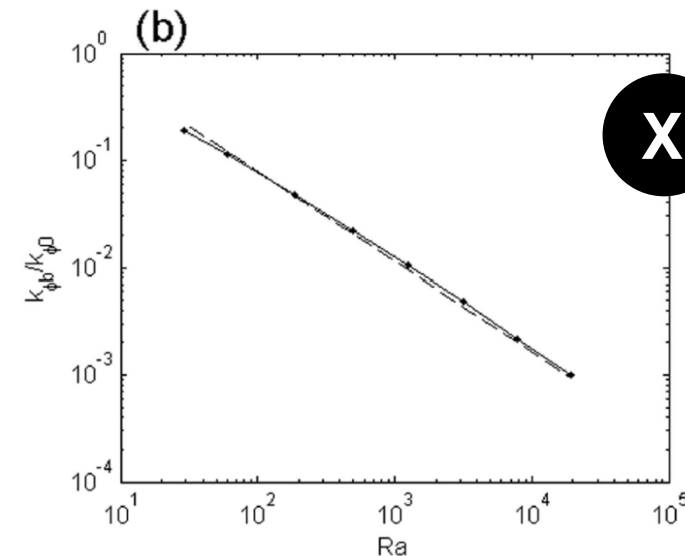
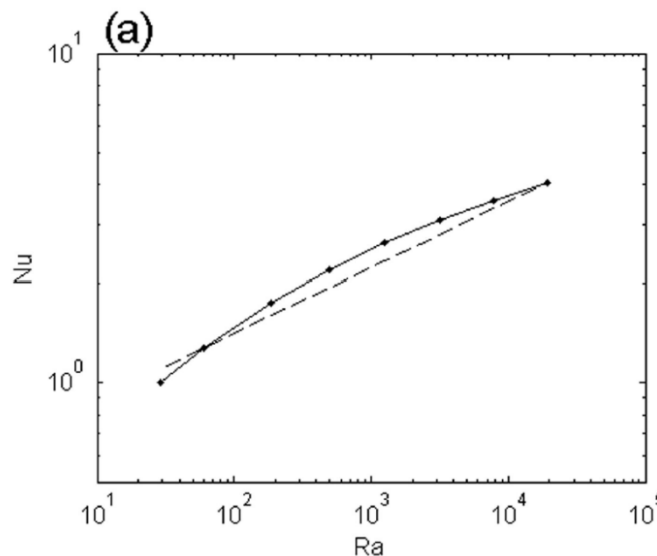
From the numerical experiments scaling laws for hydrothermal convection with depth-dependent permeability are derived.

- They show a surprisingly low exponent of 0.2 for the Nu – Ra relationship
- With increasing Ra the penetration depth increases
- With increasing Ra the conductive (measurable) contribution to the total heat flux decreases

$$\bar{Nu} = c_{Nu} Ra_{hy}^{\beta_{Nu}}, \quad \beta_{Nu} = 0.2$$

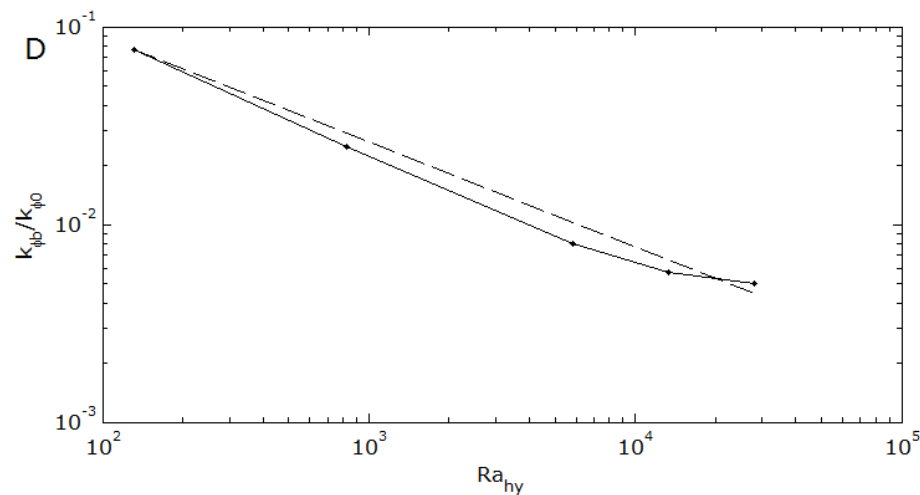
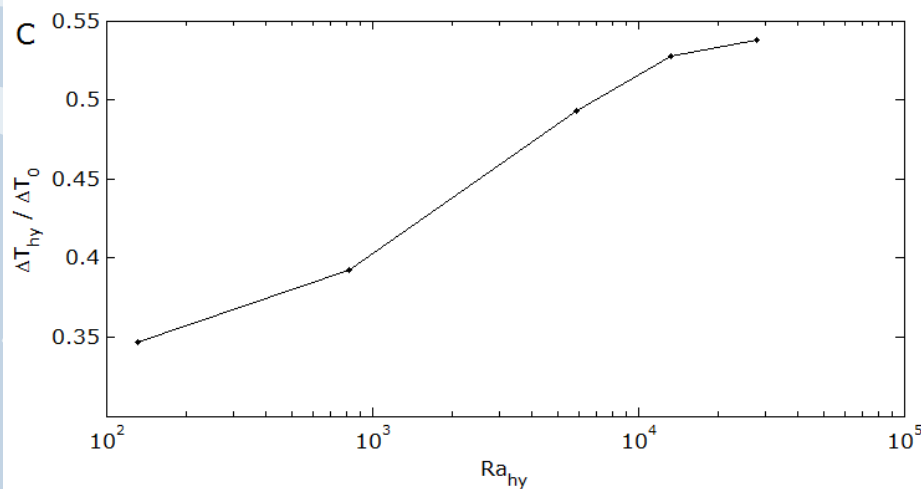
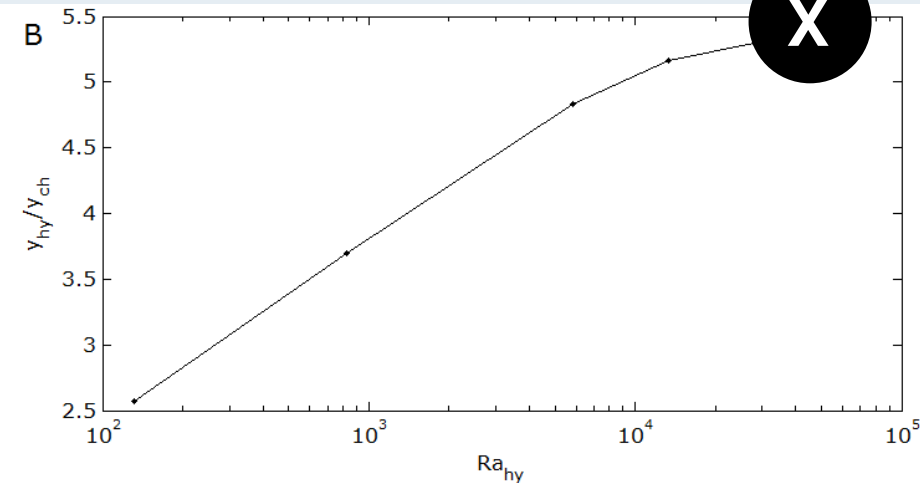
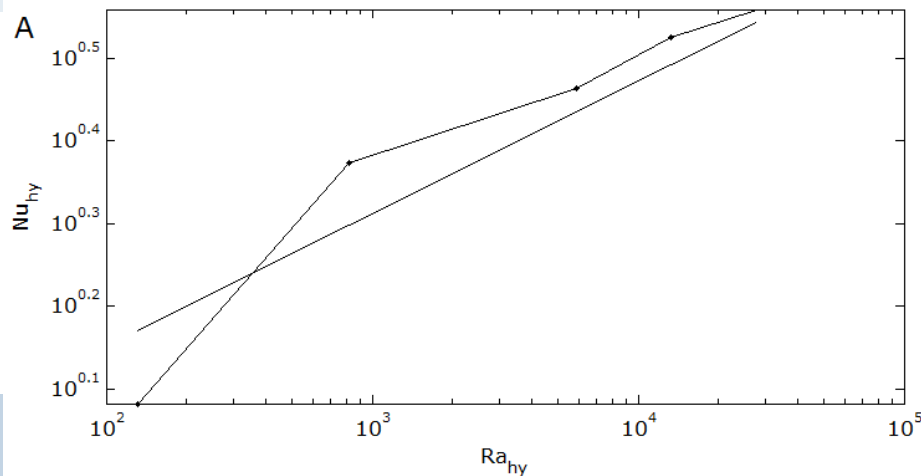
$$k_{\varphi b} / k_{\varphi 0} = c_k Ra_{hy}^{\beta_k}$$

[Click here for the impermeable case](#)



Derived scaling laws

Similar scaling laws are derived for experiments with impermeable upper boundary. They show a somewhat weaker magnitude of convection.



