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Pressure and volume changes in magmatic systems due to the vertical displacement of compressible materials

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Abstract

The problem of a pressure or volume increase due to the vertical redistribution of two substances with differing compressibilities is considered and compared with a previous treatment. Modified expressions for the pressure and volume changes are obtained using the general equation of state with constant compressibility. The effects of gas and melt compressibilities, and also of the solubility of gas in floating bubbles, on pressure and volume changes in magmatic systems are discussed. Very different conclusions from those in the previous treatment are found.

1. Introduction

This study was motivated by a recent paper (Sahagian and Proussevitch, 1993) dealing with 'an advective overpressure' in volcanic systems. The idea that these authors applied to explain the physics of gas overpressure in magma chambers consisted of a pressure increase due to the transport of a more compressible material (gas) from depth to the surface of a vessel of fixed volume. This approach is not new, and in a follow-up comment (Sahagian, 1993) one of the authors referred to the literature already existing on this subject (e.g., Steinberg et al., 1989), though his list of references is not complete. Here, I briefly review the physical phenomenon that was discussed in these earlier publications, which were aimed at understanding the physics of volcanic gas explosions. To the author's knowledge the first publications on this subject were by Czekaluk (1961, 1968), who analyzed the thermodynamics of gas solubility in a vertical liquid column in a gravity field (retrograde solubility). He described the same type of experiment as in Sahagian and Proussevitch (1993), but used mercury and an air bubble in a closed steel cylindrical vessel equipped with a manometer. As a result of the air bubble rising from the bottom to the top, the pressure in the vessel increased by $\rho g H$ where ρ is the liquid density, g is the acceleration due to gravity and H is the height of the vessel. Czekaluk argued that this mechanism of pressure increase can explain the development of overpressure in oil and gas deposits. Later, in a publication by Knothe and Litwiniszyn (1975), this experiment was repeated with the use of a cylindrical glass vessel, the bottom of which was connected to a U-tube. The liquid and the bubble were represented by water and a rubber balloon filled with air, respectively. The pressure increase was measured by the change of water level in the U-tube. Steinberg et al. (1984, 1989) elaborated this simple physical demonstration into a series of experiments dealing with the simulation of volcanic pressure growth. Their experiments aimed to understand the time dependence of pressure increase where not only a single bubble but rather a jet of bubbles ascended from the

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bottom to the top of a closed vessel.

Putting aside the history of this simple laboratory experiment based on the Boyle–Marriotte law, I would rather like to concentrate on the crudeness of this physical analogy and the inappropriatence of its application to any real magmatic system. The analogy is based on the postulates of an infinite bulk modulus of the liquid and a strictly fixed volume of the vessel. Under these conditions, as concluded by Sahagian and Proussevitch (1992), the increase of the overpressure does not depend on the amount or on the compressibility of gas which rises to the top (!). Thus, it does not even matter what material floats to the top, liquid, solid or gas. Steinberg et al. (1984, 1989) at least demonstrated that the pressure increase depends significantly on the amount of gas which has already floated to the top of the vessel, or, in other words, is a function of *the effective compressibility* in the system *gas+magma*. According to Sahagian's and Proussevitch's logic, the resulting overpressure in a magma chamber would not depend on the rate of vesiculation or on the intensity of gas transfer. The conclusion of Sahagian and Proussevitch implies that even if we displace a tiny air bubble (with a negligible volume) from the bottom to the top of a magma chamber with a height of 1 km the pressure increase will be ≈800 bar* (!) (Sahagian and Proussevitch, 1993). This nonphysical result points to some very restrictive conditions that were imposed in the laboratory experiment, and which are unrealistic in a large-scale geological environment. The aim of this paper is to understand this physical paradox in order to restrict future attempts at its application to geological or volcanological processes.