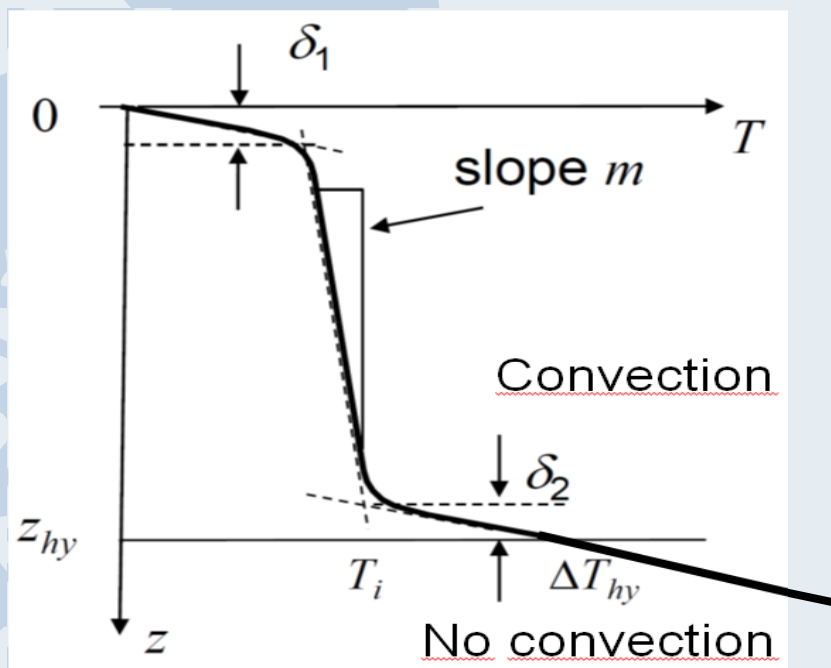
Equivalent thermal conductivity



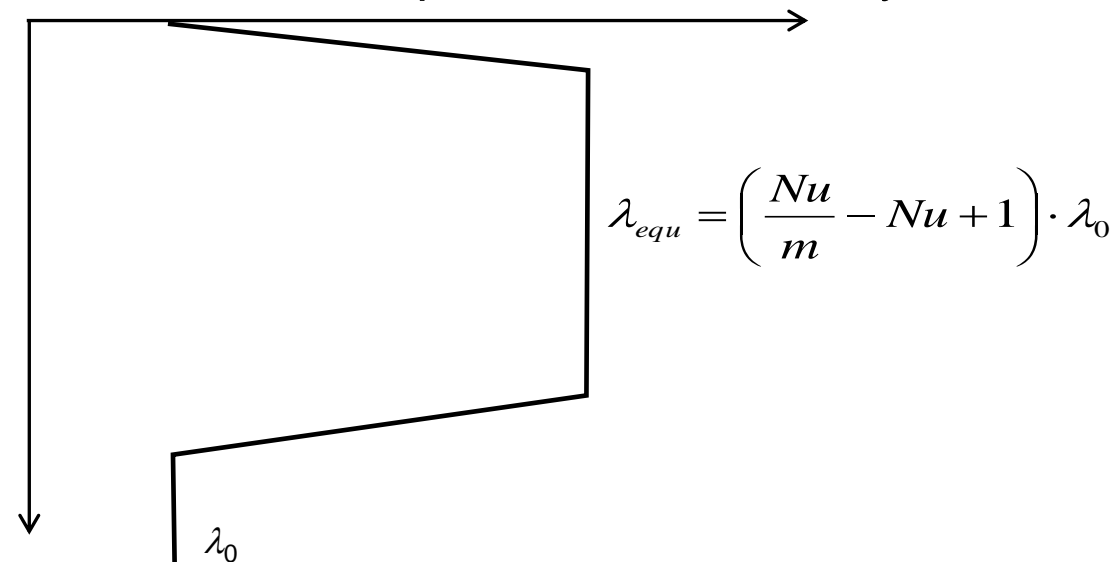
Given the Rayleigh number, z_{hy} and Nu are obtained from the scaling laws and are used to determine an equivalent thermal conductivity within the depth interval down to z_{hy} .

→ Approximate convective layer by conductive layer with an effective, higher thermal conductivity

The thermal boundary layers are correctly included by the transition regions between λ_0 and λ_{equ} . A finite slope m is assumed avoiding numerical problems in case of high Rayleigh number.



Effective, equivalent conductivity



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Solving the conductive 1D heat equation with $\lambda_{equ}(z,t)$ based on Nu, z_{hy} etc from parameterized hydrothermal convection

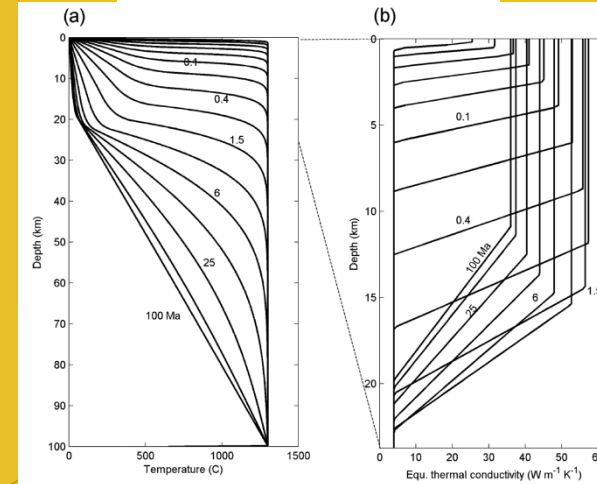
$$\frac{\partial T}{\partial t} = \frac{1}{\rho c_p} \frac{\partial}{\partial z} \left[\lambda_{eq}(T, z) \frac{\partial T}{\partial z} \right]$$

Controlling parameters:

- $k_{\phi 0}$ surface permeability ($8 \cdot 10^{-14} \text{ m}^2$ (2.4 %))
- a_r Crack aspect ratio (0.01)
- Initial temperature 1300°C
- Surface temperature 0°C

Results: click on figures

Temperatur profiles



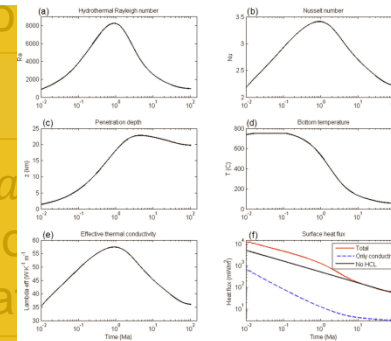
Result

Cooling plate with hydrothermal convection

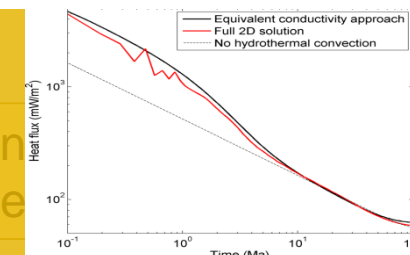
Sedimentary sealing of the ocean floor at a given age

Deviation from $\sqrt{t} - \text{law}$
 → Measurable heat flux
 → Increased total heat flux

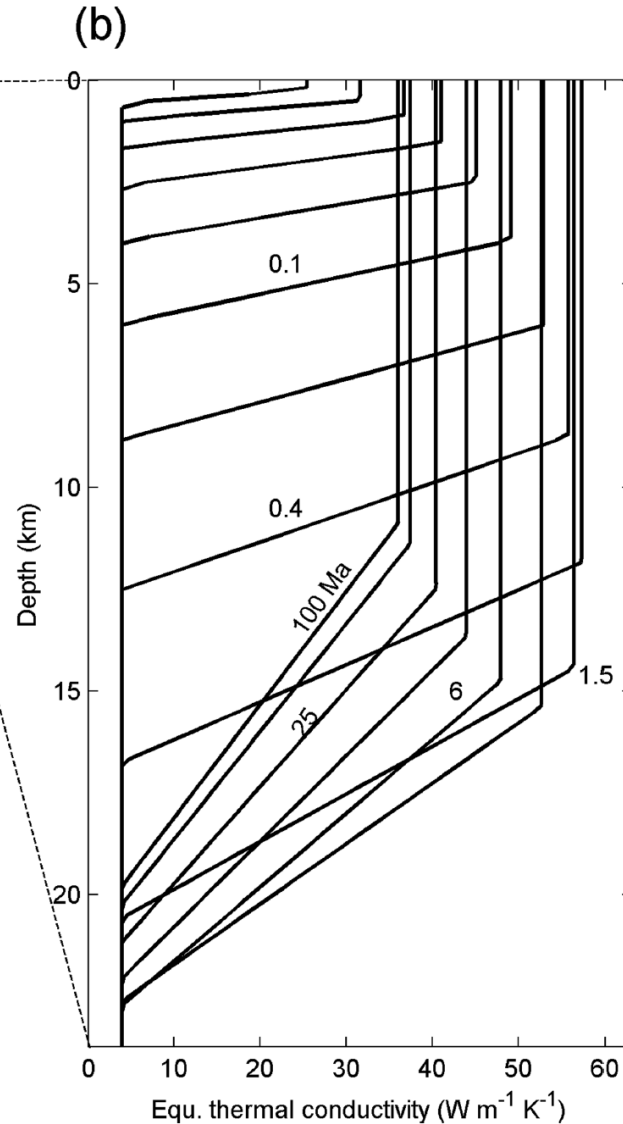
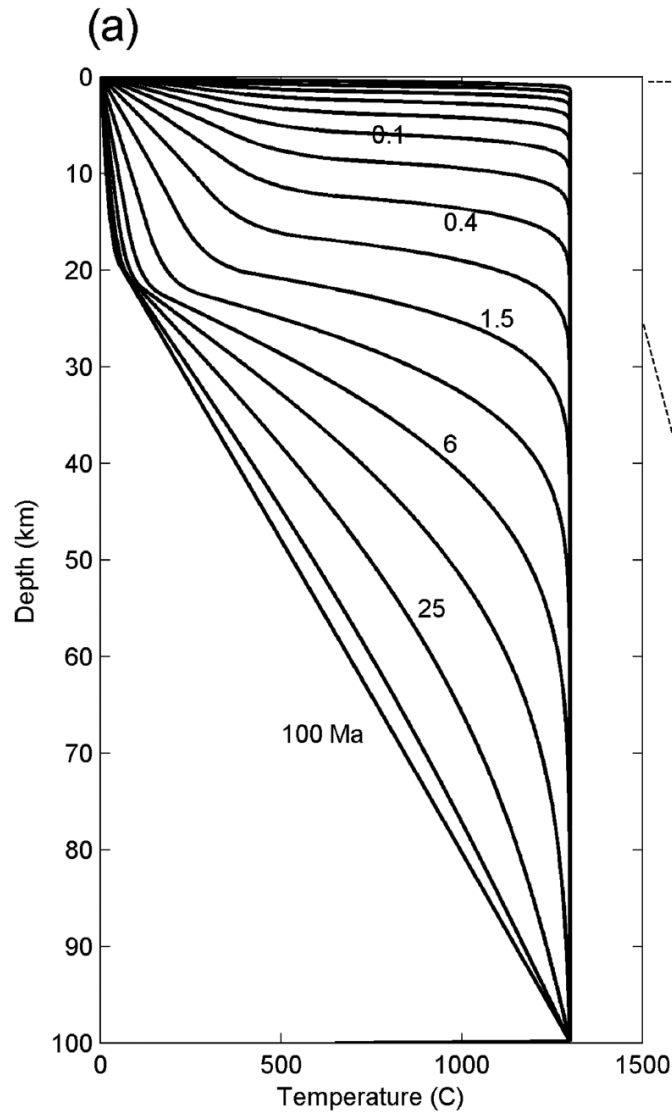
Evolution of parameters



Comparison with full 2D solution



Example of cooling plate with simulated hydrothermal convection



Phase 1 (0 Ma)

Plate is hot, no hydrothermal convection

Phase 2 (0.02 – 1.5 Ma)

Cooling opens pore space \rightarrow convection layer thickens \rightarrow Ra increases \rightarrow effective conductivity increases \rightarrow shallow cooling increases

Maximum effective conductivity is reached

Phase 3 (2 Ma)

Maximum penetration depth is reached due to pressure effect on permeability

Phase 4 ($> 2\text{Ma}$)

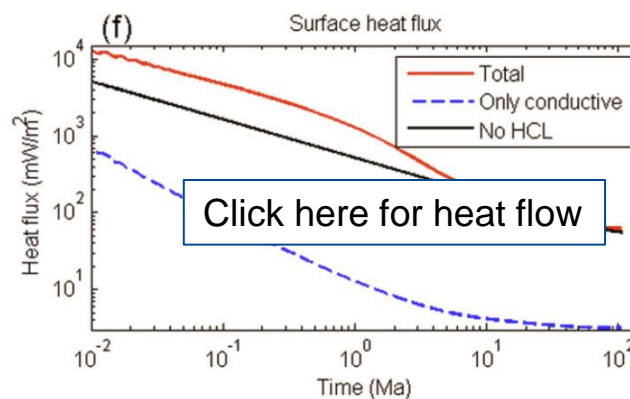
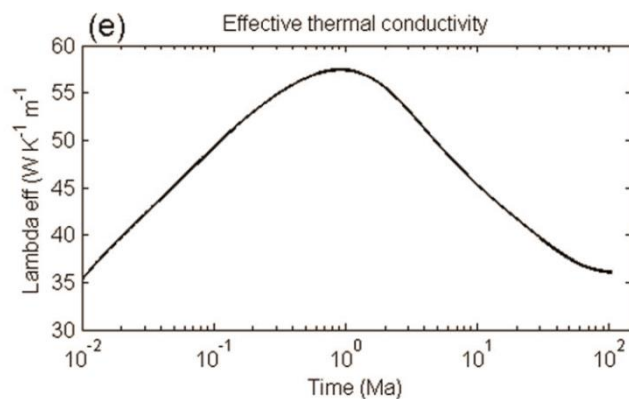
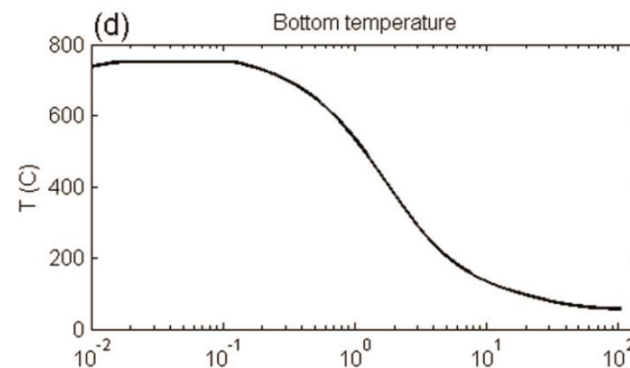
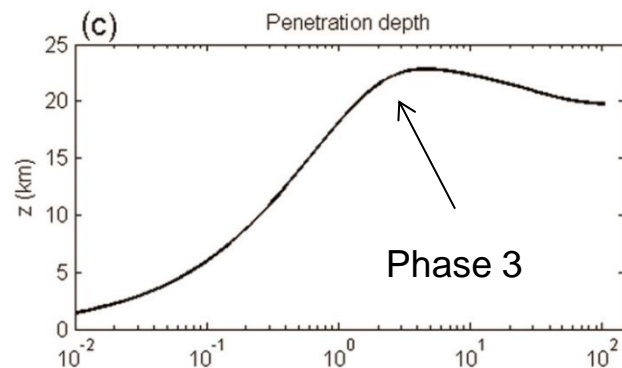
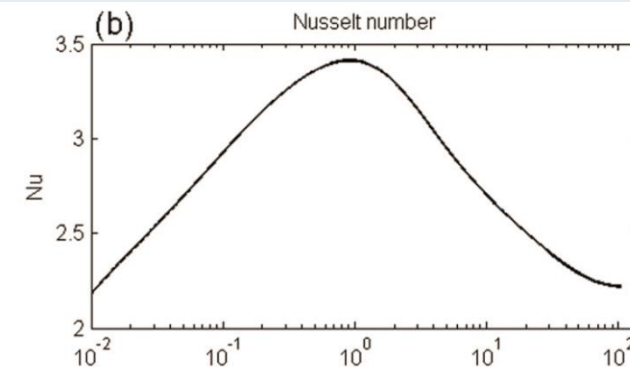
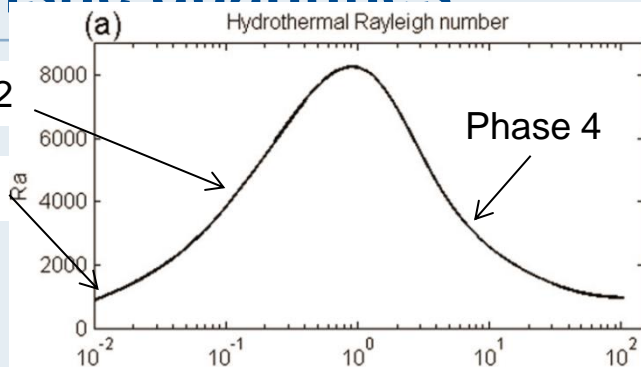
Bottom temperature of convection layer cools \rightarrow Ra drops \rightarrow hydrothermal convection slows down, but remains active even at late ages

Evolution of characteristic quantities

Evolution of hydrothermal Rayleigh number, penetration depth, Nusselt number and bottom temperature shows the 4 phases.

Phase 2

Phase 1



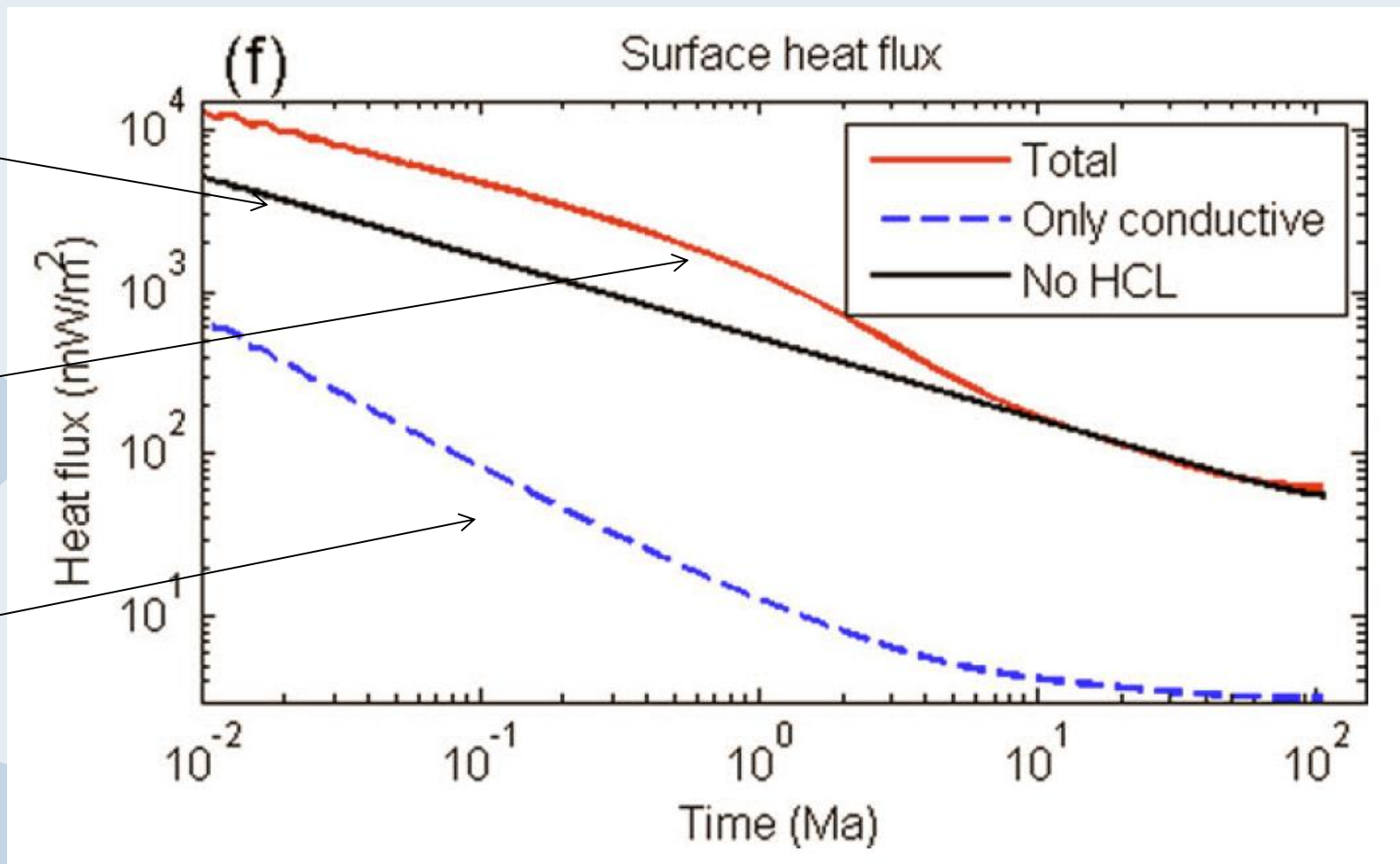
Evolution of characteristic quantities



Black curve shows the surface heat flux in the absence of hydrothermal convection. It follows the square root t law. At 100 Ma little flattening due to finite plate thickness

Red curve shows the total heat flux, which is increased due to hydrothermal convection. Between 1 – 10 Ma a clear deviation from the square root t law is evident

Blue dashed curve shows the conductive contribution of the heat flux, i.e. most of the heat is lost by advection. Through the top boundary. This is the heat flux measurable by thermal gradients



Comparison with full 2D solution

The red curve shows the full 2D solution obtained with the FEM code COMSOL for a cooling plate including the porous convection equations:

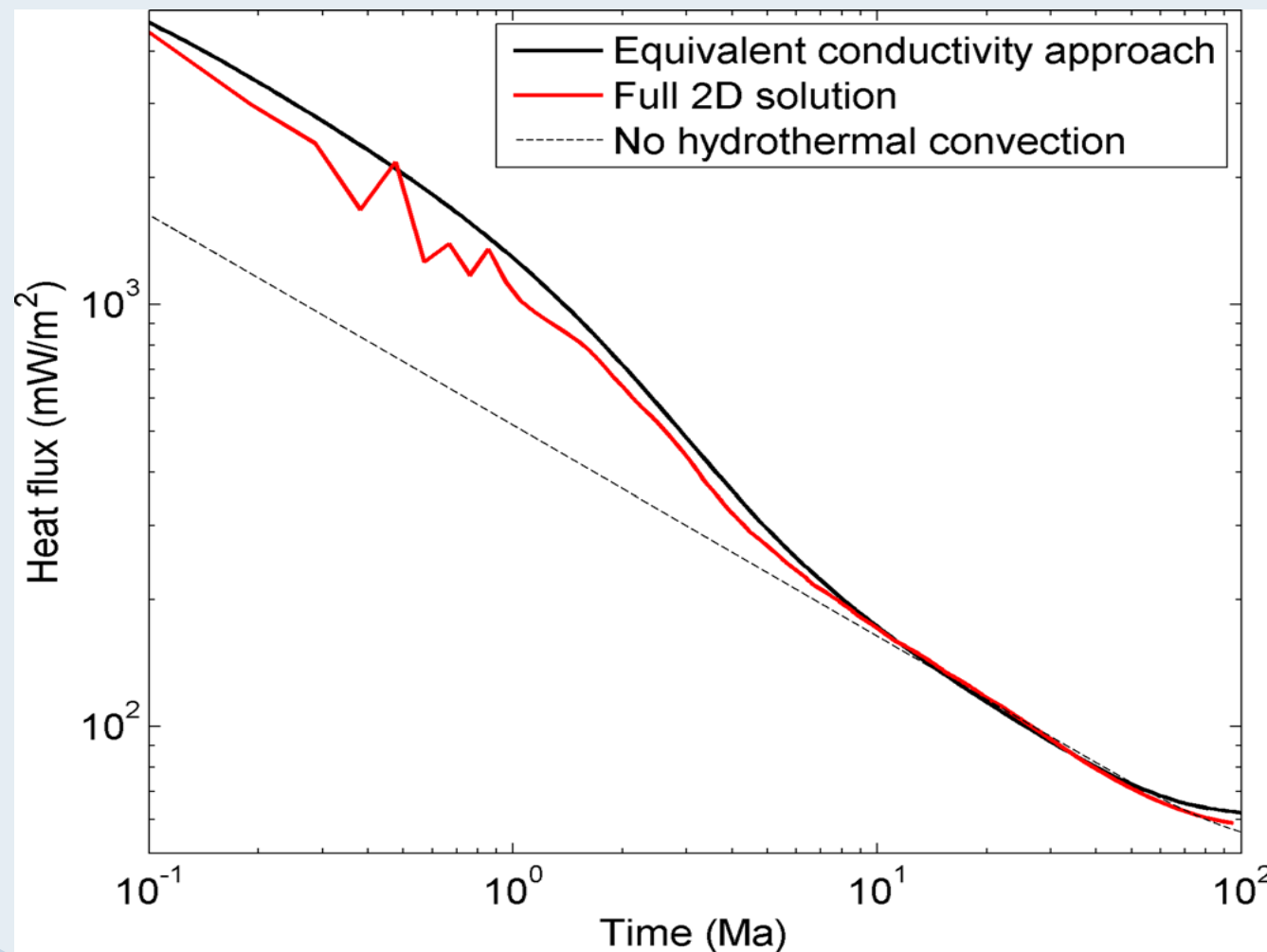
$$\rho_0 c_p \frac{\partial T}{\partial t} + \tilde{\rho}_f c_{pf} \vec{u} \cdot \vec{\nabla} T = \lambda_m \nabla^2 T,$$

$$\vec{\nabla} \cdot \tilde{\rho}_f \vec{u} = 0,$$

$$\vec{u} = -\frac{k_\phi}{\eta_f} \left(\vec{\nabla} P_f - \tilde{\rho}_f g \vec{e}_z \right),$$

$$\tilde{\rho}_f = \rho_f [1 - \alpha_f (T - T_0)],$$

The agreement with the approximate solution using the equivalent thermal conductivity is satisfactory

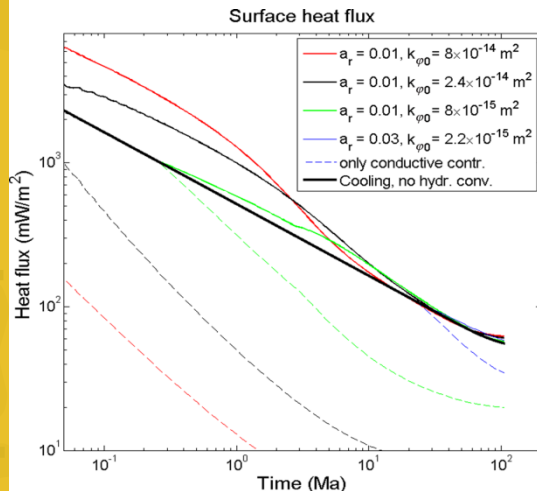


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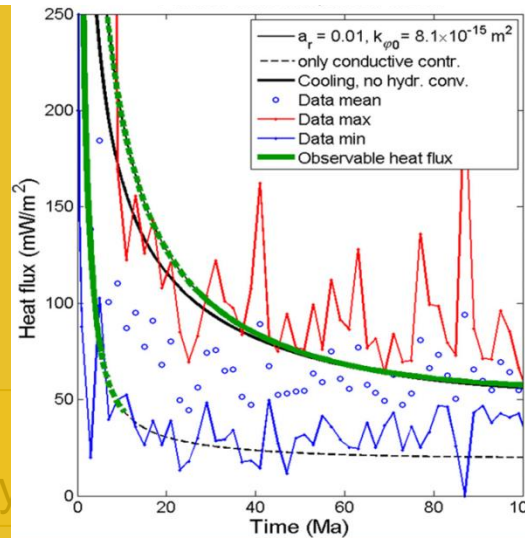
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Strong deviation from \sqrt{t} - law for various porosity parameters

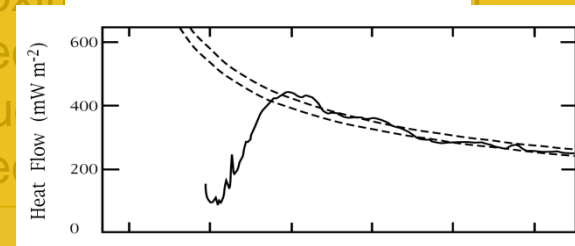


Comparison with heat flow data suggests importance of hydrothermal convection with a transition from permeable to impermeable surface conditions

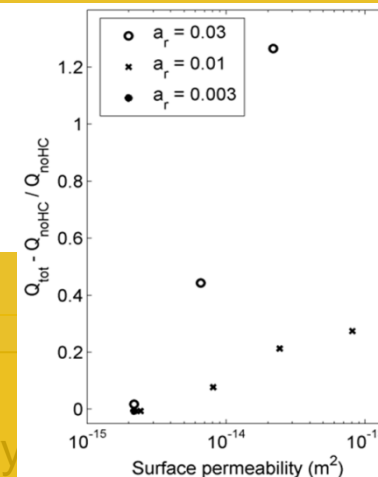


New approach
Approximate
convection
conduction
convection

A local comparison



Total heat loss is significantly increased compared to the conductive \sqrt{t} - law



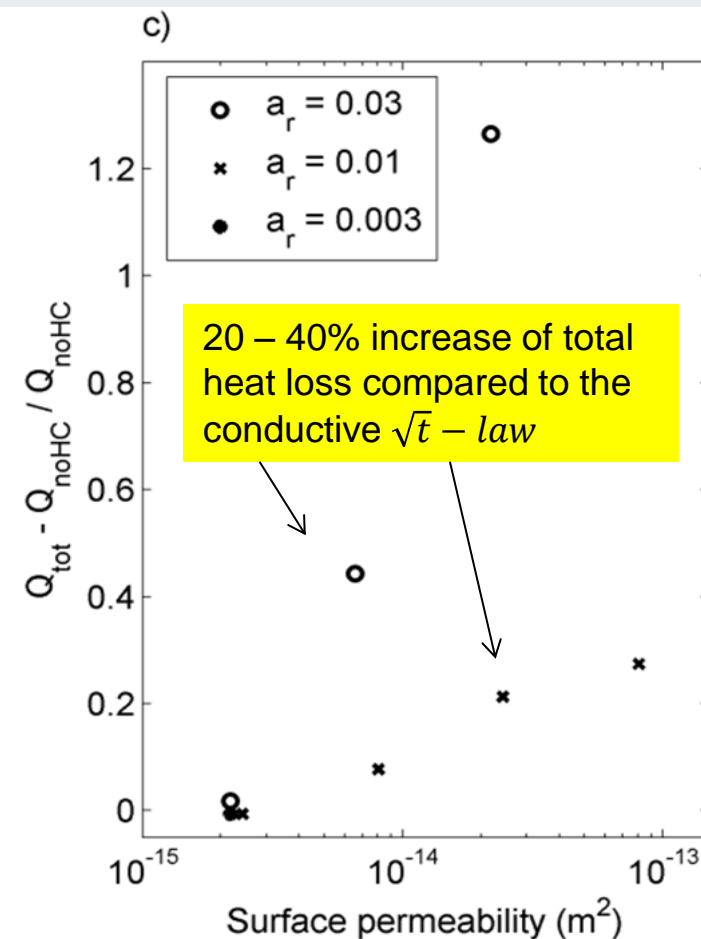
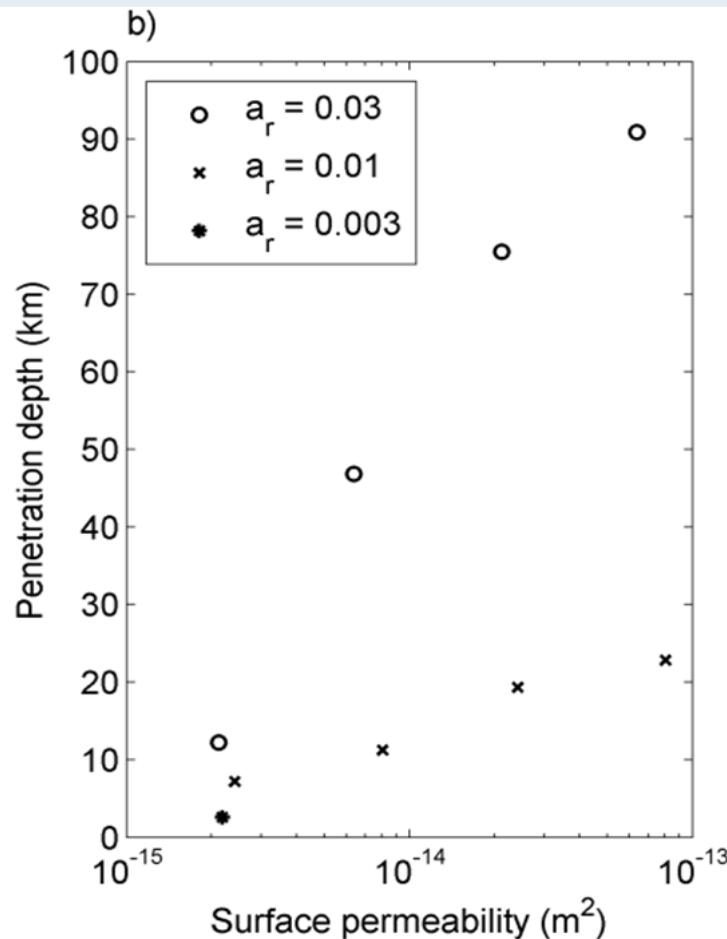
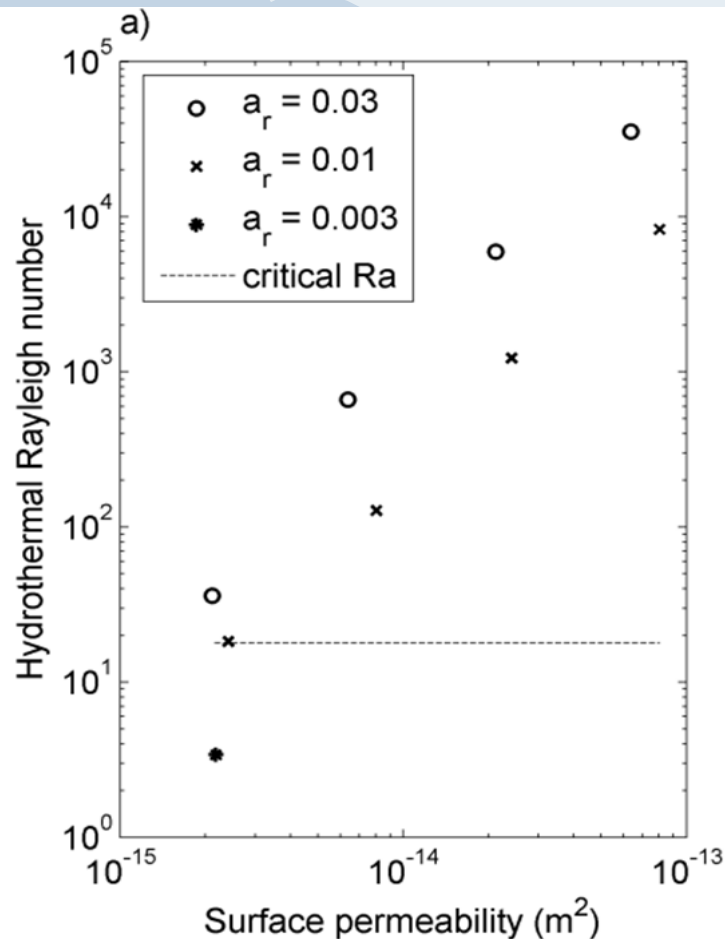
Sedimentary sealing of the ocean floor at a given age