2. Main conception

Consider a general case in which there are two substances with different densities and compressibilities in a vertical column (Fig. 1). As a result of gravity they are compressed: let M_1 and M_2 be their masses, K_1, K_2 their bulk moduli and $\rho_1(P_o)$, $\rho_2(P_o)$ their densities at some standard pressure (for example, atmospheric pressure). X_1S, X_2S are the volumes occupied by the two substances before displacement (S is the cross-sectional area of the column). Due to the difference in buoyancy $[\rho_2(X)<\rho_1(X)]$ material 2 floats and material 1 sinks. The first assumption is the equation of state for the two materials. For simplicity, the case of constant compressibility, independent of pressure (P) is considered. Thus, for both materials we can write:

$$\mathrm{d}\rho/\rho=\mathrm{d}P/K \tag{1}$$

More detailed analyses may be done for other equations of state involving pressure dependent compressibility. For example, when the floating substance is a gas, it is obvious that its compressibility should depend on pressure. For an ideal gas the compressibility is equal to the pressure and the equation of state is:

$$\rho=\rho_o P/P_o \tag{1'}$$

The second assumption is that of hydrostatic equilibrium of the materials at the beginning (Fig. 1a) and end of the displacement process (Fig. 1b and c), i.e.:

$$\mathrm{d}P=\rho(x)\,g\,\mathrm{d}x \tag{2}$$

where the acceleration due to gravity is assumed to be constant.

There are two possibilities for the vertical displacement of the two materials. The first is the case where the total volume is fixed and as a result of floating of the light material and sinking of the heavy one we get a change of pressure ΔP which has to be added to the hydrostatic pressure $P(x)$ in the whole volume. The second possibility is a change of volume ΔV due to the redistribution of differently compressible materials in the vertical column. In the first case, the height (volume) of the column is constant, i.e.:

$$X_1+X_2=X_1'+X_2' \tag{3}$$

and we estimate the increase of pressure from Eq. (3). In the second case, the expression:

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*I suspect that in this estimation Sahagian and Proussevitch assumed a very high magma density. If magma density is 2500 kg/m^3 the pressure increase will be $\rho g\Delta H\sim 245$ bar.

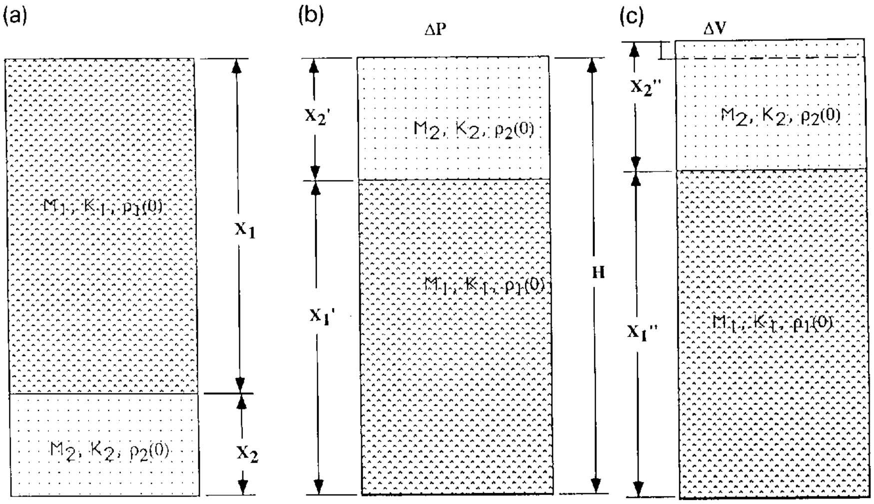

Fig. 1. Schematic representation of the vertical displacement of two substances in a vertical column. (a) The initial stage. (b) The final stage when the volume of the system is fixed. (c) The final stage when the system is free to expand.

$$\frac{\Delta V}{S}=(X'_1+X'_2)-(X_1+X_2) \tag{4}$$

gives the volume change. These two assumptions result in the following dependence of density on depth (X) and pressure (P):

$$\frac{1}{\rho(X_o)}-\frac{1}{\rho(X)}=\frac{g(X-X_o)}{K} \tag{5}$$

$$\rho(X)=\rho(X_o)\exp\left(\frac{P-P_o}{K}\right) \tag{6}$$

where $P_o=\rho(X_o)gX_o$.