Deviation from \sqrt{t} - law:
→ Measurable heat flow
→ Increased total heat loss

Effect on Bathymetry

Varying porosity parameters

Here the effect of varying porosity parameters on the hydrothermal rayleigh number, penetration depth and excess heat loss is shown



\sqrt{t} – law ?

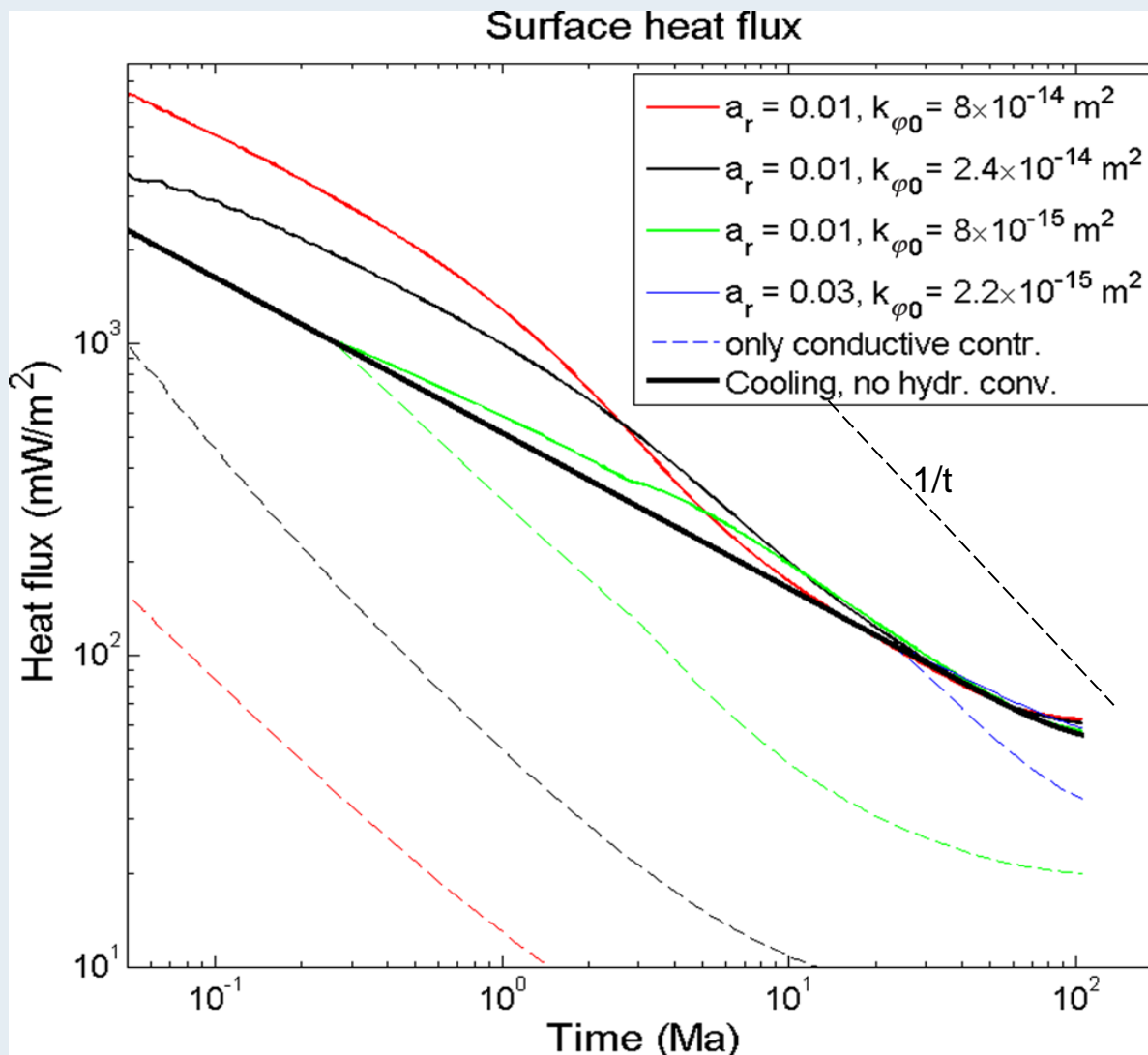
Strong deviation from \sqrt{t} – law for various porosity parameters:

The crack aspect ratio a_r and the surface permeability $k_{\varphi 0}$ are varied. They influence the magnitude of the deviation from the \sqrt{t} – law as well as the timing.

Compare the slopes with the $1/t$ slope

Effective cooling due to hydrothermal convection reduces the total cooling time, which leads to an earlier onset of flattening due to a finite plate thickness at already 60 Ma.

Dashed colored curves give the measurable conductive contribution



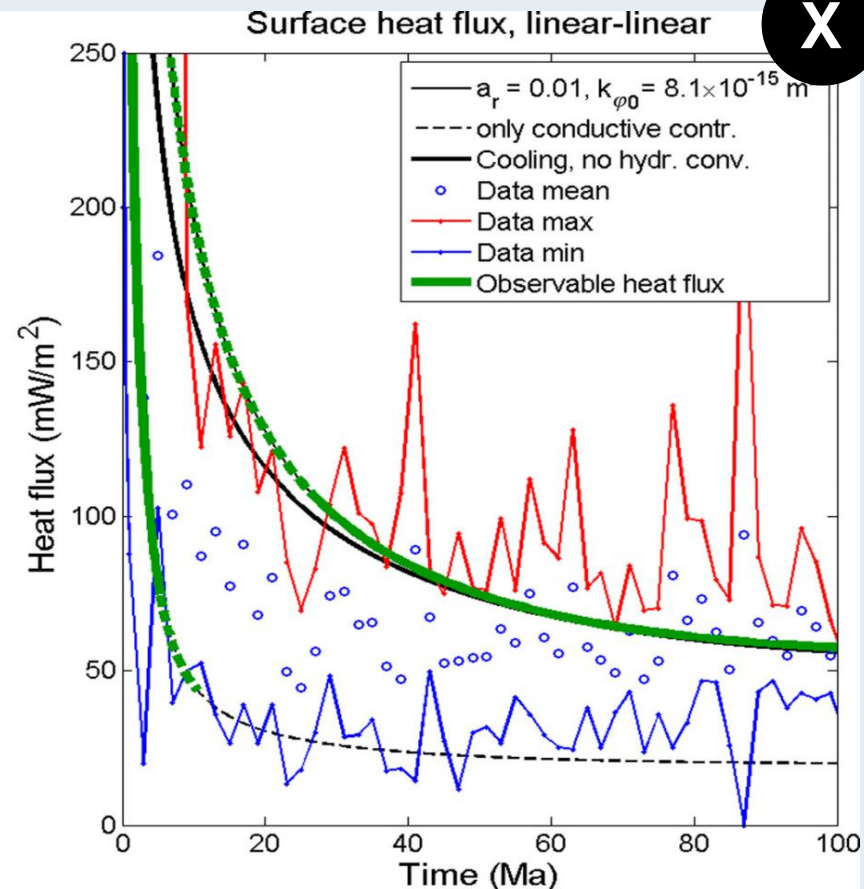
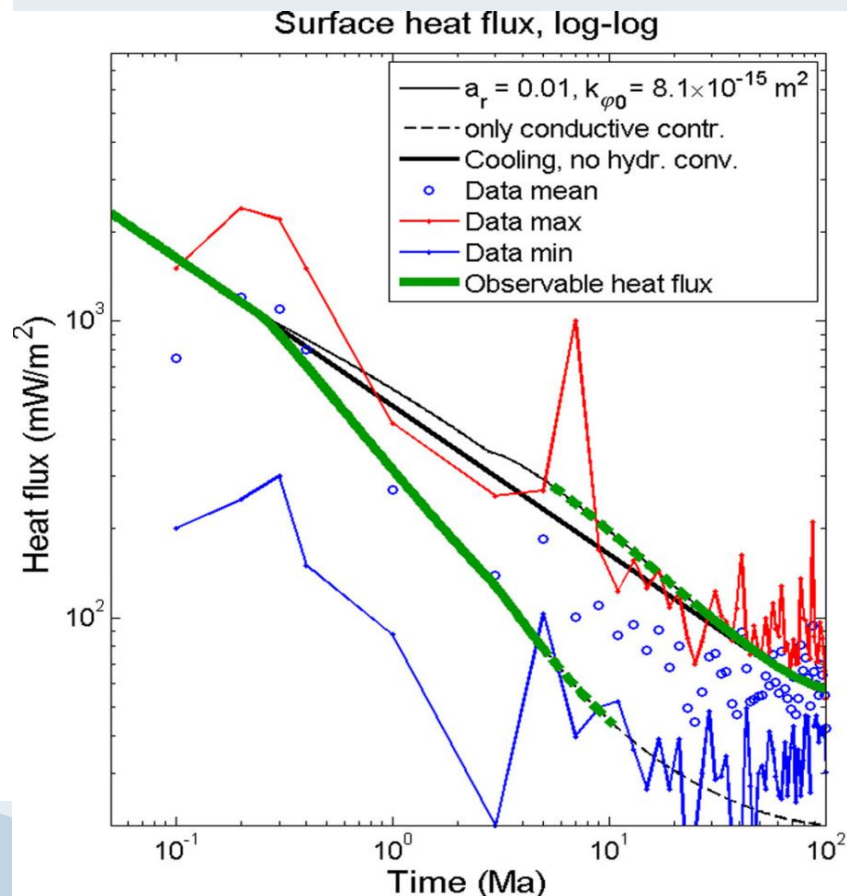
\sqrt{t} – law , comparison with data

Model and observational data from Stein and Stein (1994) shown in a log-log and a linear – linear plot

Global heat flow measurements plotted as a function of age (dots) with their upper (red) and lower (blue) bounds.