Using the equation of state of the ideal gas the dependence of pressure on depth is:

$$\ln\left[\frac{P(X_2)}{P(X_1)}\right]=\frac{\rho_o}{P_o}g(X_2-X_1) \tag{6'}$$

3. Results

Pressure at any level in a vertical hydrostatic column can be obtained from the pressure of the overlying mass plus some constant pressure which corresponds to the top of the column. For example, the pressure at depth X_1 (Fig. 1a) is equal to $M_1g/S+P_o$, the pressure at the level $X'_1+X'_2$ (Fig. 1b) is equal to $(M_1+M_2)\,g/S+\Delta P+P_o$, etc.

By combining Eqs. (5) and (6) the heights (X_1, X_2, X'_1, X'_2, ..., etc. in Fig. 1) can be expressed in terms of the material parameters of the two substances. The analytical expressions for these variables are listed in Table 1. If we have a fixed volume (Fig. 1b) we get the following expression for the excess pressure using Eq. (3):

$$\frac{K_1\rho_2[1-\exp(-M_1g/K_1S)]}{K_2\rho_1[1-\exp(-M_2g/K_2S)]}\left[1-\exp\left(-\frac{\Delta P}{K_1}-\frac{M_2g}{K_1S}\right)\right]=\exp\left(-\frac{\Delta P}{K_2}\right)-\exp\left(-\frac{M_1g}{K_2S}\right) \tag{7}$$

In the case of an open vessel (Fig. 1c) we use Eq. (4), and the excess of volume due to the displacement of masses with different compressibilities can be derived from the following expression:

$$\Delta V=\frac{S}{g}\left\{\frac{K_2}{\rho_2}\left[1-\exp\left(-\frac{M_1 g}{K_2 S}\right)\right]\left[1-\exp\left(-\frac{M_2 g}{K_2 S}\right)\right]-\frac{K_1}{\rho_1}\left[1-\exp\left(-\frac{M_1 g}{K_1 S}\right)\right]\left[1-\exp\left(-\frac{M_2 g}{K_1 S}\right)\right]\right\} \quad (8)$$

The sign of ΔV may be either positive or negative depending on the relative K/ρ ratios of the two materials and their masses. If we assume that $\rho 2 \ll \rho 1$, and $M_2 g/S \ll K_2$, $M_1 g/S \ll K_1$, which means that the sinking material has much higher density than the rising one, and the materials are not significantly compressed by their own weights, we can approximate the exponents on the left side of Eq. (7) by Taylor series and come to the conclusion that the right side of Eq. (7) is approximately zero. The rising material in experiments is supposed to be a gas. An ideal gas has its bulk modulus equal to the pressure. This means that in the conditions of a laboratory experiment ($M_1 g/S \sim \Delta P < P_o \sim K_2$) each exponent on the right side of Eq. (7) is $< e^{-1}$, and so we can also expand them as Taylor series. Thus, the first approximation for the pressure change is:

$$\Delta P \approx \frac{M_1 g}{S}\,\frac{1-(K_2\rho_2/K_1\rho_1)}{1+(K_2\rho_2 M_1/K_1\rho_1 M_2)} \quad (9)$$

The term $M_1 g/S$ is equal to $\rho g H$. Thus, the expression (9) for the pressure increase coincides with the result demonstrated in laboratory experiments ($K_1 \rightarrow \infty$, $M_2 \rightarrow 0$, $K_2 \sim P_o > M_1 g/S \approx \Delta P$). According to Eq. (9) the pressure increase obtained vanishes when the mass of gas (M_2) is 0. The greater the contrast in the compressibilities of the sinking and floating substances, the greater is the pressure increase. Under the same assumptions the expression for the volume increase in an open vessel is found from Eq. (8) to be:

$$\Delta V \approx \frac{M_1 g}{S}\,\frac{M_2}{K_2\rho_2}\left(1-\frac{K_2\rho_2}{K_1\rho_1}\right) \quad (10)$$

Depending on the relative compressibilities of the sinking and floating materials the volume change can be either positive, negative or zero (when $K_2\rho_2 = K_1\rho_1$).

Table 1
Analytical expressions for heights before and after the vertical displacement of compressible materials (see Fig. 1)

$$X_1=\frac{K_1}{g\rho_1(P_o)}\left[1-\exp\left(-\frac{M_1 g}{K_1 S}\right)\right]$$

$$X_2=\frac{K_2}{g\rho_2(P_o)}\exp\left(-\frac{M_1 g}{K_2 S}\right)\left[1-\exp\left(-\frac{M_2 g}{K_2 S}\right)\right]$$

$$X'_2=\frac{K_2}{g\rho_2(P_o)}\exp\left(-\frac{\Delta P}{K_2}\right)\left[1-\exp\left(-\frac{M_2 g}{K_2 S}\right)\right]$$

$$X'_1=\frac{K_1}{g\rho_1(P_o)}\exp\left(-\frac{\Delta P}{K_1}-\frac{M_2 g}{K_1 S}\right)\left[1-\exp\left(-\frac{M_1 g}{K_1 S}\right)\right]$$

$$X''_1=X'_1(\Delta P=0)$$

$$X''_2=X'_2(\Delta P=0)$$

4. Conclusions

4.1. Effect of compressibility

I obtained expressions (9) and (10) from (7) and (8) using the assumptions of a laboratory experiment which may not be fulfilled in large-scale geological environments. First, the bulk modulus of magma at depth (at pressure) may be comparable to or even higher than the bulk modulus of the surrounding rocks. Thus, in reality the assumption of the fixed volume cannot be strictly fulfilled. In this case, the effect of pressure increase may be significantly smaller than that predicted by Eq. (9). At a depth of several kilometers the effect of gas expansion is much smaller than near the surface (K_2 increases), and, thus, the result of the laboratory experiment (Sahagian and Proussevitch, 1992) cannot be extrapolated to these conditions.

In a magma possessing a yield strength (σ_y), small bubbles with sizes less than a critical size ($R_{cr} \sim 3\sigma_y/4\Delta\rho g$) can avoid floating upwards through the melt because the buoyancy forces cannot overcome the yield stress (Sparks et al., 1978). The presence of these small bubbles can significantly decrease the effective bulk modulus of magma. In this situation big floating bubbles ($R > R_{cr}$) cannot significantly increase the pressure of the system because the bulk modulus of the high temperature magmatic fluids may be high enough to be comparable with the bulk modulus of the vesicular melt (melt + bubbles).

If the amount of rising gas is small in comparison with the volume of the magma chamber the size of the pressure or volume increase vanishes ($M_2 \rightarrow 0$ in Eqs. 9 and 10) whereas Sahagian and Proussevitch (1993) concluded that the same increase occurs whatever the volume of the gas.

4.2. Effect of gas solubility

In the first section of the discussion I specifically avoided the problem of solubility or mixing between substances during the process of the vertical displacement. This important aspect of bubble rise may drastically alter the final pressure or volume changes. In case of $Re \ll 1$ (slow motion) the flow around a rising bubble is viscous and the character of gas dissolution (or accumulation) depends on the Peclet number Pe (Levich, 1962):

$$Pe = \frac{2\Delta\rho g R^3}{9\eta D} \qquad (11)$$

where R is the bubble radius, $\Delta\rho g$ is the gas buoyancy, η is the melt viscosity and D is the gas diffusivity. If $Pe \ll 1$ the gas transport from the bubble is diffusional and the total flux is:

$$I = 4\pi DR\Delta C \qquad (12)$$

where ΔC is the difference in concentrations of gas on the surface of a bubble (C_s, the equilibrium solubility) and far away from it (C_∞, the actual gas content in the melt). If $Pe \gg 1$ the advective mechanism of gas transport dominates and the total gas flux from the bubble is higher:

$$I = 4\pi DR\Delta C(1 + 0.64 Pe^{1/3}) \qquad (13)$$

At $Re > 1$ the total flux is only advective (Levich, 1962):