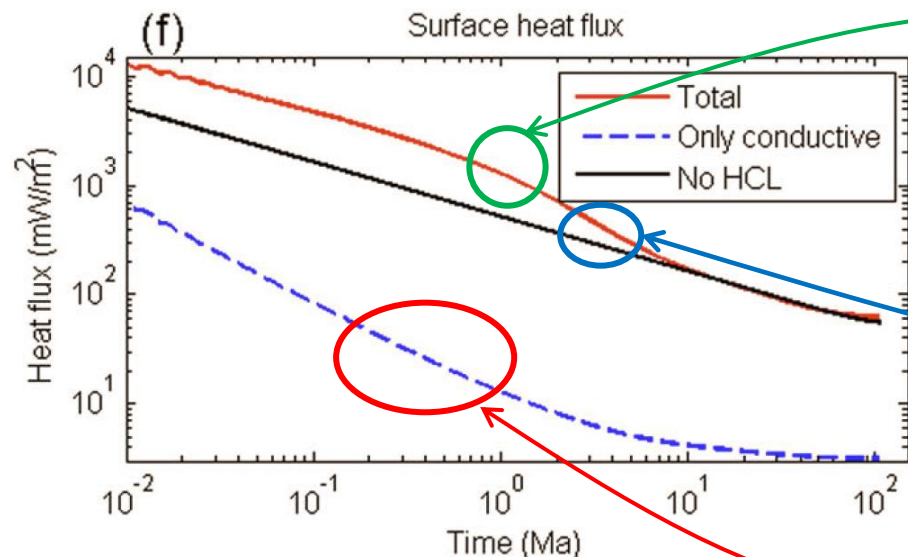
Black (thin) curve shows the total heat flux of a model with hydrothermal cooling, dashed (thin) is its measurable, conductive contribution.

The green thick curves indicated the curve sections which are observable in case of a permeable surface at young plates, and an impermeable surface at older ages in case of a sealing sediment layer

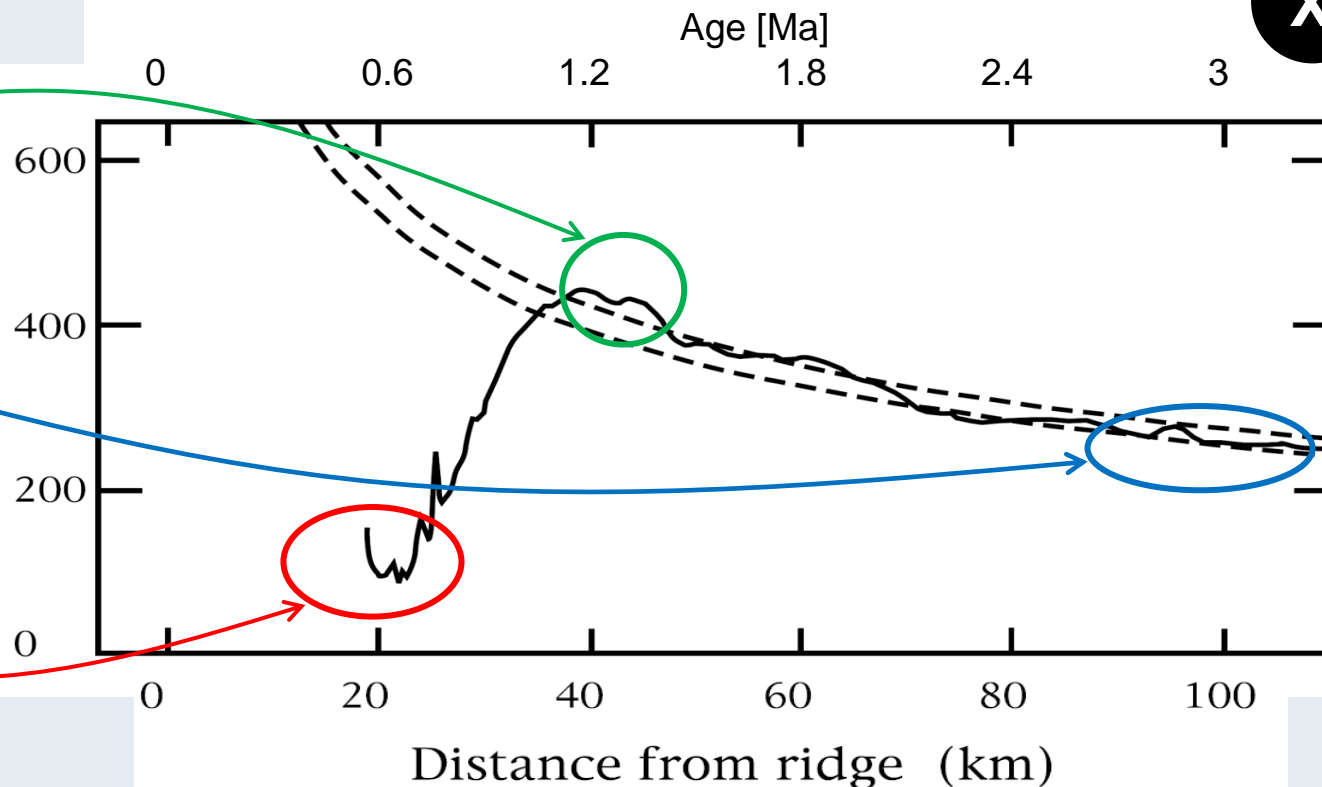


Comparison with a heat flow profile near the Juan de Fuca ridge

Typical model



Observations (Davies et al. 1999)



Tentative correspondence between model and observation indicated by arrows

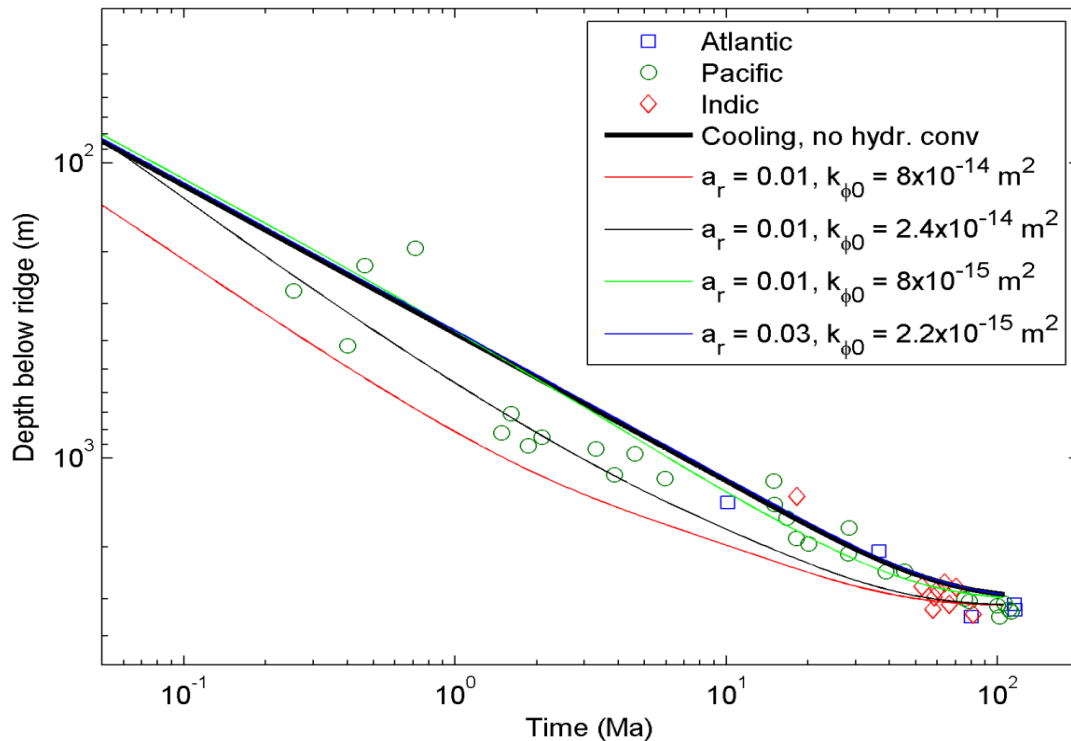
- At 0.6 Ma convective contribution not measurable
- At 1.2 Ma sedimentary sealing allows the total convective + conductive heat become measurable
- At 3 Ma hydrothermal contribution decreases

Fig. 4. High resolution heat flow profile near the Juan de Fuca ridge. From *Davis et al. [1999]*. Dashed lines stand for two predictions of the half-space cooling model with constant C_Q in the $\tau^{-1/2}$ heat flux-age relationship equal to 470 and 510 (with heat flux in mW/m^2 and age in Ma).

Hydrothermal cooling of the oceanic lithosphere and the square root age law

H. Schmeling¹ and G. Marquart²,
¹Goethe University Frankfurt, ²RWTH Aachen

Comparison of global bathymetry as a function of age with cooling models with hydrothermal convection for various parameters



The subsidence equation including the effect of porosity

$$w = \frac{\rho_m \alpha_l}{\rho_m - \rho_f} \left(T_m h - \int_0^h T dz \right) - \int_0^h \phi dz$$

Bathymetry of the Atlantic, Pacific and Indic show significant deviations from the \sqrt{t} - law. Between 0.3 – 1Ma the bathymetry seems to subside faster, between 1 – 15 Ma it seems to subside slower than with the \sqrt{t} - law. Models with hydrothermal cooling show a faster qualitatively a similar behavior.