$$I = 4\pi DR\Delta C Pe \qquad (14)$$

The sign of $\Delta C = C_\infty - C_s$ determines the dissolution of the bubble into the melt ($C_s > C_\infty$ in an undersaturated melt) or the accumulation of gas from the melt into the bubble ($C_s < C_\infty$ in an supersaturated melt). In a silicate melt the equilibrium gas concentration is related to the ambient pressure through Henry's law:

$$C_s \sim P^{(1-m)/n} \qquad (15)$$

for example, in the case of water solubility $m = -1/2$, $n = 2$ (Toramaru, 1989). In an open chamber a bubble ascending in a supersaturated melt will always accumulate volatiles from the melt ($Cs < C_\infty$) because the equilibrium solubility (C_s) decreases with distance from bottom to top according to Eq. (15). In a closed chamber the ambient pressure will increase due to the bubble ascent to the top (C_s increases) and at some moment the bubble will begin to dissolve. This can happen when the initial conditions on the bubble surface (C_s) and the equilibrium solubility at the bottom of the chamber (H) satisfy the condition:

$$C_\infty - C_s = C_o < K(\rho g H)^{(1-m)/n} \qquad (16)$$

where K is a constant. According to Eq. (16) and the water solubility data in felsic magmas (Burnham and Jahns, 1962), for small degrees of initial supersaturation ($H_2O \sim 0.5$ wt.%) the pressure increase due to the bubble ascent may dampen the effect of the 'advective overpressure' in a magma chamber with $H \approx 600$ m.

The complete problem of the shrinkage (or growth) of a rising bubble containing a multicomponent gas should include the effects of diffusing and nondiffusing species, surface tension, variable viscosity, etc., and is beyond the scope of this paper (but see the Appendix).

4.3. Effect of bubble nucleation

In the preceding discussion of the bubble ascent process I tacitly assumed some mechanism of bubble nucleation near the bottom of magma chamber. It may occur when hot basic magma contacts cool rhyolite melt containing volatiles (Sparks et al., 1977). The traditional mechanism of magma boiling is the decompression of saturated magma due to pressure release in a volcanic edifice and the subsequent exsolution of gas throughout the whole volume of the magma body (Wilson and Head, 1981; Fink and Kieffer, 1993). In this case the bubbles nucleated on the top will create a compressible layer of "foam" which will also dampen "an advective overpressure" in the magma chamber.

Thus, the conditions of the laboratory demonstration of Sahagian and Proussevitch (1993) are so restricted that the result cannot be directly applied to any geological process. Nevertheless, the problem of pressure or volume changes due to the migration of substances with different compressibilities has considerable importance for the Earth sciences. For example, during the process of mantle–core differentiation the iron blobs sinking from the silicate mantle into the core can result in volume changes of the Earth (a planetary contraction or expansion model?). Such problems should be solved precisely in every particular case with the use of an appropriate equation of state and realistic boundary conditions. They cannot be simply reduced to the laboratory demonstrations of water or mercury incompressibility. Thus, the discussion of the bubble rise effect in magma chamber by Sahagian and Proussevitch (1992) leads the reader into the false impression that a new mechanism of volcanic overpressure has been discovered.

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Appendix

If r is the reduced radius of a rising bubble (defined as R/R_o, the present radius R divided by initial radius R_o) the rate of radius change at $Pe \gg 1$ is described by:

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{r^3(1 - C_\infty/C_s) + (2\sigma r^2/R_o - P_n)/P}{r^{3/2}[r + 2\sigma/(3R_oP)]} \tag{1A}$$

where σ is the surface tension, P is the ambient pressure, and P_n is the initial partial pressure of a nondiffusing gas (Weinberg et al., 1990). The equation (1A) is based on a Henry's Law, i.e. the concentration of gas on the bubble interface is proportional to its pressure in the bubble. According to (1A) the presence of a nondiffusing gas component results in faster bubble growth. Even if $C_\infty/C_s = 1$, the rate of bubble growth $\mathrm{d}r/\mathrm{d}t$ is >0 when $P_n > 2\sigma/R_o$.

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