Effect on Bathymetry

given age

→ Increased total heat loss

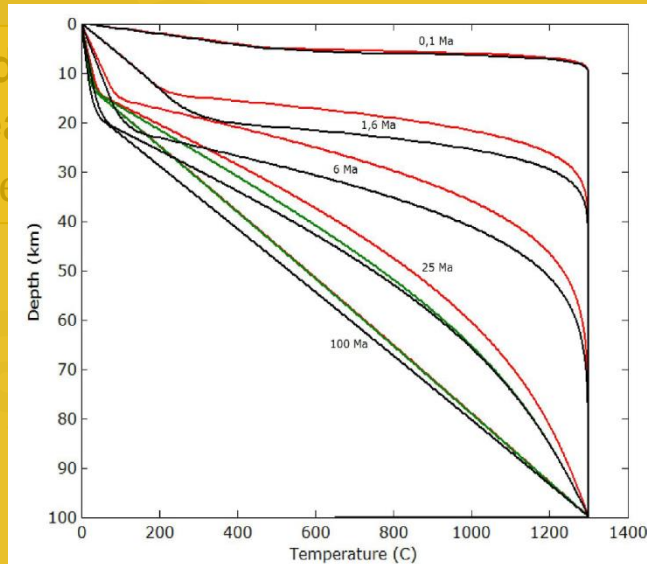
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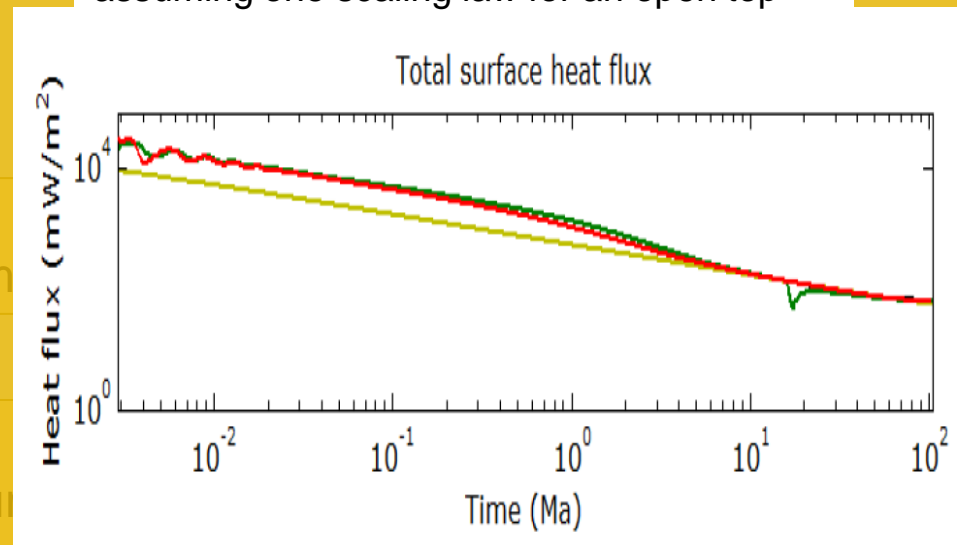


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Two scaling laws are used: one with open surface boundary, one with impermeable boundary

The effect is not very strong compared to assuming one scaling law for an open top

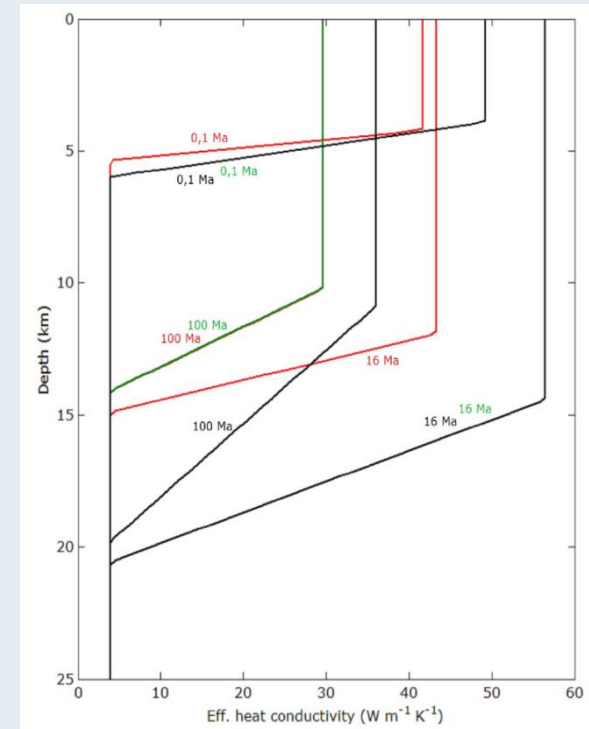
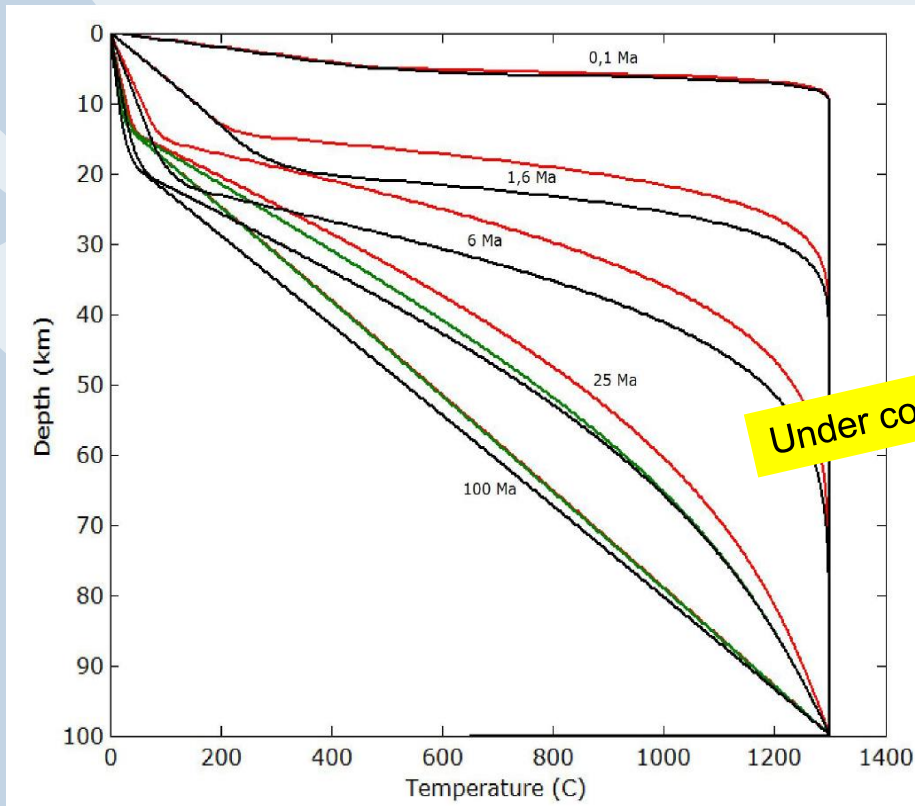


Cooling plate with hydrotherm

Sedimentary sealing of the ocean floor at a given age

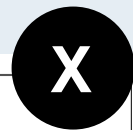
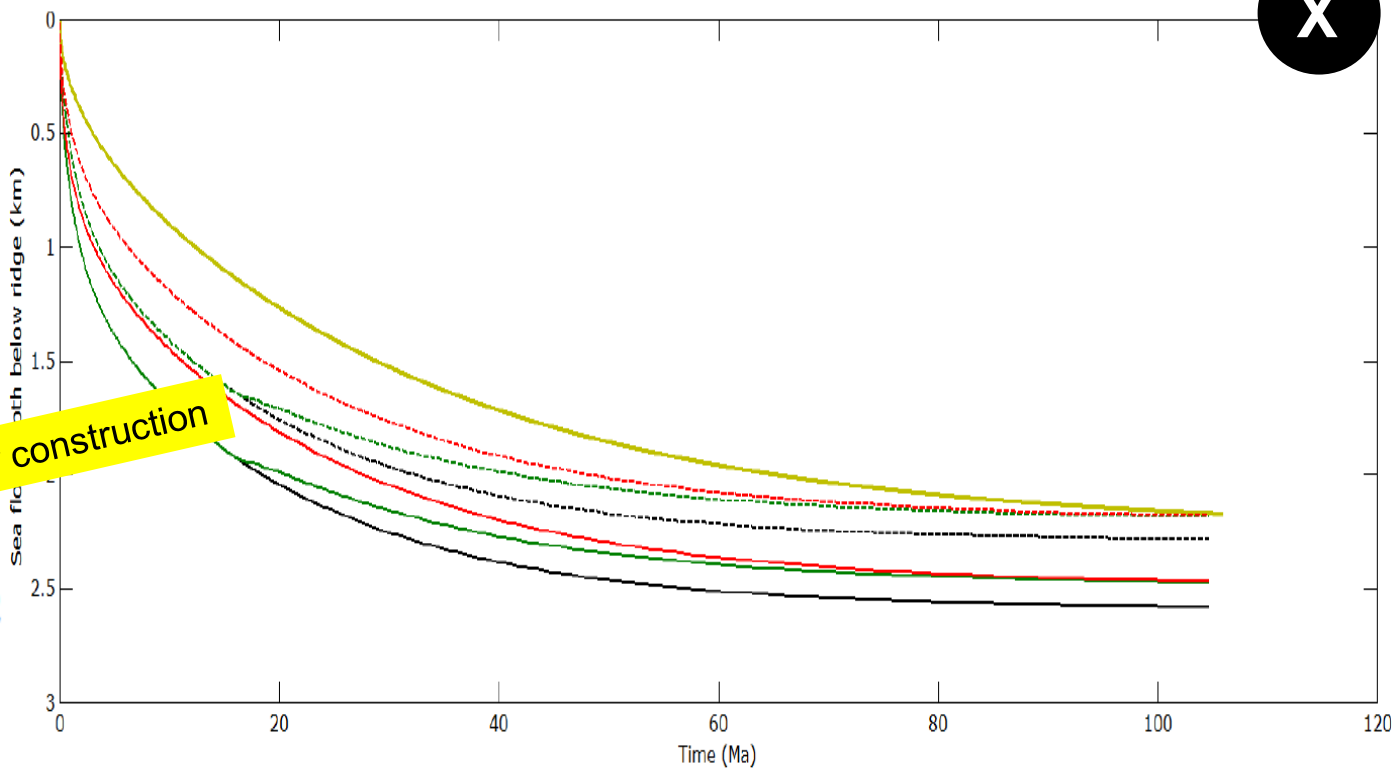
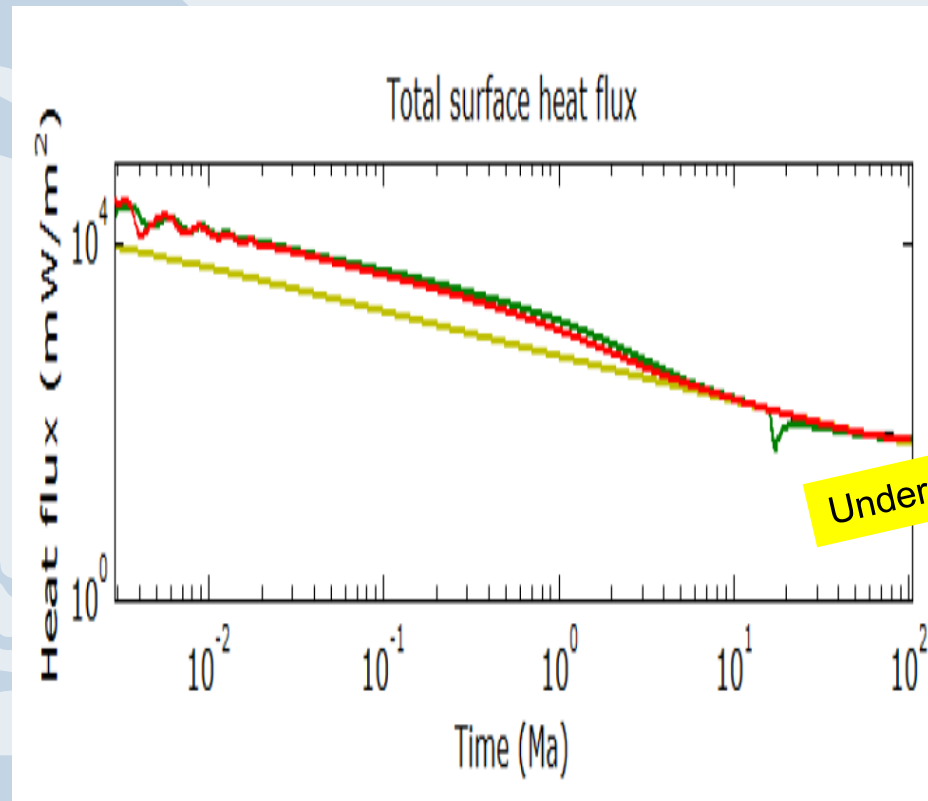
Deviation
→ Measur
→ Increased total heat loss

Changing cooling due to sediment cover at 15 Ma



Changing cooling due to sediment cover at 15 Ma

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Conclusion

- ...for the first time: Cooling plate model with self-consistent cooling due to parameterized hydrothermal convection (HC)
- HC strongly affects \sqrt{t} - law:
 - Total heat loss higher than without HC
 - Heat flux vs. age: steeper than $1/\sqrt{t}$ slope (up to $1/t$) for young lithosphere,
 - Flatter slope for older lithosphere
- HC also important for old parts of plates
- Scaling law with depth-dependent permeability:
- Future improvements:
 - crack closure due to plastic deformation
 - sediment cover
 - crack closure due to cementation

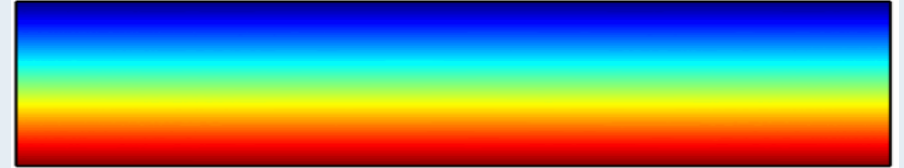
$$Nu \propto Ra_{hy}^{0.2}$$

Comparison

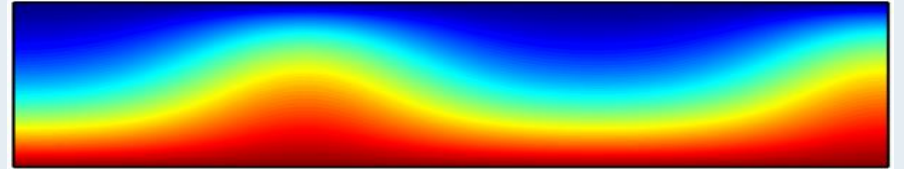
For comparison:
Constant permeability,
constant T at bottom

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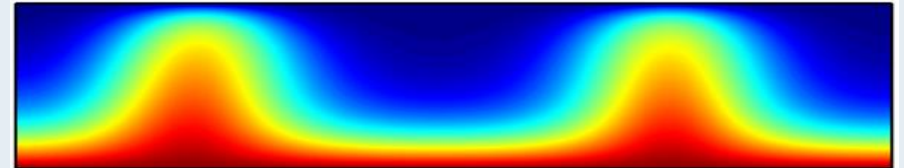
Ra = 29



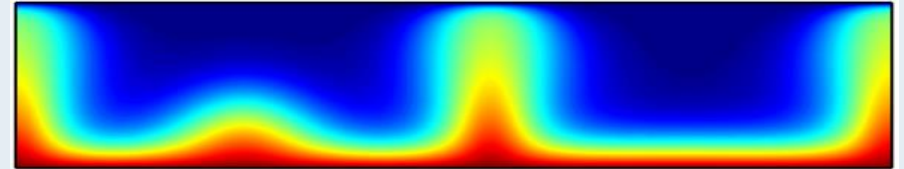
Ra = 30



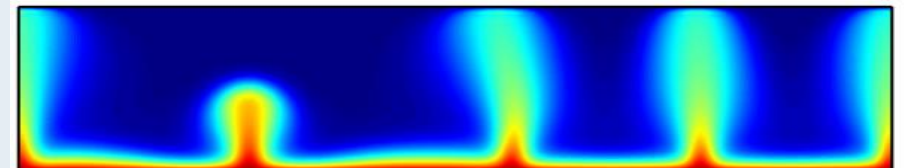
Ra = 50



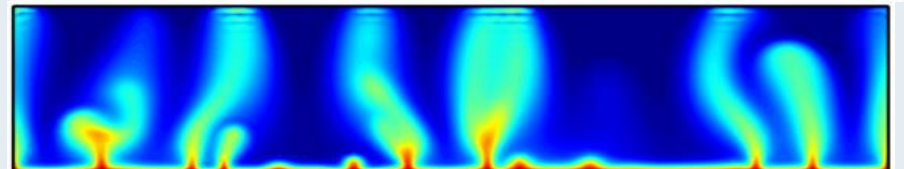
Ra = 100



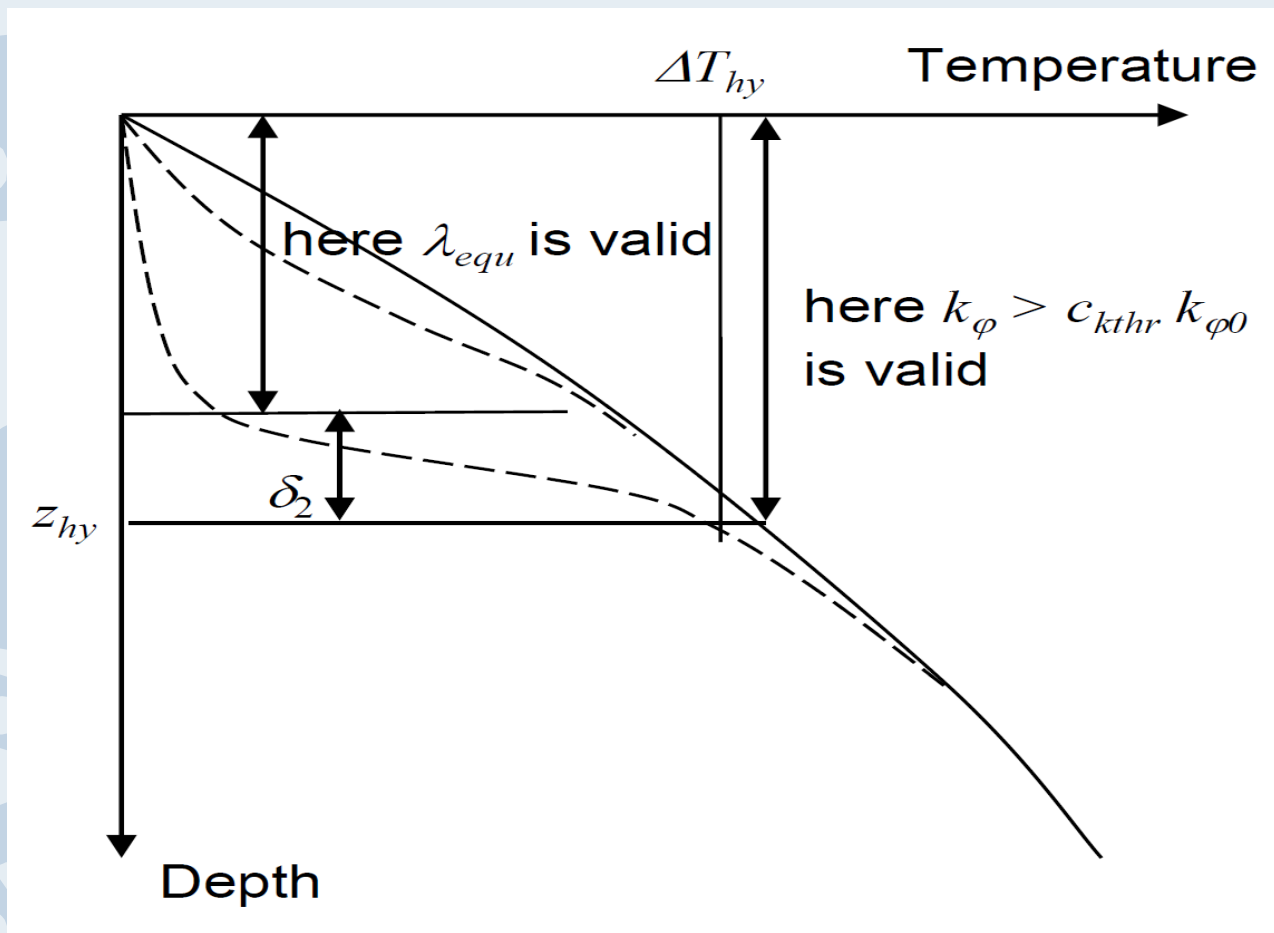
Ra = 300



Ra = 1000



Applying the concept of equivalent thermal conductivity



c_{kthr} allowing for hydrothermal convection (HC)?

→ Scaling law from numerical models of HC

